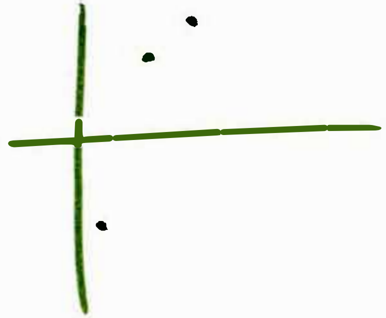
t	1	2	3
y	−6	5	8



Cálculo de un polinomio interpolador por el método de Lagrange

$$p(t) = -6 \cdot L_1(t) + 5 \cdot L_2(t) + 8 \cdot L_3(t)$$

$$L_1(t) = \frac{(t-2)(t-3)}{(1-2)(1-3)} = \frac{t^2}{2} - \frac{5t}{2} + 3$$

$$L_2(t) = \frac{(t-1)(t-3)}{(2-1)(2-3)} = -t^2 + 4t - 3$$

$$L_3(t) = \frac{(t-1)(t-2)}{(3-1)(3-2)} = \frac{t^2}{2} - \frac{3t}{2} + 1$$

$$p(t) = -6\left(\frac{t^2}{2} - \frac{5t}{2} + 3\right) + 5\left(-t^2 + 4t - 3\right) + 8\left(\frac{t^2}{2} - \frac{3t}{2} + 1\right)$$

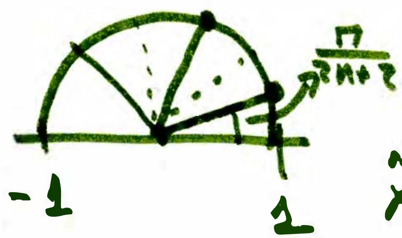
$$= -4t^2 + 23t - 25 \qquad p(0) = -25 \text{ m}$$

Cálculo de los nodos de Chebyshev en un intervalo $[a,b]$

Queremos 5 nodos en $[-2,3]$

$n=4$

$$\left.\begin{array}{l}
\hat{x}_0 = \cos\left(\frac{\pi}{10}\right) \\
\hat{x}_1 = \cos\left(\frac{3\pi}{10}\right) \\
\hat{x}_2 = \cos\left(\frac{5\pi}{10}\right) \\
\hat{x}_3 = \cos\left(\frac{7\pi}{10}\right) \\
\hat{x}_4 = \cos\left(\frac{9\pi}{10}\right)
\end{array}\right\} \text{ en } [-1,1]$$



$n+1$ nodos

$$\hat{x}_k = \cos\left(\frac{2k+1}{2n+2}\pi\right)$$

$k=0\ldots n$

$$x_k = \hat{x}_k \cdot \underbrace{\frac{b-a}{2}}_{2.5} + \underbrace{\frac{b+a}{2}}_{0.5} \longrightarrow$$

$$x_0 = 0.5 + 2.5\cos\frac{\pi}{10} = 2.877614$$

$$x_1 = 0.5 + 2.5\cos\frac{3\pi}{10} = 1.96946$$

$$x_2 = 0.5 + 2.5\cos\frac{5\pi}{10} = 0.5$$

$$x_3 = 0.5 + 2.5\cos\frac{7\pi}{10} = -0.96946$$

$$x_4 = 0.5 + 2.5\cos\frac{9\pi}{10} = -1.87764$$

~~Aproximación~~ de una función por el método de mínimos cuadrados

x	0	1	3	5	6
y	0	2.9	10.1	17.8	21.5

$y \approx ax + b\sqrt{x}$

$$\begin{pmatrix} 0 & 1 & 3 & 5 & 6 \\ 0 & 1 & \sqrt{3} & \sqrt{5} & \sqrt{6} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 3 & \sqrt{3} \\ 5 & \sqrt{5} \\ 6 & \sqrt{6} \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 & 1 & 3 & 5 & 6 \\ 0 & 1 & \sqrt{3} & \sqrt{5} & \sqrt{6} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2.9 \\ 10.1 \\ 17.8 \\ 21.5 \end{pmatrix}$$

$$\begin{pmatrix} 71 & 32.073 \\ 32.073 & 15 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 251.20 \\ 112.86 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{36.295} \cdot \begin{pmatrix} 15 & -32.073 \\ -32.073 & 71 \end{pmatrix} \cdot \begin{pmatrix} 251.20 \\ 112.86 \end{pmatrix} = \begin{pmatrix} 4.0832 \\ -1.2069 \end{pmatrix}$$

$$y \approx 4.0832x - 1.2069\sqrt{x}$$

Aproximación de la 1ª y 2ª derivada de una función a partir de una tabla de valores

x	0	0.1	0.2	0.3	0.4
y	2	2.11	2.24	2.4	2.58

$$y'(0.1) = \frac{y(0.2) - y(0)}{0.2} = \frac{2.24 - 2}{0.2} = 1.2$$

$$y'(0.3) \simeq \frac{y(0.4) - y(0.2)}{0.2} = \frac{2.58 - 2.24}{0.2} = 1.7$$

$$y'(0) \simeq \frac{4\cdot y(0.1) - 3\cdot y(0) - y(0.2)}{0.2} = \frac{4\cdot 2.11 - 3\cdot 2 - 2.24}{0.2} = 1$$

$$y'(0.4) = \frac{-4\cdot y(0.3) + 3\cdot y(0.4) + y(0.2)}{0.2} = \frac{-4\cdot 2.4 + 3\cdot 2.58 - 2.24}{0.2} = 1.9$$

$$y'(0.2) \simeq \frac{y(0) - 8y(0.1) + 8y(0.3) - y(0.4)}{1.2} = \frac{2 - 8\cdot 2.11 + 8\cdot 2.4 - 2.58}{1.2} = 1.45$$

$$y''(0.1) \simeq \frac{y(0.2) + y(0) - 2\cdot y(0.1)}{0.01} = \frac{2.24 + 2 - 2\cdot 2.11}{0.01} = 2$$

$$y''(0) \simeq 2$$

Cálculo aproximado de una integral definida mediante las reglas de punto medio/trapecio/Simpson

$$\int_{-\pi/3}^{\pi/4} \cos(x^2)\,dx \overset{P.M.}{\approx} \left(\frac{\pi}{4} - \left(-\frac{\pi}{3}\right)\right)\cdot \cos\left(\left(\frac{\frac{\pi}{4}-\frac{\pi}{3}}{2}\right)^2\right) = 1.8323$$

$$\int_{-\pi/3}^{\pi} \cos(x^2)\,dx \approx \frac{\ }{4} - -\frac{\ }{3} \cdot \frac{\cos\left(\left(-\frac{\pi}{3}\right)^2\right) + \cos\left(\left(\frac{\pi}{4}\right)^2\right)}{2} = 1.16$$

$$\int_{-\pi/3}^{\pi/4} \cos(x^2)\,dx \approx \frac{\pi - -\frac{\pi}{3})}{\ } \cdot \left[1\cdot\cos\left(\left(-\frac{\pi}{3}\right)^2\right) + 4\cdot\cos\left(\left(\frac{\frac{\pi}{4}-\frac{\pi}{3}}{2}\right)^2\right) + 1\cdot\cos\left(\left(\frac{\pi}{4}\right)^2\right)\right] = 1.6102$$

Valor exacto: 1.684

Cálculo aproximado de una integral mediante la regla de Simpson compuesta

$$\int_0^1 e^{\sqrt{x}}\,dx = \underbrace{\int_0^{1/3} e^{\sqrt{x}}\,dx}_{I_1} + \underbrace{\int_{1/3}^{2/3} e^{\sqrt{x}}\,dx}_{I_2} + \underbrace{\int_{2/3}^{1} e^{\sqrt{x}}\,dx}_{I_3} \approx 0.48878 + 0.67535 + 0.83037$$

$$= 1.9945$$

$$I_1 \approx \frac{1}{3\cdot 6}\cdot\left(e^0 + 4e^{\sqrt{1/6}} + e^{\sqrt{1/3}}\right) = 0.48878$$

$$I_2 \approx \frac{1}{3\cdot 6}\cdot\left(e^{\sqrt{1/3}} + 4e^{\sqrt{1/2}} + e^{\sqrt{2/3}}\right) = 0.67535$$

$$I_3 \approx \frac{1}{3\cdot 6}\cdot\left(e^{\sqrt{2/3}} + 4e^{\sqrt{5/6}} + e^1\right) = 0.83037$$

$$\int_0^1 e^{\sqrt{x}}\,dx \approx \frac{1}{3\cdot 6}\cdot\left[e^0 + 4\cdot e^{\sqrt{1/6}} + 2e^{\sqrt{1/3}} + 4e^{\sqrt{1/2}} + 2e^{\sqrt{2/3}} + 4e^{\sqrt{5/6}} + e\right] = 1.9945$$

Respuesta exacta: $\int_0^1 e^{\sqrt{x}}\,dx = 2$

Cálculo de una integral por el método de Gauss-Legendre

$$\int_1^2 \cos(\ln(x))\,dx \underset{\substack{G-L \\ 3\text{ nodos}}}{\approx} 0.277778\cdot\left(\cos(\ln(1.1127)) + \cos(\ln(1.8873))\right) + 0.444444\cos(\ln(1.5)) = 0.90821$$

$$\int_{-1}^{1} f(x)\,dx \approx \sum_{i=1}^{3} w_i\cdot f(x_i)$$

$n=3 \rightarrow$

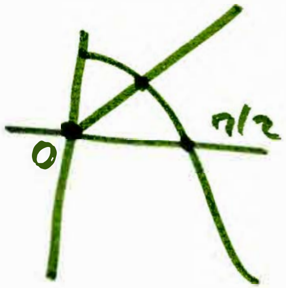
x_i	± 0.774597	0
w_i	0.555556	0.888889

Nodos para $[1,2]$: $\rightarrow \pm 0.774597\cdot\dfrac{(2-1)}{2} + \dfrac{2+1}{2} = \begin{cases} 1.1127 \\ 1.8873 \end{cases}$

$$\rightarrow 0\cdot\frac{2-1}{2} + \frac{2+1}{2} = \frac{3}{2} = 1.5$$

Pesos correspondientes $\begin{cases} 0.555556\cdot\dfrac{1}{2} = 0.277778 \\ 0.888889\cdot\dfrac{1}{2} = 0.444444 \end{cases}$

Resolución de una ecuación por el método de bisección



$$x = \cos x$$

$$f(x) = \cos x - x \quad ¿x : f(x) = 0?$$

$f(x) > 0$	$f(x) < 0$
0	$\pi/2$
0	$\pi/4$
$\pi/8$	$\pi/4$
$\frac{3\pi}{16}$	$\pi/4$
$\frac{7\pi}{32}$	$\pi/4$
$\frac{15\pi}{64}$	$\pi/4$
$\frac{15\pi}{64}$	$\frac{31\pi}{128}$

$$f\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} - \frac{\pi}{4} < 0$$

$$f\left(\frac{\pi}{8}\right) = \cos\frac{\pi}{8} - \frac{\pi}{8} > 0$$

$$f\left(\underbrace{\frac{\frac{\pi}{4} + \frac{\pi}{8}}{2}}_{3\pi/16}\right) = f\left(\frac{3\pi}{16}\right) > 0$$

$$f\left(\frac{\frac{3\pi}{16} + \frac{\pi}{4}}{2}\right) = f\left(\frac{7\pi}{32}\right) > 0$$

$$f\left(\frac{\frac{7\pi}{32} + \frac{\pi}{4}}{2}\right) = f\left(\frac{15\pi}{64}\right) > 0$$

$$f\left(\frac{\frac{15\pi}{64} + \frac{\pi}{4}}{2}\right) = f\left(\frac{31\pi}{128}\right) < 0$$

Respuesta:

$$x = \frac{\frac{15\pi}{64} + \frac{31\pi}{128}}{2} = \frac{61\pi}{256} \simeq 0.74858 \;//\; f(x) = -0.0159$$

Resolución de una EDO por el método de Euler y el método de Euler modificado.

$$\begin{cases} \dot{x} = f(t,x) = t^2 + x^2 \\ x(0) = 0 \end{cases}$$

Euler: sea $h = 0.1$.

$x(0) = w_0 = 0$

$x(h) \approx w_1$

$x(2h) \approx w_2$

$\ldots$

$$\left\{ w_{n+1} = w_n + h \cdot f(nh, w_n) \right.$$
$$= w_n + 0.1 \cdot ((n \cdot 0.1)^2 + w_n^2)$$

$w_0 = 0$

$w_1 = 0$

$w_2 = 0.001$

$w_3 = 0.005$

$w_4 = 0.014003$

$x(0.4) \approx 0.014003$

Euler mod: sea $h = 0.1$

w_0 ✓

$$w_{n+1} = w_n + h \cdot f\left(\left(n+\tfrac{1}{2}\right) \cdot h,\ w_n + \tfrac{h}{2} \cdot f(nh, w_n)\right)$$
$$= w_n + h \cdot f\left(\left(n+\tfrac{1}{2}\right)h,\ w_n + \tfrac{h}{2}\left((nh)^2 + w_n^2\right)\right)$$
$$= w_n + h\left[\left(n+\tfrac{1}{2}\right)^2 h^2 + \left(w_n + \tfrac{h}{2}\left((nh)^2 + w_n^2\right)\right)^2\right]$$

$w_0 = 0$

$w_1 = 0.00025$

$w_2 = 0.0025$

$w_3 = 0.008752$

$w_4 = 0.02102$

$x(0.4) \approx 0.02102$

Casi exacto: 0.021965

Transformación de un sistema de EDOs de 2º orden en uno de 1er orden

$$r = \begin{pmatrix} r_x \\ r_y \end{pmatrix} \qquad \ddot{r} = \dot{v} = \begin{pmatrix} 0 \\ -g \end{pmatrix} - \underbrace{\alpha(r_y)}_{\alpha_0 - K \cdot r_y} \cdot \frac{v}{\|v\|} \cdot \|v\|^2$$

$$\begin{cases} \ddot{r} = \begin{pmatrix} 0 \\ -g \end{pmatrix} - (\alpha_0 - K \cdot r_y) \cdot \|v\| \cdot v \\ r(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \dot{r}(0) = \begin{pmatrix} 600 \\ 100 \end{pmatrix} \end{cases}$$

Sea $W = \begin{pmatrix} r \\ v \end{pmatrix} = \begin{pmatrix} r \\ \dot{r} \end{pmatrix} = \begin{pmatrix} r_x \\ r_y \\ v_x \\ v_y \end{pmatrix}$.

$$\begin{cases} \dot{W} = \begin{pmatrix} \dot{r} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} v \\ \begin{pmatrix} 0 \\ -g \end{pmatrix} - (\alpha_0 - K r_y) \cdot \|v\| \cdot v \end{pmatrix} = \begin{pmatrix} W_3 \\ W_4 \\ \begin{pmatrix} 0 \\ -g \end{pmatrix} - (\alpha_0 - K \cdot W_2) \cdot \sqrt{W_3^2 + W_4^2} \cdot \begin{pmatrix} W_3 \\ W_4 \end{pmatrix} \end{pmatrix} \\ W_0 = \begin{pmatrix} 0 \\ 0 \\ 600 \\ 100 \end{pmatrix} \end{cases}$$