Tabla de integrales inmediatas

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$
2. $\int u^{-1} du = \int \frac{du}{u} = \log|u| + C$
3. $\int a^u du = \frac{a^u}{\log a} + C \quad (a \neq 0); \quad \text{si } a = e \Rightarrow \int e^u du = e^u + C$
4. $\int \operatorname{sen} u du = -\cos u + C$
5. $\int \cos u du = \operatorname{sen} u + C$
6. $\int \tan u du = -\log|\cos u| + C \quad (a \neq 0)$
7. $\int \cot u du = \log|\operatorname{sen} u| + C$
8. $\int \frac{du}{\cos^2 u} = \tan u + C$
9. $\int \frac{du}{\operatorname{sen}^2 u} = -\cot u + C$
10. $\int \operatorname{senh} u du = \cosh u + C$
11. $\int \cosh u du = \sinh u + C$
12. $\int \tanh u du = \log(\cosh u) + C$
13. $\int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{arc\,sen} \frac{u}{|a|} + C = -\arccos \frac{u}{|a|} + C \quad (a \neq 0)$
14. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$
15. $\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{|a|} + C \quad (a \neq 0)$
16. $\int \frac{du}{\sqrt{a^2+u^2}} = \operatorname{argsinh} \frac{u}{|a|} + C = \log|u + \sqrt{u^2+a^2}| + C \quad (a \neq 0)$
17. $\int \frac{du}{\sqrt{u^2-a^2}} = \operatorname{argcosh} \frac{u}{|a|} + C = \log|u + \sqrt{u^2-a^2}| + C \quad (a \neq 0)$
18. $\int \frac{du}{a^2-u^2} = \frac{1}{a} \operatorname{argtanh} \frac{u}{|a|} + C = \frac{1}{2a} \log\left|\frac{a+u}{a-u}\right| + C \quad (a \neq 0)$

Áreas, longitudes y volúmenes

- $A = \int_a^b f(x)dx$ • $\int_{t_1}^{t_2} y(t)x'(t)dt$ • $\frac{1}{2}\int_{\theta_1}^{\theta_2} \rho^2(\theta)d\theta$
- $L = \int_a^b \sqrt{1+[f'(x)]^2}dx$ • $\int_{t_0}^{t_1} \sqrt{[x'(t)]^2+[y'(t)]^2}dt$ • $\int_{\theta_1}^{\theta_2} \sqrt{[\rho(\theta)]^2+[\rho'(\theta)]^2}d\theta$
- $V_{OX} = \pi \int_a^b y^2 dx$ • $\pi \int_{t_0}^{t_1} [y(t)]^2 x'(t)dt \parallel V_{OY} = \pi \int_a^b x^2 dy$ • $\pi \int_{t_0}^{t_1} [x(t)]^2 y'(t)dt$
- $V = \frac{2\pi}{3} \int_{\theta_1}^{\theta_2} \rho^3(\theta) \operatorname{sen}\theta d\theta$

Integración de funciones comunes

Racionales trigonométricas: $R(\operatorname{sen} x, \cos x)$

- Cambio "universal":
  $\tan \frac{x}{2} = t \Rightarrow \operatorname{sen} x = \frac{2t}{1+t^2}$ • $\cos x = \frac{1-t^2}{1+t^2}$ • $dx = \frac{2dt}{1+t^2}$
- f par en seno y coseno:
  $\tan x = t \Rightarrow \operatorname{sen} x = \frac{t}{\sqrt{1+t^2}}$ • $\cos x = \frac{1}{\sqrt{1+t^2}}$ • $dx = \frac{dt}{1+t^2}$
- f impar en seno $\to \cos x = t$ • f impar en coseno: $\to \operatorname{sen} x = t$

Irracionales: $R(x, \sqrt[n/2]{ax^2+2bx+c}), \quad n \in \mathbb{Z}$

$$ax^2 + 2bx + c = \left(\sqrt{a}x + \frac{b}{\sqrt{a}}\right)^2 + c - \frac{b^2}{a} = (px+q)^2 \pm m^2$$

- $(px+q)^2 + m^2 \to px + q = m\tan t \to \sqrt{ax^2+2bx+c} = \frac{m}{\cos t}$
- $(px+q)^2 - m^2 \to px + q = \frac{m}{\cos t} \to \sqrt{ax^2+2bx+c} = m\tan t$
- $m^2 - (px+q)^2 \to px + q = m\operatorname{sen} t \to \sqrt{ax^2+2bx+c} = m\cos t$

Método alemán: $\int \frac{P_n(x)}{\sqrt{ax^2+bx+c}} = Q_{n-1}(x)\sqrt{ax^2+bx+c} + K\int \frac{dx}{\sqrt{ax^2+bx+c}}$

Cuadrado perfecto: $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

Algunas identidades trigonométricas e hiperbólicas

$\operatorname{sen} \frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ • $\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$ • $\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha$

$$\left.\begin{array}{l}\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta \\ \operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cos\beta - \cos\alpha\operatorname{sen}\beta\end{array}\right\} \to \left\{\begin{array}{l}\operatorname{sen}\alpha\cos\beta = \frac{\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)}{2} \\ \cos\alpha\operatorname{sen}\beta = \frac{\operatorname{sen}(\alpha+\beta)-\operatorname{sen}(\alpha-\beta)}{2}\end{array}\right.$$

$$\left.\begin{array}{l}\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta \\ \cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta\end{array}\right\} \to \left\{\begin{array}{l}\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2} \\ \operatorname{sen}\alpha\operatorname{sen}\beta = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}\end{array}\right.$$

$\cosh x = \frac{e^x+e^{-x}}{2}$ • $\operatorname{senh} x = \frac{e^x-e^{-x}}{2}$
$\cosh x + \operatorname{senh} x = e^x$ • $\cosh x - \operatorname{senh} x = e^{-x}$
$\cosh^2 x - \operatorname{senh}^2 x = 1$ • $\operatorname{senh} 2x = 2\operatorname{senh} x\cosh x$ • $\operatorname{senh}^2 x = \frac{\cosh 2x + 1}{2}$
$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$ • $\tanh^2 x = \frac{\cosh 2x - 1}{\cosh 2x + 1}$