



A. BASIC CONCEPTS



FATIGUE

FATIGUE DEFINITION

- **Engineering** : type of failure in materials that implies initiation and propagation of cracks in components subjected to cyclic loading that, generally, do not exceed the yield stress of the material.
- **Science** : behaviour of a material subjected to cyclic loads that implies plastic deformations, crack nucleation and propagation and failure.



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FATIGUE IMPORTANCE

- **Basic idea:** Monotonous loads do not produce fatigue damage. Loads must be variable
- **Examples:** from 19th century (bridges in UK) to now (ships, planes,..) many registered accidents.
- **Design:** Fatigue design of structures and components supported by procedures, Eurocode, ASME, API,..



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FATIGUE ASSESSMENT

Focusing the problem

- Fatigue life assessment can be performed in two ways:
 - I. Estimation of the total life of the component, including incubation period.
 - II. Life determination through the propagation, supposing the presence of existing conditions (cracks and a stress intensity factor amplitude or variation) over the threshold ones.



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FATIGUE ASSESSMENT

Focusing the problem

I. Estimation of Total Life is the classical way (Wöhler, Basquin, Goodman).

➤ *Based on experimental and statistical studies, life can be determined from the knowledge of the applied stresses or the existent strains. The design parameter is the endurance*

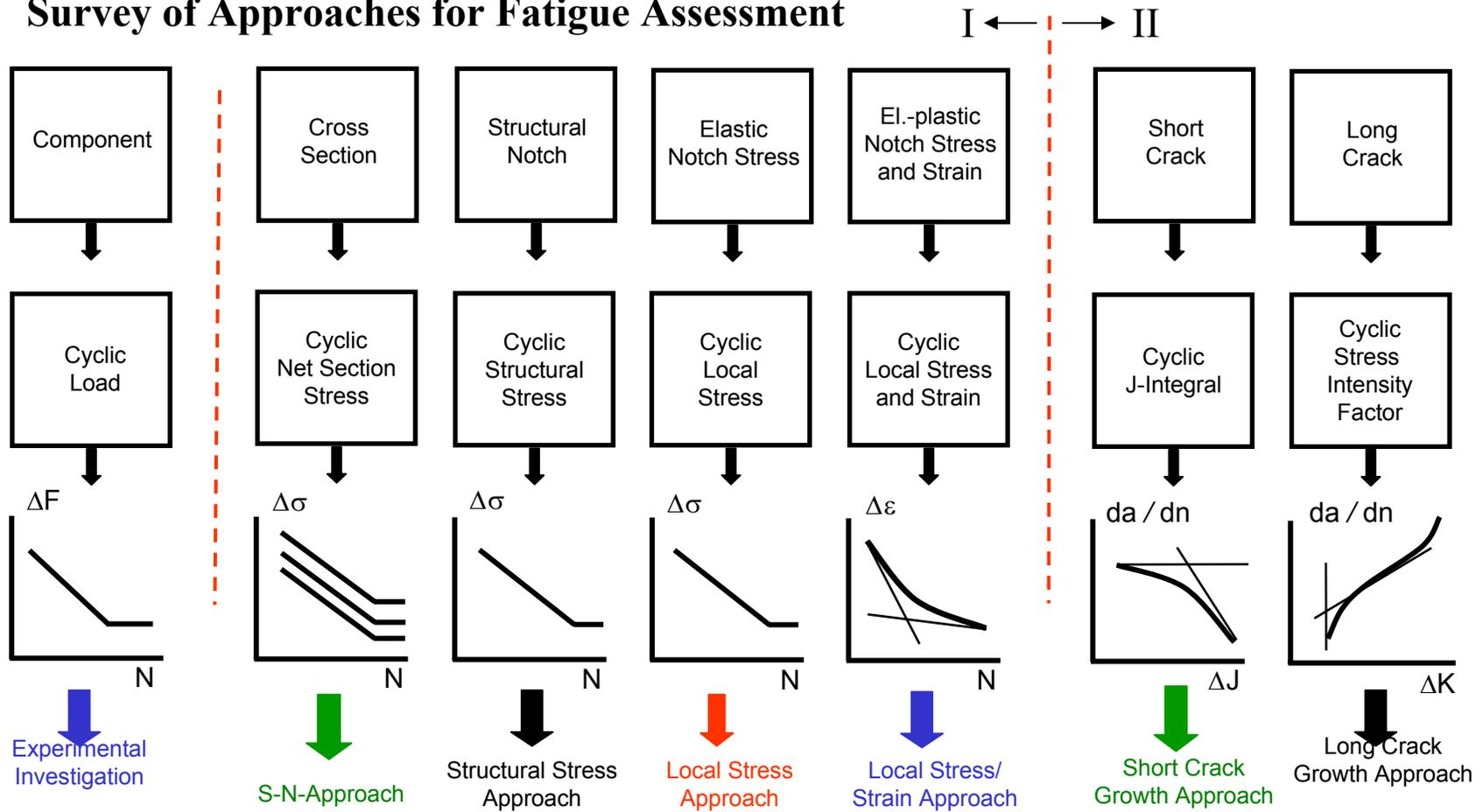
➤ *This approach distinguishes LCF (Low Cycling Fatigue) from HCF (High Cycling Fatigue). Also processes with no constant stresses can be assessed (Miner).*

II. Life determination based on crack propagation rate appears after the FM Paris works



FATIGUE FATIGUE ASSESSMENT

Survey of Approaches for Fatigue Assessment



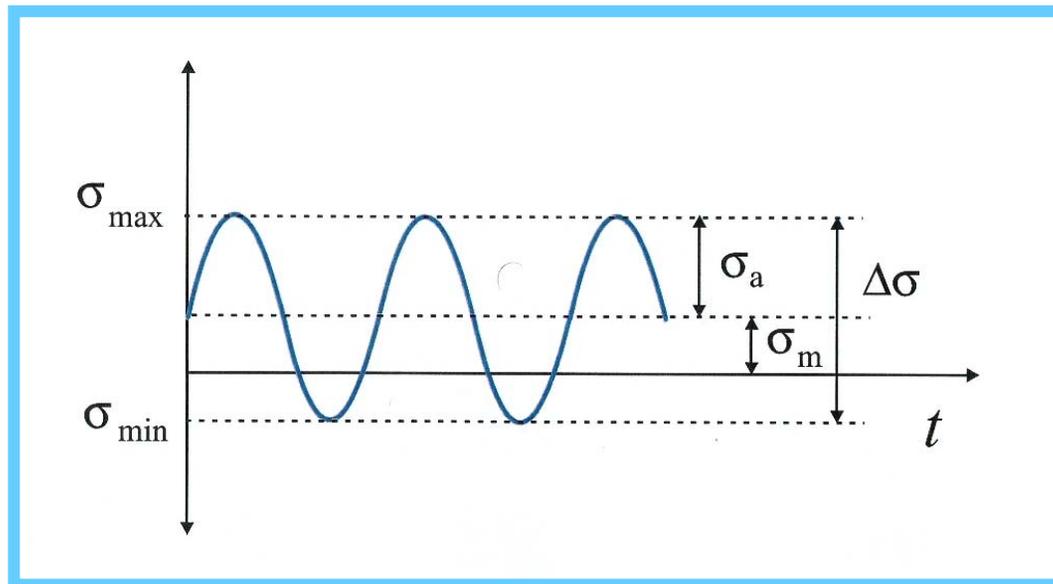


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CYCLIC LOADS

Definition and variables

- Evolution of the stresses during a constant cyclic loading process





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CYCLIC LOADS

Definition and variables

- Parameters characterising the fatigue process:

- **Stress amplitude:** $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$
- **Mean stress:** $\sigma_m = \frac{1}{2} \{ \sigma_{\max} + \sigma_{\min} \}$
- **Stress Ratio:** $R = \frac{\sigma_{\min}}{\sigma_{\max}}$
- **Frecuency:** Measured in Hz (s^{-1})

- Generally, it only influences crack growth when it is accompanied by combined environmental effects (humidity, high temperatures, aggressive environments,...)

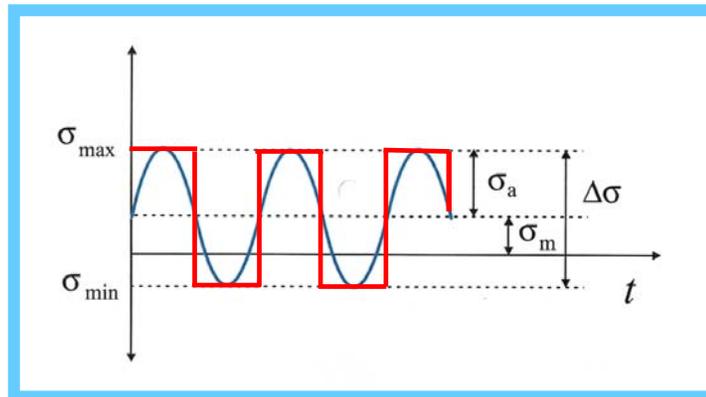


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CYCLIC LOADS

Definition and variables

- **Shape of the stress function:** Is it adjustable to a sine function, square, ...
 - its influence on the crack growth is small, except when there is some environmental effect.





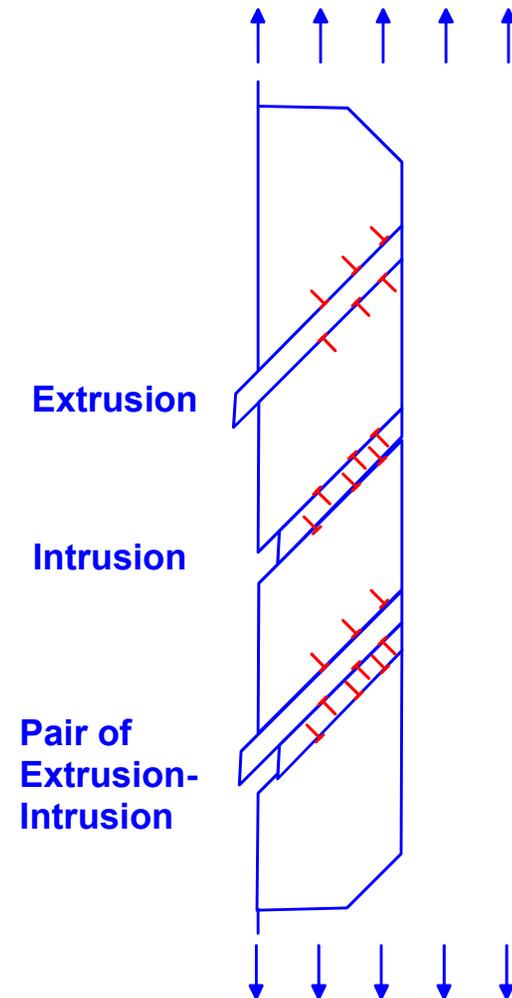
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REASONS

Cracks form due to cyclic plastic deformation.

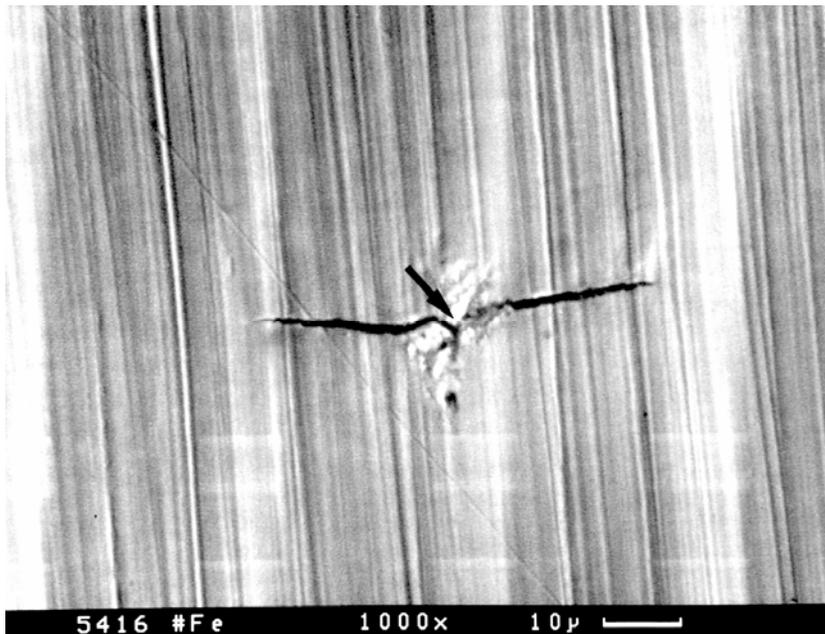
In defect free material cracks form at slip bands, at intrusions and extrusions.

Plastic deformation starts in grains where slip planes are favorably oriented in the direction of alternating shear stresses.

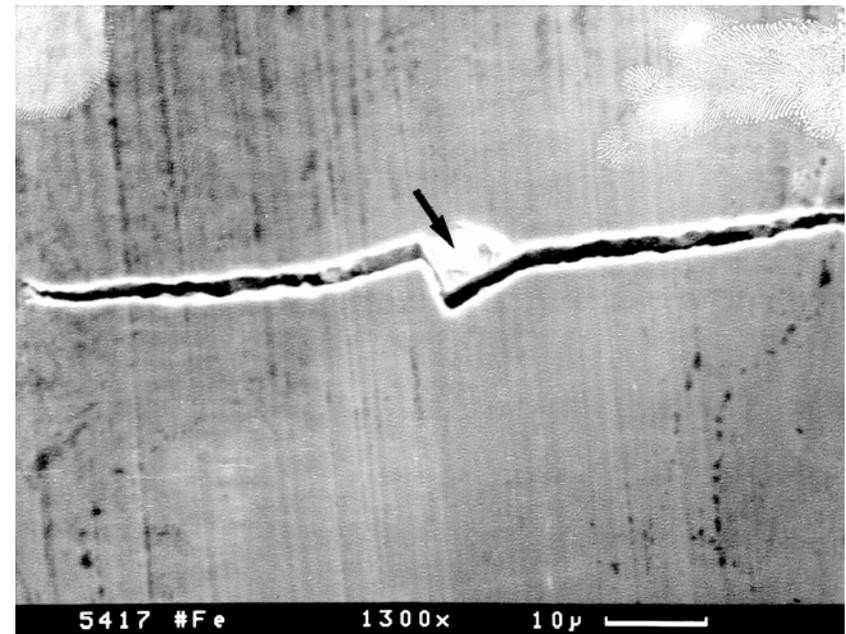


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The effect is enforced by stress raisers
(inclusions of Zirconium oxide in S690Q)



Broken Inclusion

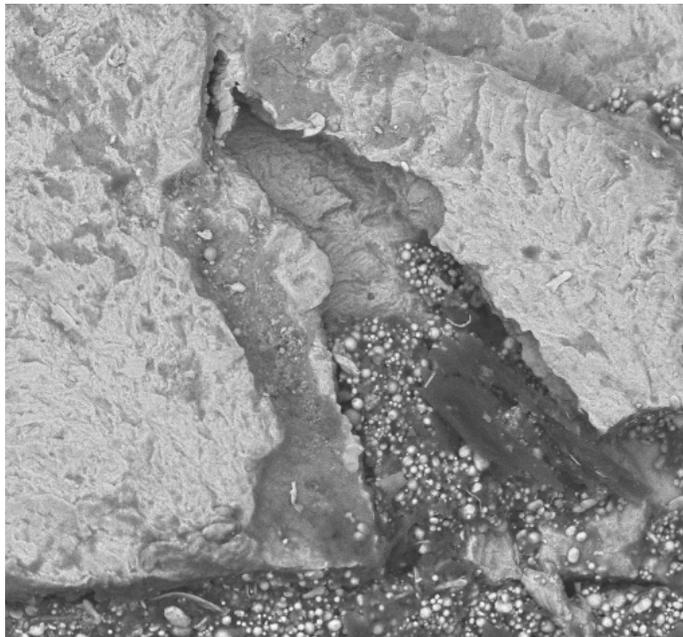


Broken Interface

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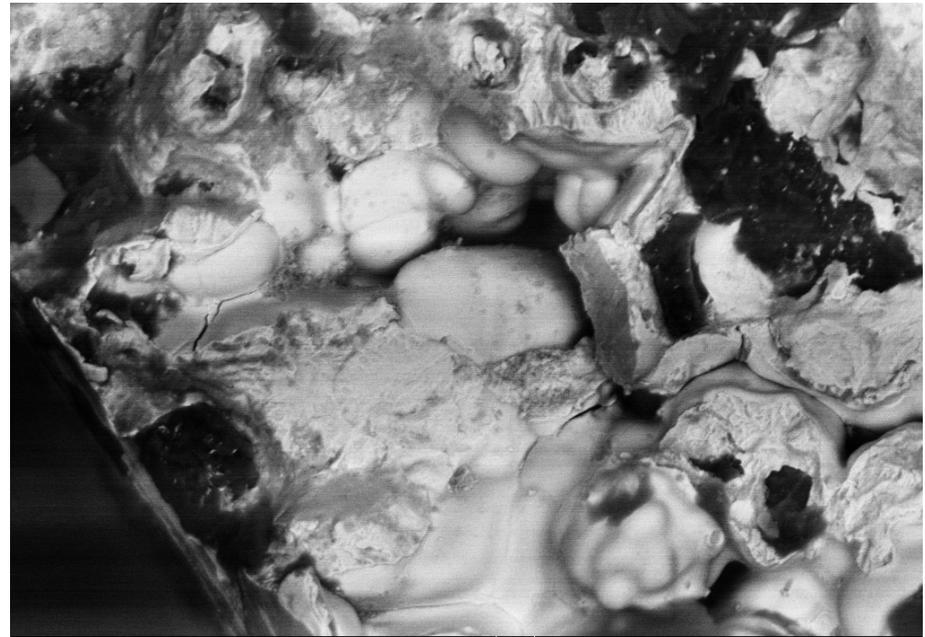
The effect is enforced by stress raisers

(Microscopical notches or pores)



— 10 μ m

Pore in a spring steel



— 10 μ m

Pore in nodular graphite iron

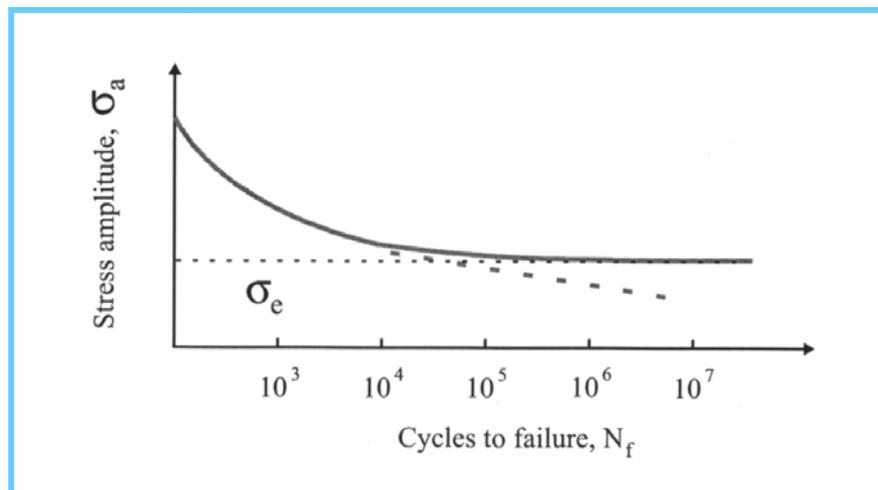


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TOTAL LIFE ESTIMATION

Based on S-N Curves

- Stress amplitude σ_a vs Number of cycles before failure (N_f)



If $\sigma_a < \sigma_e$ (fatigue limit or endurance), life is considered infinite

- σ_e approx. 0.35- 0.50 σ_u in steels and bronzes.
- Infinite life $N_f = 10^7$ cycles



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TOTAL LIFE EVALUATION

Stress approach I

Basquin 1910 ($\sigma_m = 0$; $\sigma_{\max} = -\sigma_{\min}$; $R = -1$)

$$\frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f (2N_f)^{-b}$$

- Logarithmic relation between σ_a and $2N_f$
- σ'_f is, approximately, the tensile strength (σ_n)
- b varies between 0.05 y 0.12 σ_u in steels and bronzes



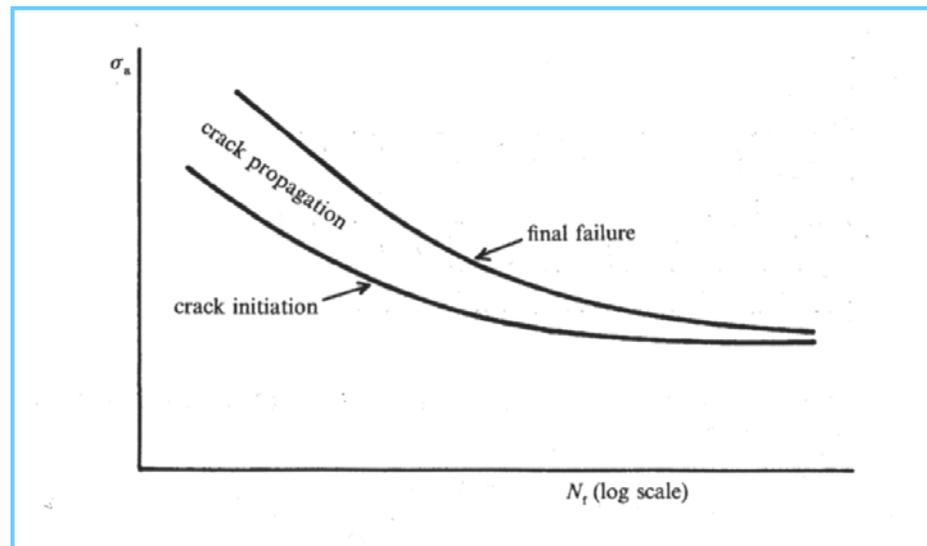
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TOTAL LIFE EVALUATION

Stress approach II

The whole life of a component has two periods:

- Crack Initiation period
- Crack Propagation period





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TOTAL LIFE EVALUATION

Stress approach III ($\sigma_m \neq 0$)

On previous considerations $\sigma_m = 0$. :

How can we design when σ_m is not equal to 0?

Corrections:

Soderberg

$$\sigma_a = \sigma_a |_{\sigma_m=0} \left\{ 1 - \frac{\sigma_m}{\sigma_y} \right\}$$

Goodman

$$\sigma_a = \sigma_a |_{\sigma_m=0} \left\{ 1 - \frac{\sigma_m}{\sigma_{TS}} \right\}$$

Gerber

$$\sigma_a = \sigma_a |_{\sigma_m=0} \left\{ 1 - \left(\frac{\sigma_m}{\sigma_{TS}} \right)^2 \right\}$$



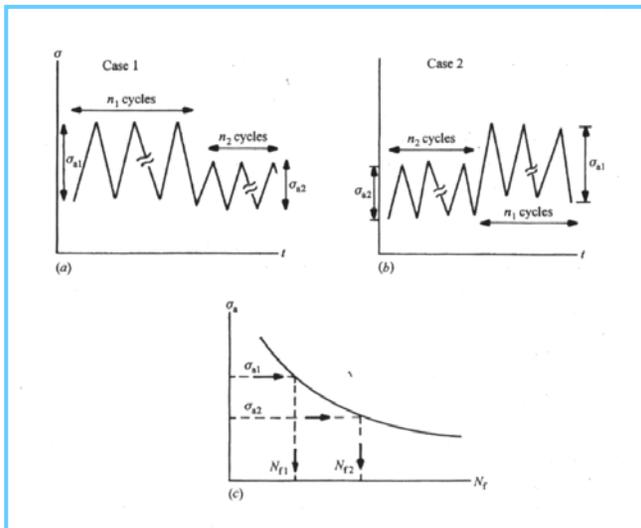
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TOTAL LIFE EVALUATION

Stress approach IV \longrightarrow Amplitude

On previous considerations σ_a is constant

If σ_a is not constant, define the damage due to each cyclic block.



$$d_i = \frac{n_i}{N_{fi}} \quad \text{Damage}$$

$$\sum_i \frac{n_i}{N_{fi}} = 1 \quad \text{Accumulated damage at life time (Miner's rule)}$$



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TOTAL LIFE EVALUATION

Strain approach I

The previous stress approach is useful with conditions which imply elastic strains (high N_f). This focus is known as High Cycling Fatigue (HCF).

In practice, there are some conditions in which fatigue is associated with high strains (high temperatures, stress concentration). Therefore, the number of cycles before failure is low.

This new focus, based on strains, is known as Low Cycling Fatigue (LCF)



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APPROXIMATION TO TOTAL LIFE

Strain approach II

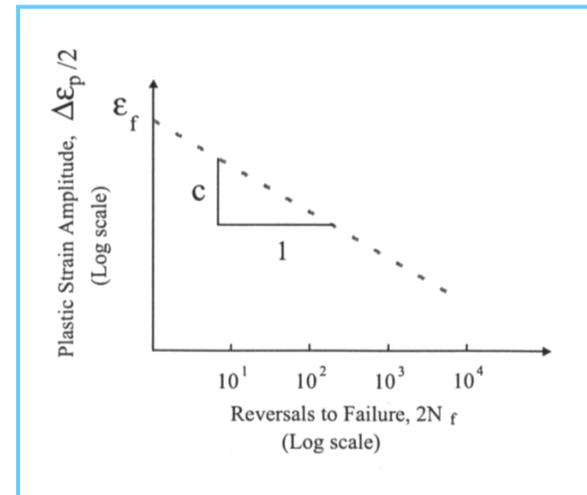
Coffin-Manson 1955

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f' (2N_f)^c$$

$\Delta\varepsilon_p/2$: Strain amplitude

ε_f' : tensile strain factor (aprox. ε_f)

c : fatigue coefficient (between 0.5 and 0.7)





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TOTAL LIFE EVALUATION

General approach: HCF/LCF

In a general case:

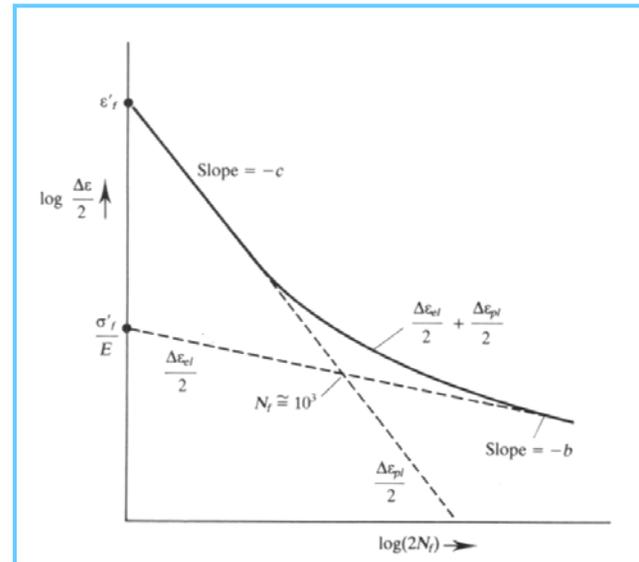
$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2}$$

HCF
$$\frac{\Delta \sigma}{2} = \sigma'_f (2N_f)^b$$

if
$$\frac{\Delta \varepsilon_e}{2} = \frac{\Delta \sigma}{2E} = \frac{\sigma_a}{E}$$

$$\frac{\Delta \varepsilon_e}{2} = \frac{\sigma'_f}{E} (2N_f)^b$$

HCF/LCF
$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$





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FATIGUE CRACK GROWTH

LEFM APPROACH

- In 1963 LEFM concepts were applied for first time to crack growth by Paris, Gómez and Anderson.
- For a given cyclic loading, ΔK is defined as $K_{\text{máx}} - K_{\text{mín}}$, which can be obtained from $\Delta\sigma$ and the geometry of the cracked element, including crack extension.
- Paris, Gómez and Anderson established that crack propagation (Δa in N cycles) depends on ΔK :

$$\frac{\Delta a}{\Delta N} \rightarrow \frac{da}{dN} = C(\Delta K)^m \quad (\text{Paris Law})$$



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FATIGUE CRACK GROWTH

LEFM APPROACH

$$\frac{da}{dN} = C(\Delta K)^m$$

- Thus, the representation (da/dN) vs. Log (ΔK) must be a straight line with a slope equal to m.
- The relation between crack growth rate and ΔK defines three regions for the fatigue behaviour:
 - A: Slow growth (near the threshold) → Region I or Regime A
 - B: Growth at a medium rate (Paris regime) → Region II or Regime B
 - C: Growth at a high rate (near to fracture) → Region III or Regime C



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FATIGUE CRACK GROWTH

Three states

State I (Regime A)

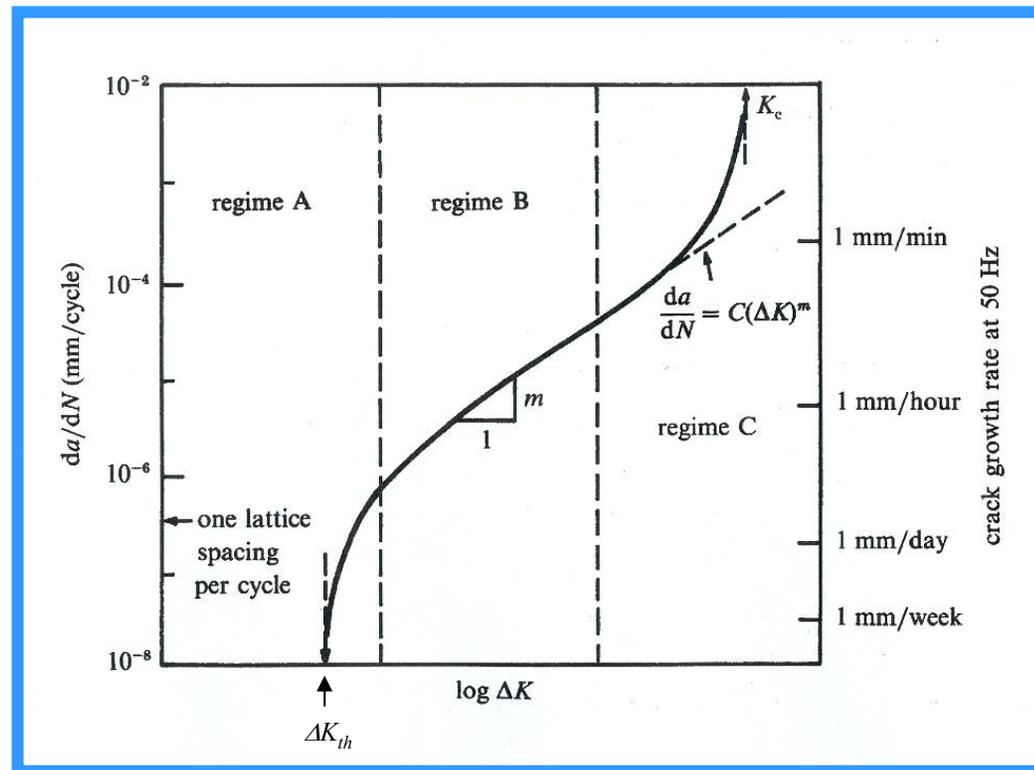
$$\Delta K_{th}$$

State II (Regime B)

$$\frac{da}{dN_{II}} = C(\Delta K)^m$$

State III (Regime C)

near failure, where K_c is achieved





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FATIGUE CHARACTERISATION

Obtaining the Paris law

Methodology: Based on the LEFM, the crack propagation rate is determined as a function of ΔK .

1. Selection of specimen (FM type as CT, SENB,...)
2. Loading application system (Constant amplitude.)
3. Follow Crack propagation as a function of time or N.
4. Obtain crack propagation rate in zone II (mean value).
5. Determine the threshold, ΔK_{th}
6. Represent da/dN - $\log \Delta K$ and adjust with Paris parameters

Standard: ASTM E-647



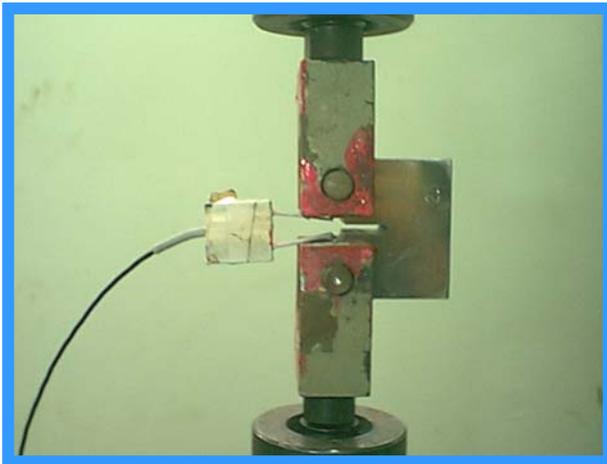
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FATIGUE CHARACTERISATION

Obtaining the Paris law

• *Example: Obtaining da/dN and Paris law*

1. Selection of the specimens in (FM type, such as CT, SENB,...)
2. Loading application system (Constant amplitude)



$$\Delta K = \frac{\Delta P}{B\sqrt{W}} f\left(\frac{a}{W}\right)$$



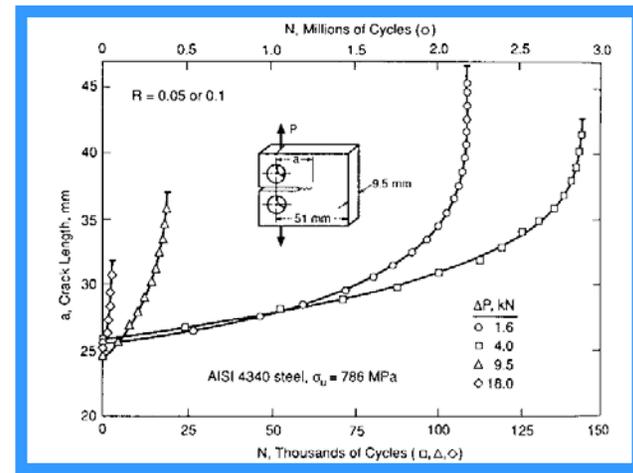
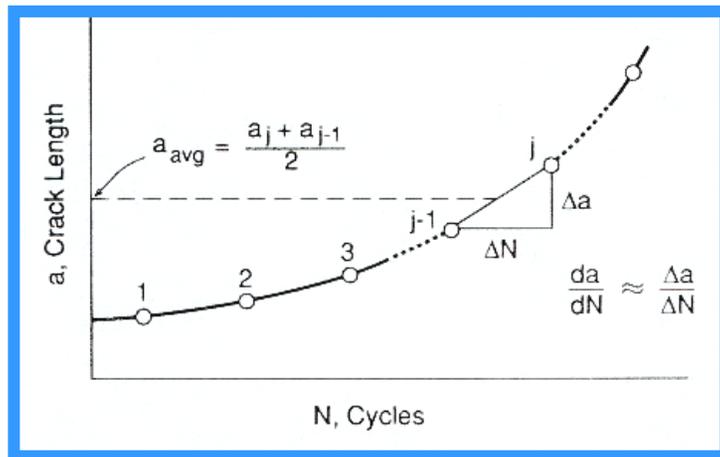
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FATIGUE CHARACTERISATION

Obtaining the Paris law

• *Example: Obtaining da/dN and Paris law*

3. Determining crack propagation as a function of time or N cycles: by optical microscope or any other method





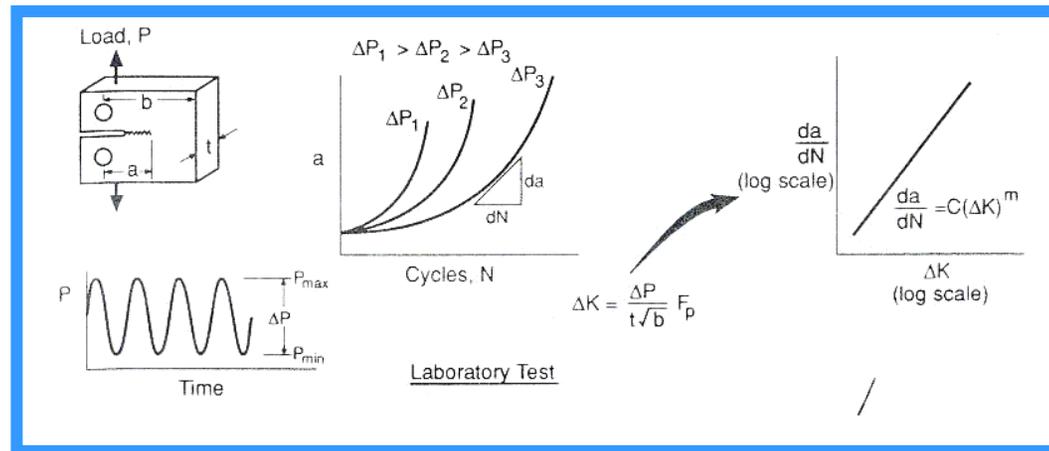
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FATIGUE CHARACTERISATION

Obtaining the Paris law

• *Example: Obtaining da/dN and Paris law*

4. Obtaining crack propagation rate law in zone II (Paris law).
5. Threshold determination, ΔK_{th} (i.e ASTM E647,...)



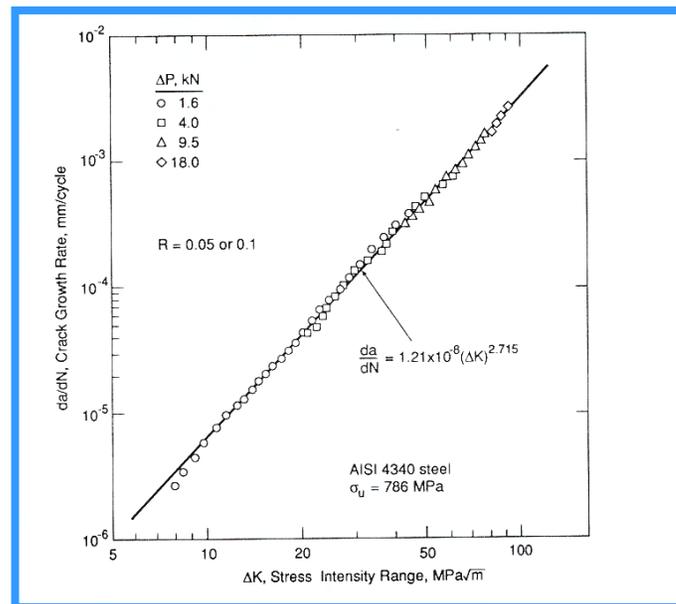


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FATIGUE CHARACTERISATION

Obtaining the Paris law

- Example: Determination of da/dN_{II} , m and C on AISI4130 steels





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FATIGUE CHARACTERISATION

Variables affecting $(da/dN)_{II}$:

- **Environmental effects**
 - Corrosion – fatigue
 - Temperature
- **Loading effects**
 - Stress ratio $R = \sigma_{\min}/\sigma_{\max}$
 - Variable amplitude. (Miner's rule).
 - Frequency
- **Limitations : LEFM**
 - Short cracks
 - Thickness
 - Plastic zone extension



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FATIGUE CRACK GROWTH

Three regimes

Regime	A Slow growth	B Paris zone	C Quick growth
Fracture Microscopy	Mode II (Shear) Brittle facets	Striations (mode I) Beach Marks	Cleavages, Microvoids (failure)
Influence of microstructure	High	Low	High
R effect	High	Low	High
Environment effect	High	*	Low
Plastic zone	$r_y < d_g$ (grain size)	$r_y > d_g$	$r_y \gg d_g$
*It depends on environment, frequency and material SCC,CF.			



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FATIGUE CRACK GROWTH

Regime A (I)

-Threshold concept, ΔK_{th} :

– When ΔK is equal or lower to ΔK_{th} , crack propagation rate is extremely slow and so, it is considered that crack doesn't propagate or that it propagates at non-detectable rates.

– **Practical definition:** When crack propagation rate is less than 10^{-8} mm/cycle, it is considered that propagation has stopped and ΔK is called ΔK_{th} .



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Regime A (II)

-This propagation rate is smaller than one interatomic distance per cycle.
How is it possible?

- It is considered that there is a large amount of cycles on which there is no propagation. Crack grows one interatomic space in a cycle and then it stabilises for some cycles.
- There are experimental difficulties to determine crack propagation rates at these values.



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Regime B (I)

- In regime B (Paris Zone) the number of cycles before failure can be calculated using the Paris law:

$$\frac{da}{dN} = C(\Delta K)^m$$

ΔK is defined as a function of $\Delta\sigma$

$$\Delta K = Y\Delta\sigma\sqrt{\pi a}$$

Y is a geometric factor

m and C are characteristic parameters of the material and they are obtained experimentally. For metallic materials, m varies between 2 and 4 and for ceramics and polymers it can reach values up to 100.



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Regime B (II)

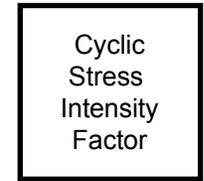
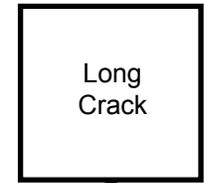
- Therefore, the Paris law can be written in this way:

$$\frac{da}{dN} = C \left(Y \Delta \sigma \sqrt{\pi a} \right)^m$$

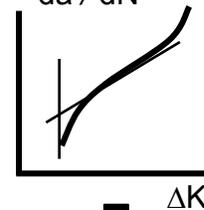
- If Y is constant, both sides of the expression can be integrated:

$$\int_{a_0}^{a_f} \frac{da}{a^{m/2}} = C Y^m (\Delta \sigma)^m \pi^{m/2} \int_0^{N_f} dN$$

If Y depends on crack length, it is necessary to solve the problem numerically.



da / dN



Long Crack Growth Approach



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Regime B (III)

If $m > 2$:

$$N_f = \frac{2}{(m-2)CY^m (\Delta\sigma)^m \pi^{m/2}} \left[\frac{1}{a_0^{(m-2)/2}} - \frac{1}{a_f^{(m-2)/2}} \right]$$

If $m = 2$:

$$N_f = \frac{1}{CY^2 (\Delta\sigma)^2 \pi} \operatorname{Ln} \frac{a_f}{a_0}$$



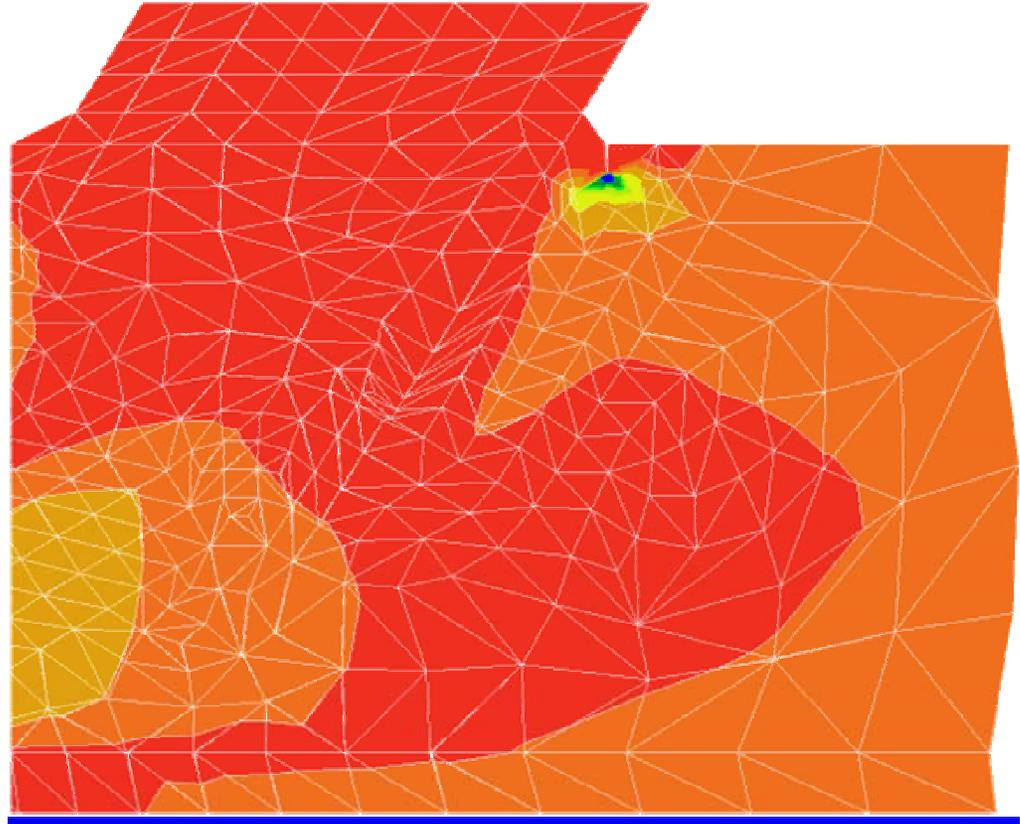
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Regime B (IV)

Determining Y:

- Search in handbooks (Tada, Rooke&Cartwright, Murakami)
- Perform (FE-) calculations





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Regime B (V)

-If $\Delta\sigma$ is not a constant value, the methods that are used to determine the number of cycles before failure are based on the application of [Miner Rule](#) (traditional method), considering the foreseen crack propagation rate law by Paris and following these steps :

- Reduce the load spectrum to blocks with constant amplitude (block_j)
- Estimate the foreseen N_f for each block (N_{fi})
- Apply Miner's rule
- Previous plastification history of the material must be taken into account



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Regime B (VI)

- In order to solve the problem of life estimation (N_f), it is necessary to obtain the initial crack length, a_0 , and the final crack length, a_f (usually called critical crack length).

How can we determine the initial crack length?

- There are various techniques, from visual inspection to ultrasonics or X rays. If no crack is detected with these methods, it is considered that crack length is equal to the resolution of inspection equipments.



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Regime B (VII)

How can we calculate the expected final crack length?

- Cracks grow until fracture occurs. Then, at failure:

$$K_{\max} = K_c$$

- In other terms:

From $Y\sigma_{\max}\sqrt{\pi a_f} = K_c$ we can estimate a_f in this way: $a_f = \frac{1}{\pi} \frac{K_c^2}{Y^2 \sigma_{\max}^2}$



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Regime B (VIII)

- Based on the previous analysis, a very important idea appears :
Even when cracks are detected in a component or structure, it is not necessary to replace it!
- We must assess the remaining life. The component can be used if it is periodically inspected.

Then assessment concepts as

- *Admissible crack - Admissible damage*
- *Inspection period - Life time*

should be considered



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FATIGUE CRACK GROWTH

Regime C

The failure of a structure or component after a fatigue process can be produced in two different ways:

- For high ΔK , crack propagation rate increases a lot until sudden fracture occurs when fracture toughness is reached

Ex: Brittle failure conditions at low temperatures

- Plastification and failure of the remaining section

Ex: Plastic collapse ductile conditions



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FRACTOGRAPHIC ASPECTS

Regime B

- When a crack propagates because of a fatigue process, it produces marks which are known as **striations** or **beach marks**. These marks are usually the main proof of a failure caused by fatigue.
- Striations are the marks that crack propagation produces on the failure surface in various cycles.





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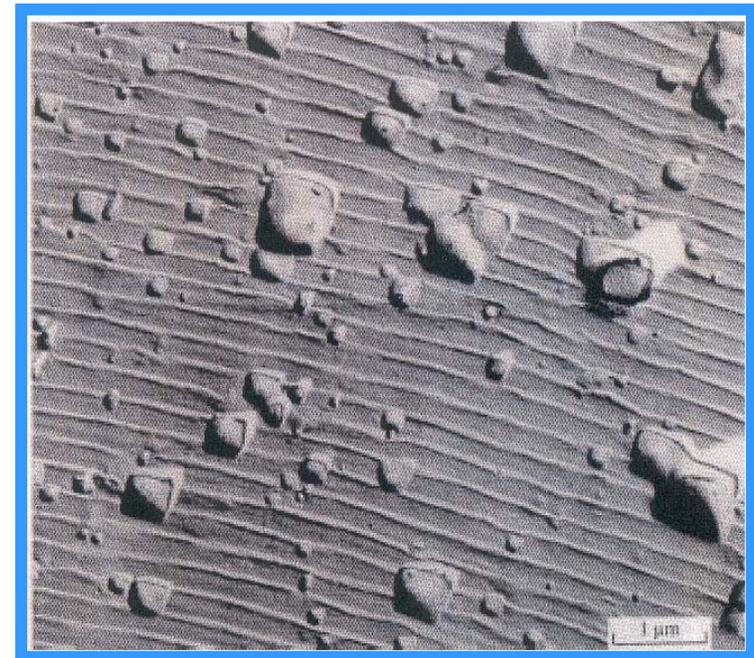
FRACTOGRAPHIC ASPECTS

Regime B

EXAMPLE:

Fatigue striations on the fracture surface of a 2024-T3Al alloy.

In some materials, each line is identified with the propagation Δa per cycle.





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FRAC TOGRAPHIC ASPECTS

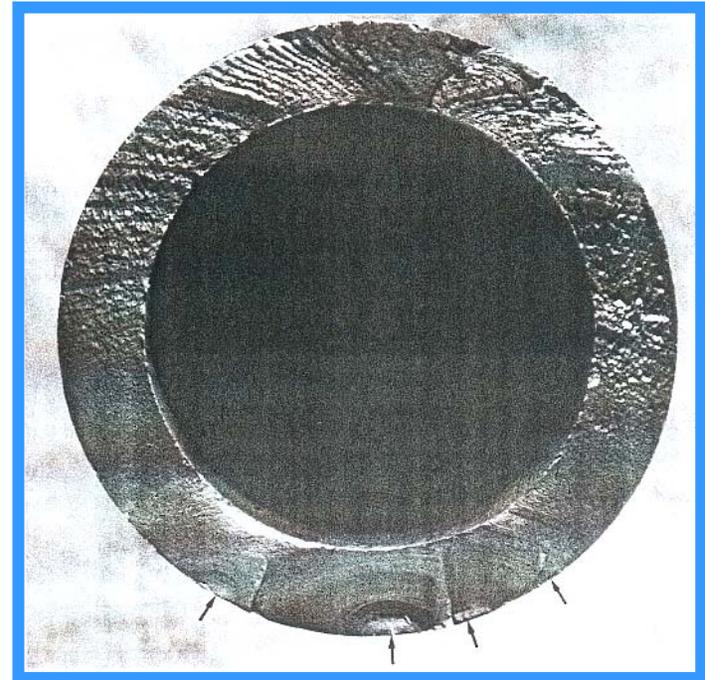
Regime C

Striations disappear in the final failure section and the following can appear:

1. Cleavage micromechanisms and tearing if fracture is brittle

or

2. Microvoids if fracture occurs because of the plastification process of the remaining section (ductile failure).



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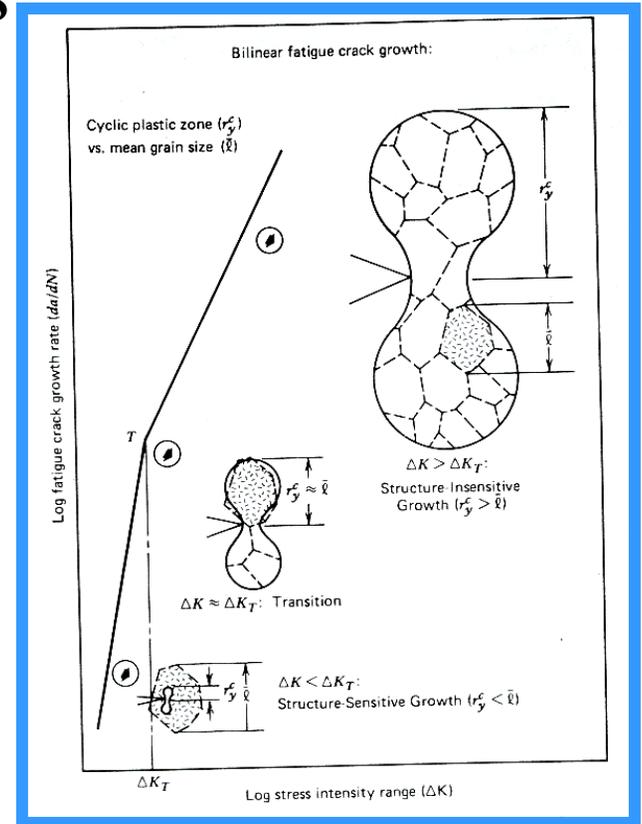
CRACK PROPAGATION MECHANISMS

Regimes A and B

Propagation models:

a) Plastic field extends inside a grain or occupies only a few grains ($r_y < d$). Propagation through sliding planes. (Regime A)

b) Plastic zone with a considerable size ($r_y > d$). Propagation occurs through a straight line (Regime B)



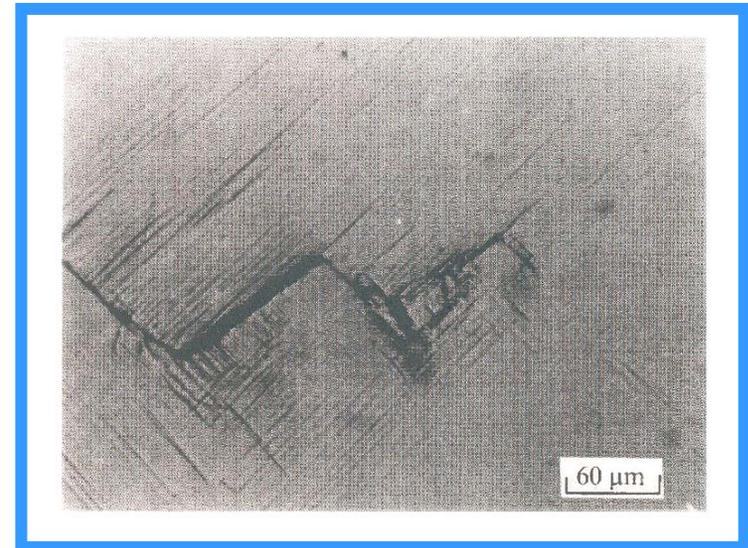
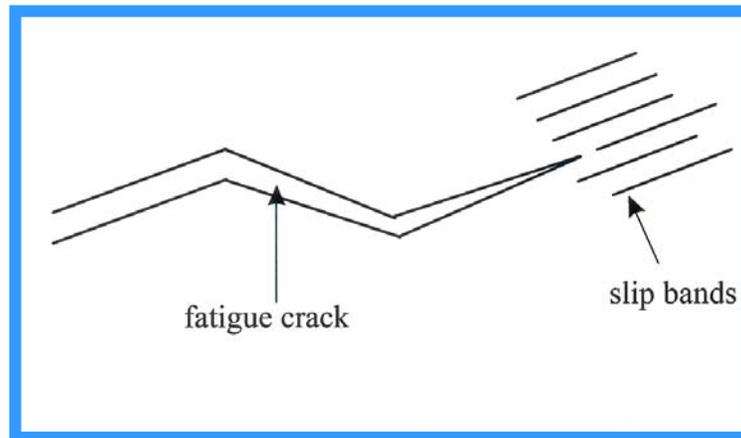
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CRACK PROPAGATION MECHANISMS

Regime A: Threshold zone: $r_y < d$.

Propagation modes:

Propagation through sliding planes. Fracture Mode II (Shear)





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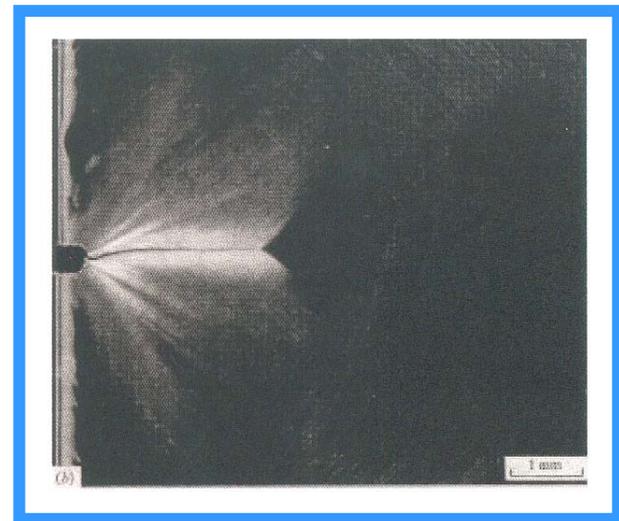
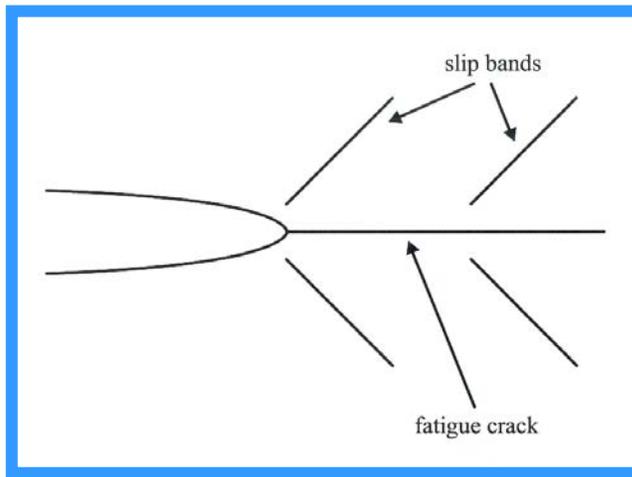
CRACK PROPAGATION MECHANISMS

Regime B: State II Paris Law: $r_y > d$.

Propagation modes:

There are many sliding planes implied, so crack propagates through the intersection between them. Fracture Mode I (tension).

Sometimes striations are observed.



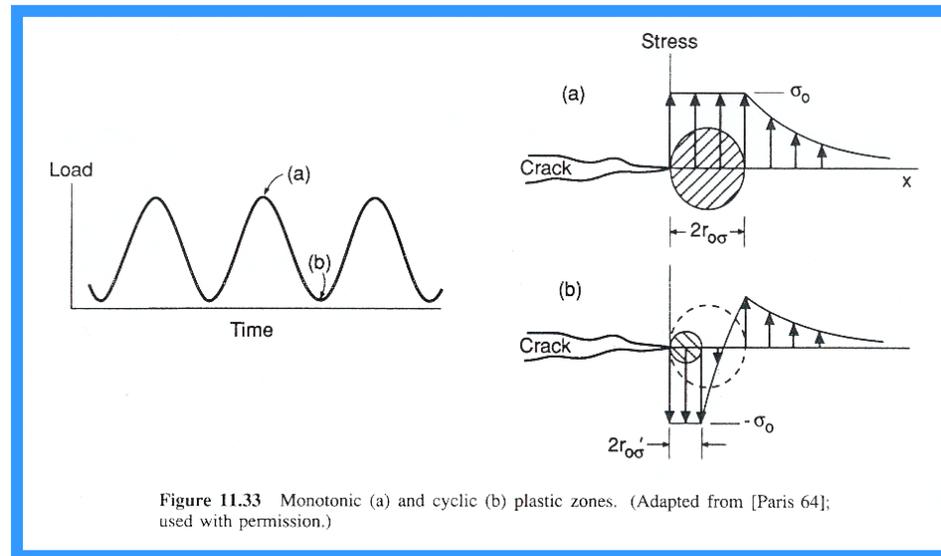
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CRACK PROPAGATION MECHANISMS

Regime B: State II Paris Law: $r_y > d$.

Physical models of crack propagation :

1 . Sliding irreversibility



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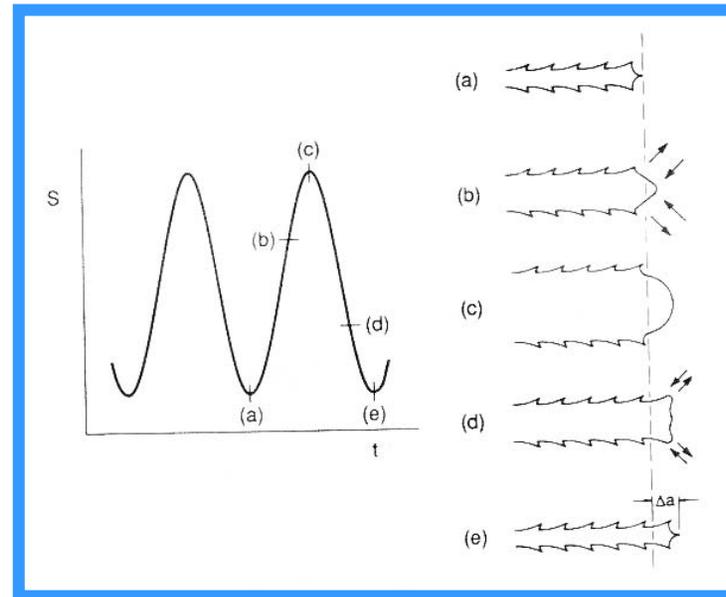
CRACK PROPAGATION MECHANISMS

Regime B: State II Paris Law: $r_y > d$.

Physical models of crack propagation at Paris zone:

1 . Sliding irreversibility

Laird Model
(1967)





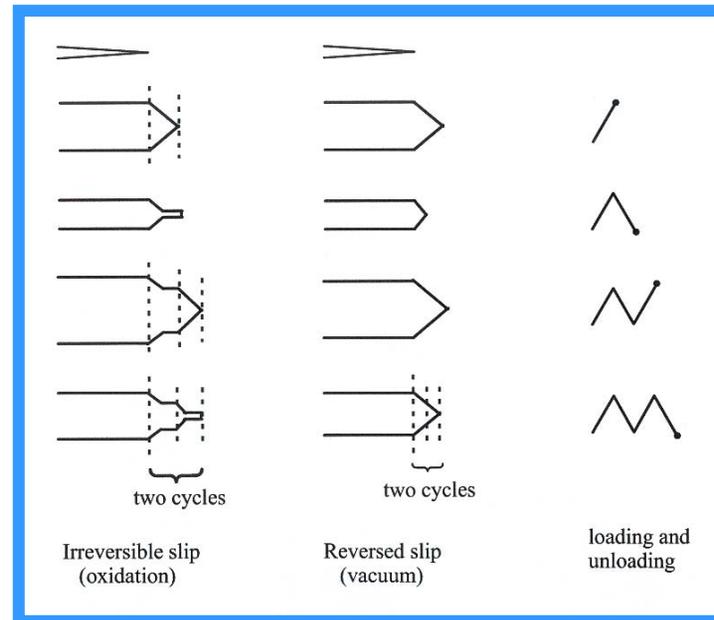
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CRACK PROPAGATION MECHANISMS

Regime B: State II Paris Law: $r_y > d$.

Physical models of crack propagation at Paris zone:

2. Environmental effects





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CRACK PROPAGATION MECHANISMS

Regime B. State II Paris Law

A model for the Paris law based on CTOD (δ_t)

$$da/dN = (\Delta a)_{1 \text{ cycle}} \approx \delta_t = \beta \frac{(\Delta K)^2}{\sigma'_y E'}$$

Important: This implies $m = 2$ in the Paris law

Advantages of models based on CTOD:

1. Physical justification
2. Application to multiaxial fatigue.



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FATIGUE DESIGN

Safe-life

- **Philosophy:** Elements without cracks
- **Steps:**
 - Load spectrum determination.
 - Life estimation for the material through laboratory tests (from an initial crack size).
 - Application of a safety factor.
 - When estimated life finishes, the component is replaced, even though it could continue in service for a considerable time under safety conditions.
 - Periodic inspection
 - Ex: pressure vessels.



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FATIGUE DESIGN

Fail-safe

- **Philosophy:** Cracks acceptable until they reach a critical size.
- **Periodic inspections:** Inspection period design in order to detect cracks before they reach their critical size.
- **Steps:**
 - The component is replaced when its estimated life finishes: Detectable crack smaller than critical are allowed.
 - Ex: aeronautical industry.

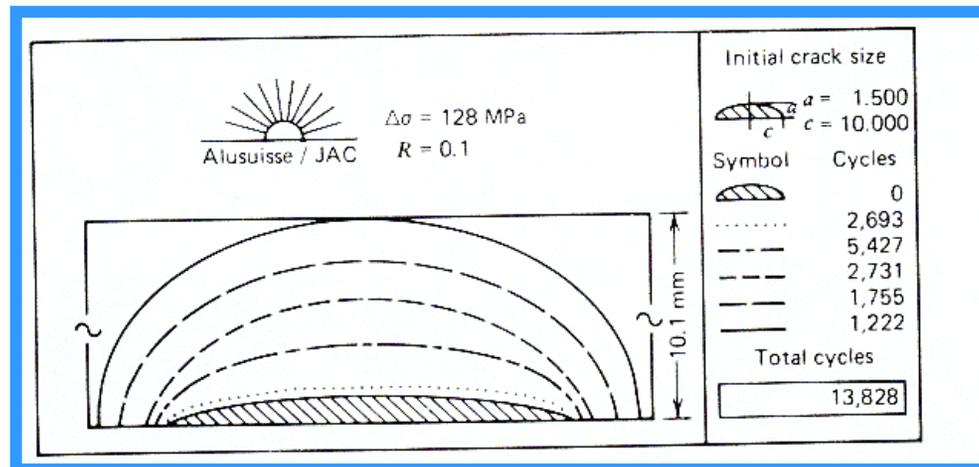


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Leak before break

- Application to pipelines and pressure vessels
- Material and geometry selection in such a way that crack becomes a through thickness crack before the component fails.





FATIGUE

SHORT CRACK GROWTH

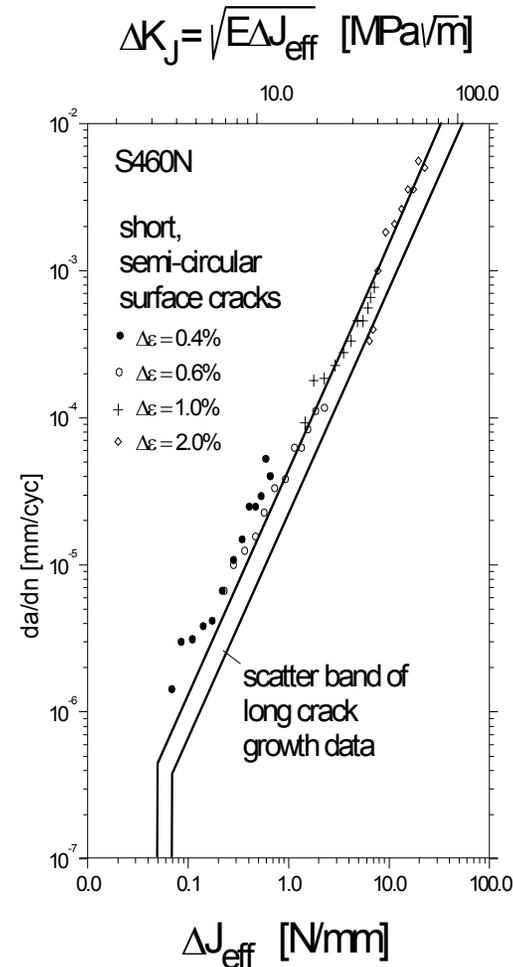
Short cracks can grow only under high stresses

Plastic zones are no longer much smaller than the crack size

The concepts of the Linear Elastic Fracture Mechanics are usually not applicable

Replace ΔK by ΔJ

$$\frac{da}{dN} = C \cdot (\Delta J_{\text{eff}})^m$$





FATIGUE

SHORT CRACK GROWTH

Short crack's closure behaviour differs from long crack behaviour.

Approximation formulas:

$$\sigma_{op} = \begin{cases} \sigma_{max} \cdot (A_0 + A_1 \cdot R + A_2 \cdot R^2 + A_3 \cdot R^3) & \text{for } R > 0 \\ \sigma_{max} \cdot (A_0 + A_1 \cdot R) & \text{for } R \leq 0 \end{cases}$$

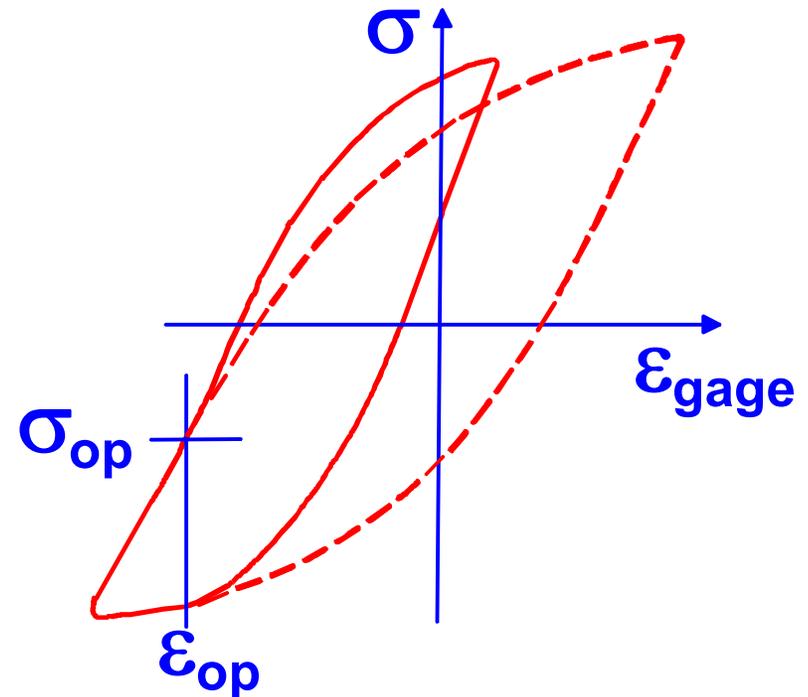
$$A_0 = 0.535 \cdot \cos\left(\frac{\pi}{2} \cdot \frac{\sigma_{max}}{\sigma_F}\right) + a_{mitt}$$

$$A_1 = 0.344 \cdot \frac{\sigma_O}{\sigma_F} + a_{mitt}$$

$$A_3 = 2 \cdot A_0 + A_1 - 1$$

$$A_2 = 1 - A_0 - A_1 - A_3$$

$$\sigma_Y = \frac{1}{2} (\sigma'_{0.2} + \sigma_{UTS})$$



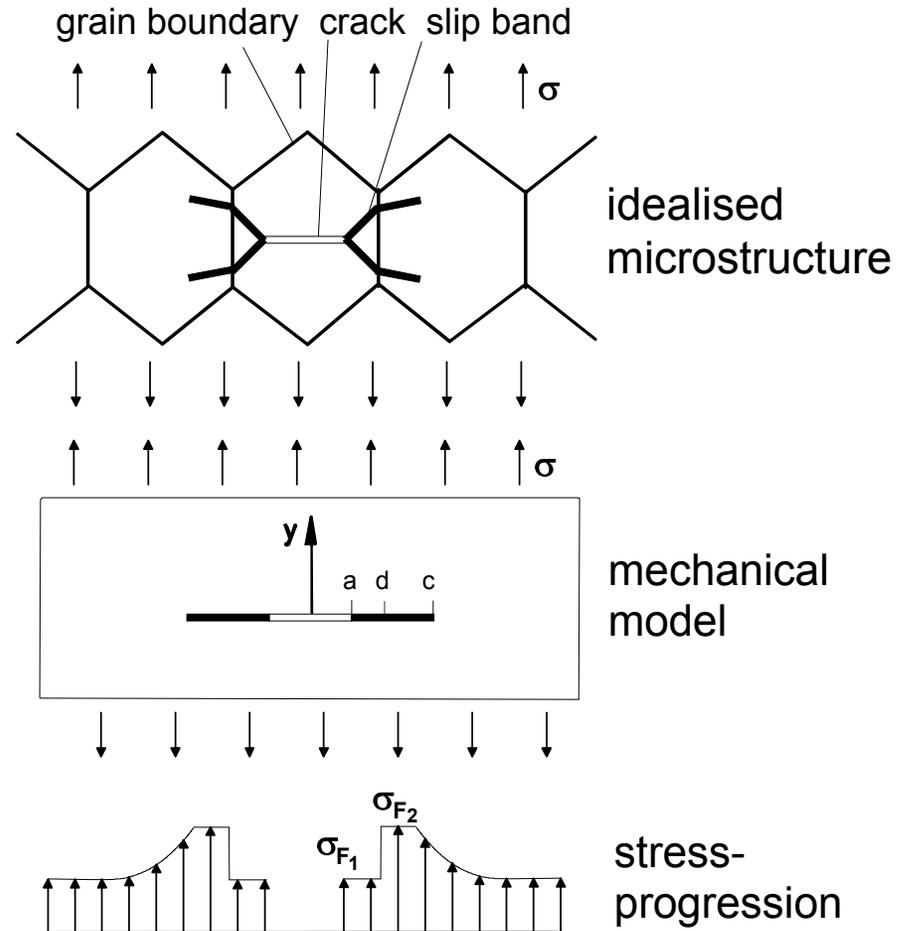


FATIGUE

SHORT CRACK GROWTH

Short crack growth is influenced by the microstructure

Principles can be studied using Tanaka's model



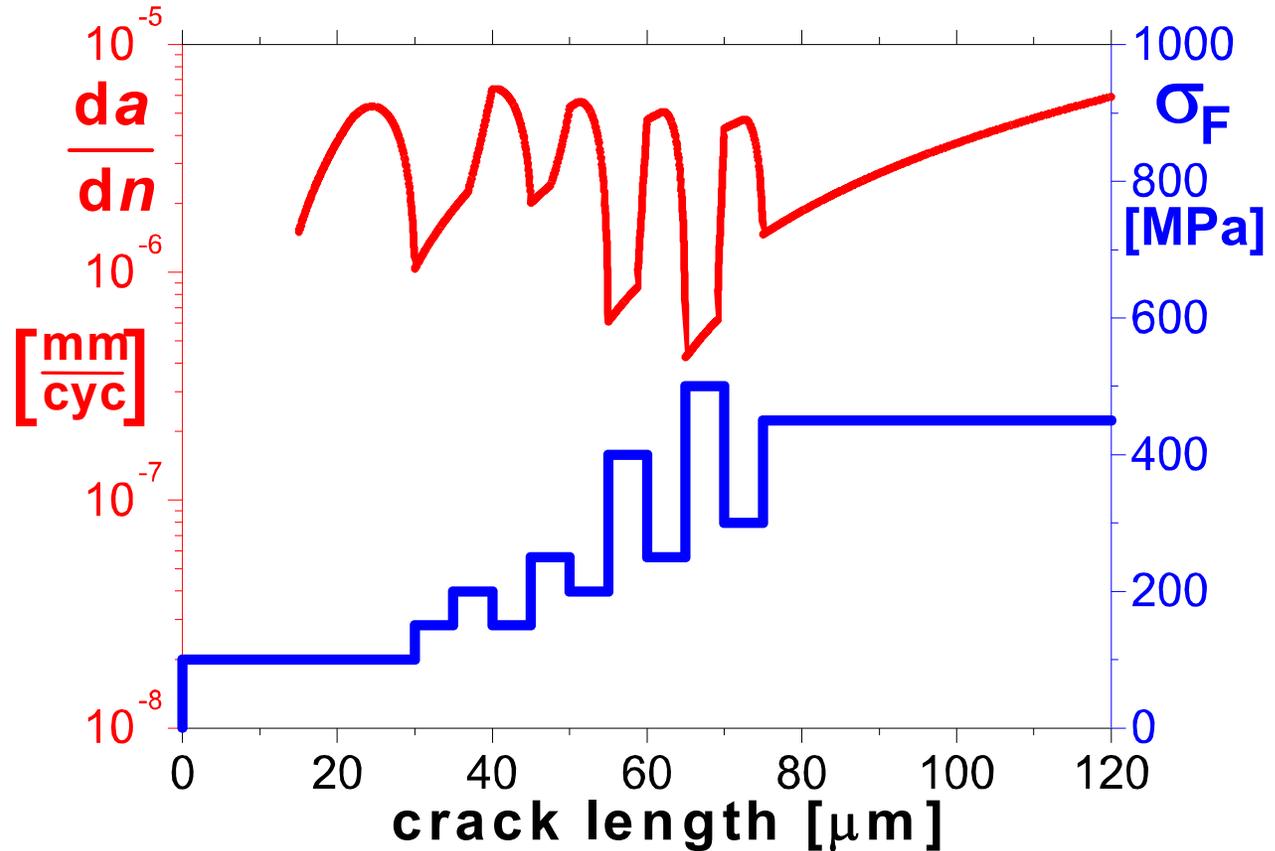


FATIGUE

SHORT CRACK GROWTH

$$\frac{da}{dN} = \left(0.63 \cdot \frac{CTOD}{\text{mm}} - 5.7 \cdot 10^{-5} \right)^{1.5}$$

Example of short crack growth through inhomogeneous microstructure calculated applying Tanaka's model





FATIGUE

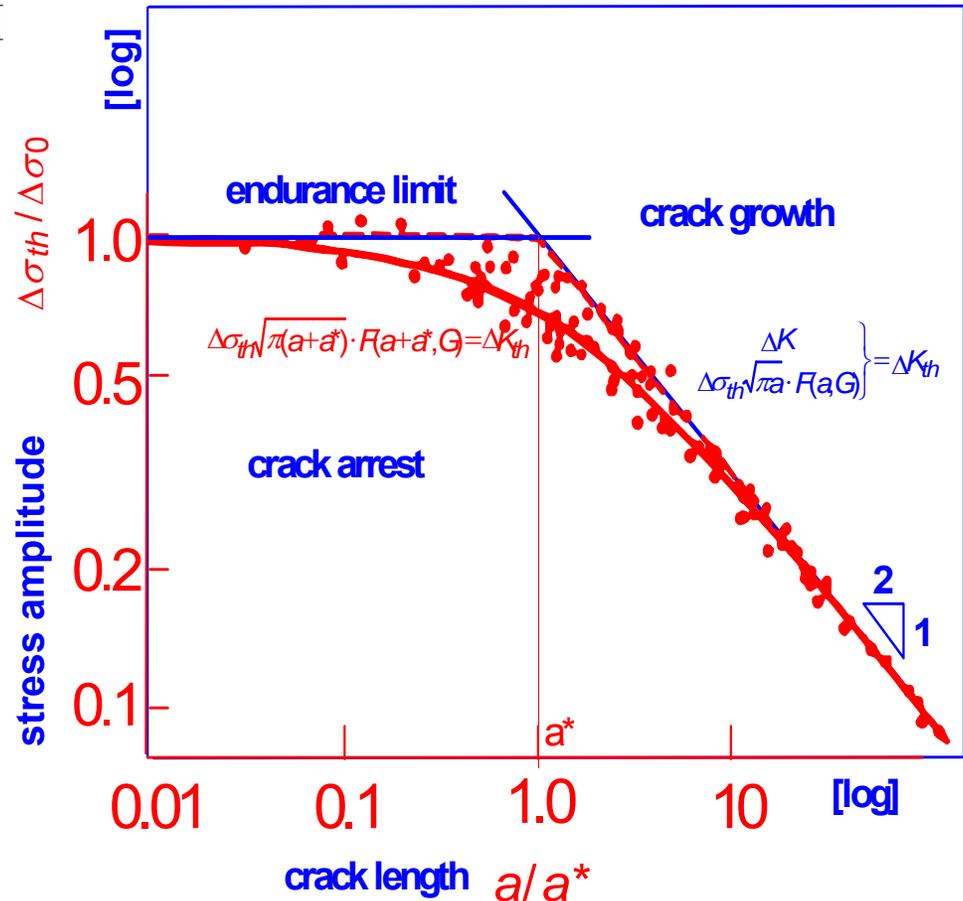
SHORT CRACK GROWTH

Microstructural influence dominates near the endurance limit.

Continuum mechanics based concepts need adjustment.

This leads to the introduction of an intrinsic crack length a^* .

The crack length dependend endurance limit is often shown in a **Kitagawa** plot.



FATIGUE

SHORT CRACK GROWTH

Short cracks are usually semi-circular surface cracks

There are approximation formulas to calculate J.

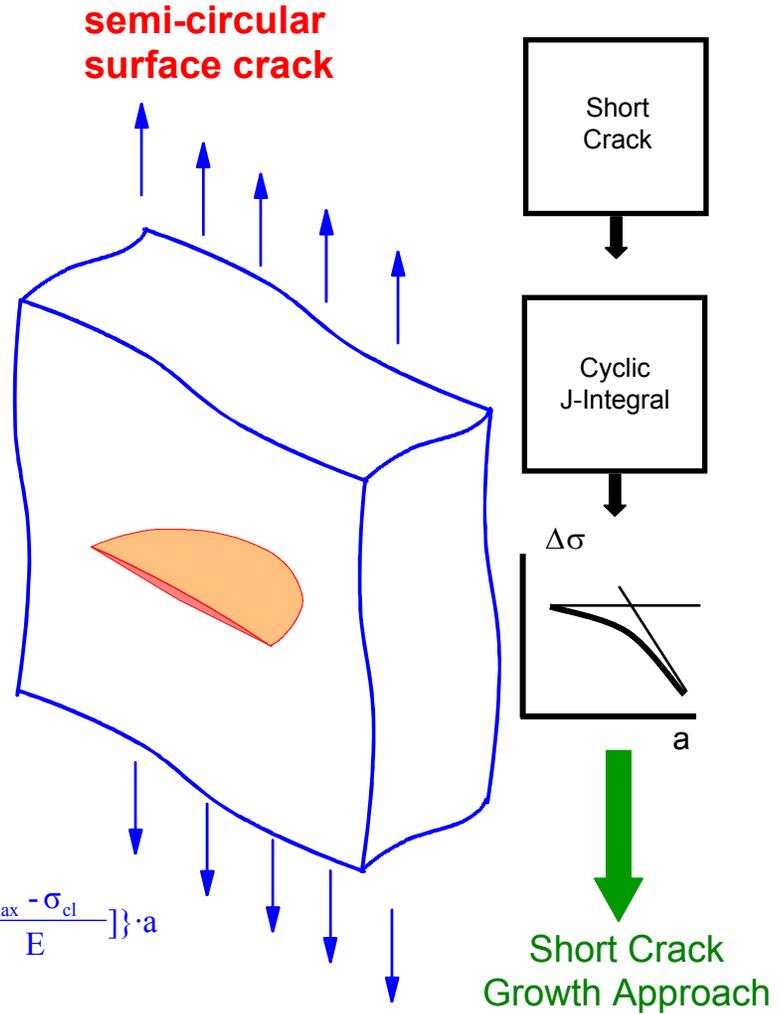
For

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n}}$$

holds

$$J \approx \left(1.24 \cdot \frac{\sigma^2}{E} + \frac{1.02}{\sqrt{n}} \cdot \sigma \cdot \varepsilon_p\right) \cdot a$$

$$\Delta J_{\text{eff}} = \left\{ 1.24 \cdot \frac{(\sigma_{\text{max}} - \sigma_{\text{cl}})^2}{E} + \frac{1.02}{\sqrt{n}} \cdot (\sigma_{\text{max}} - \sigma_{\text{cl}}) \left[(\varepsilon_{\text{max}} - \varepsilon_{\text{cl}}) - \frac{\sigma_{\text{max}} - \sigma_{\text{cl}}}{E} \right] \right\} \cdot a$$



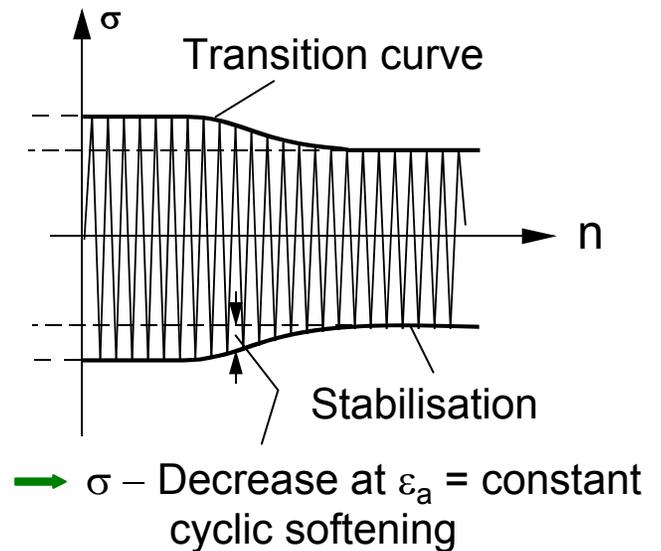
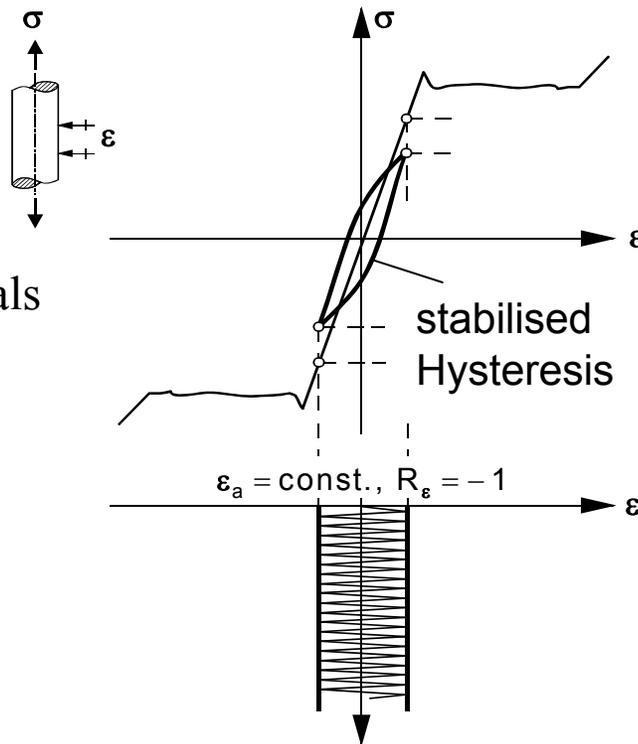


FATIGUE

CRACK INITIATION LIFE ESTIMATION

(without crack growth calculation)

Metallic materials show cyclic hardening or softening.



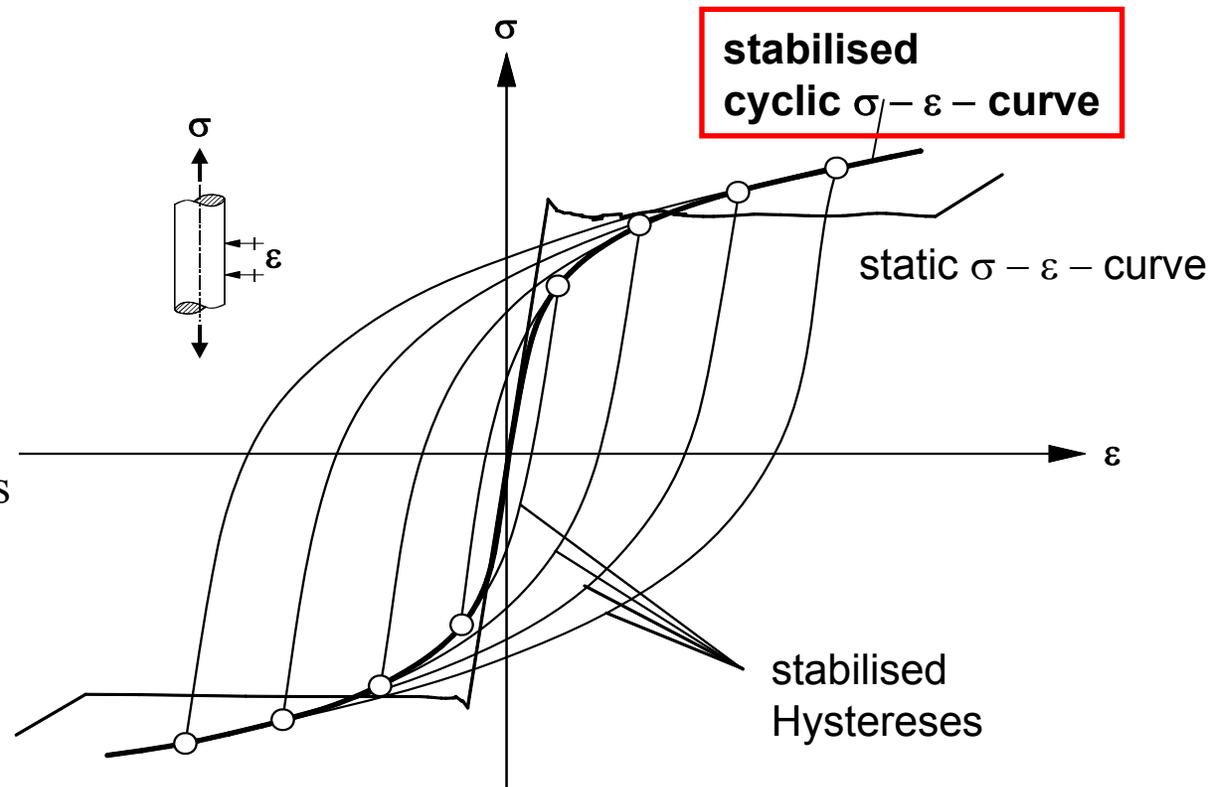
FATIGUE

CRACK INITIATION LIFE ESTIMATION

Until a stabilization is reached:

The stabilized cyclic stress-strain-curve can be used like usual static stress-strain curves.

However, amplitudes are calculated.





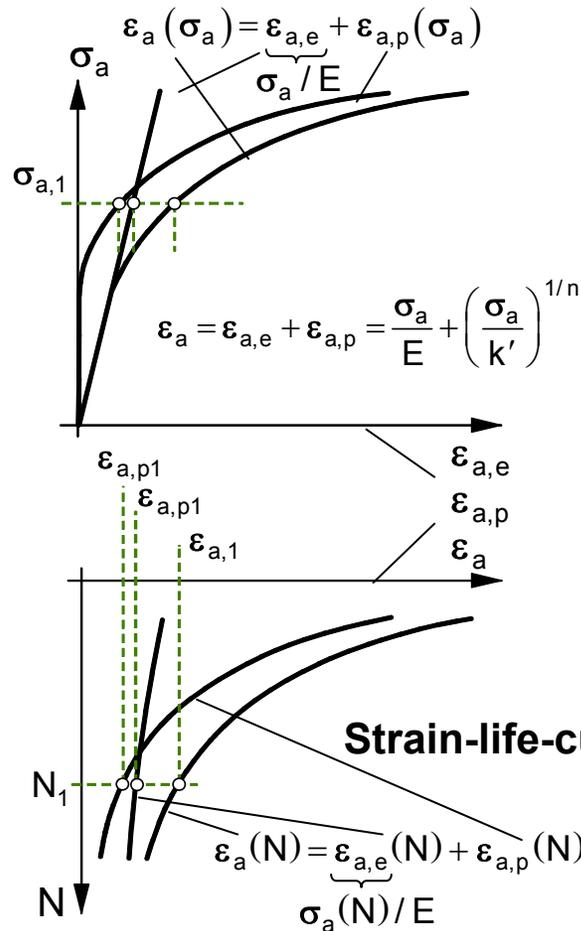
FATIGUE

CRACK INITIATION LIFE ESTIMATION

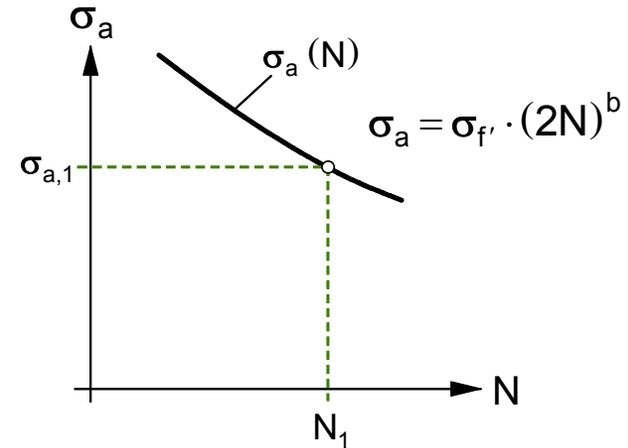
Stress- and strain-life curves give the number of cycles at the particular amplitudes.

Equations according to [Coffin](#), [Manson](#), [Morrow](#), [Basquin](#).

stabilised cyclic $\sigma - \varepsilon -$ curves



Stress-life-curve



Compatibility among $\varepsilon_{a,p}(\sigma_a)$, $\sigma_a(N)$ and $\varepsilon_{a,p}(N)$



FATIGUE

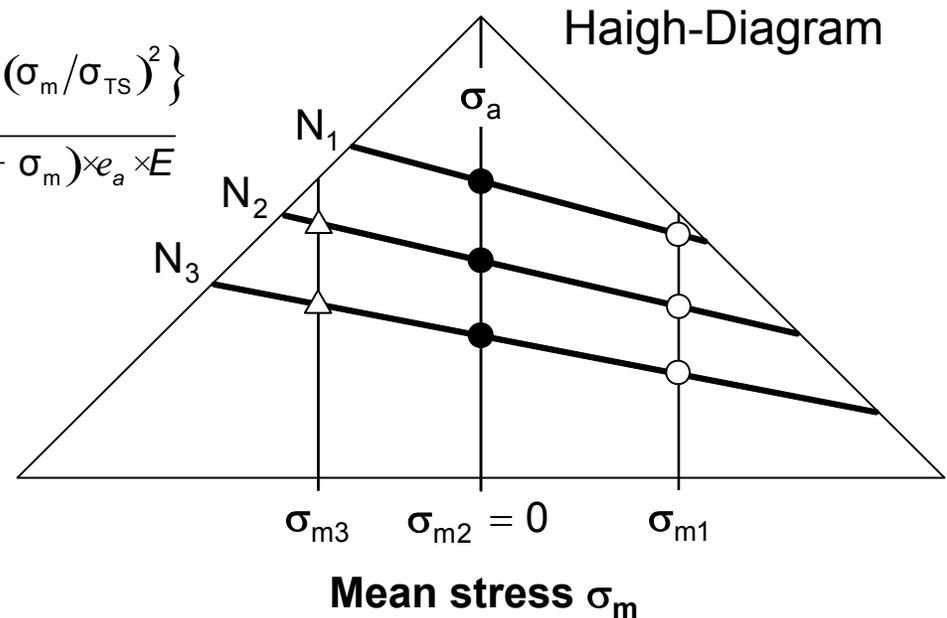
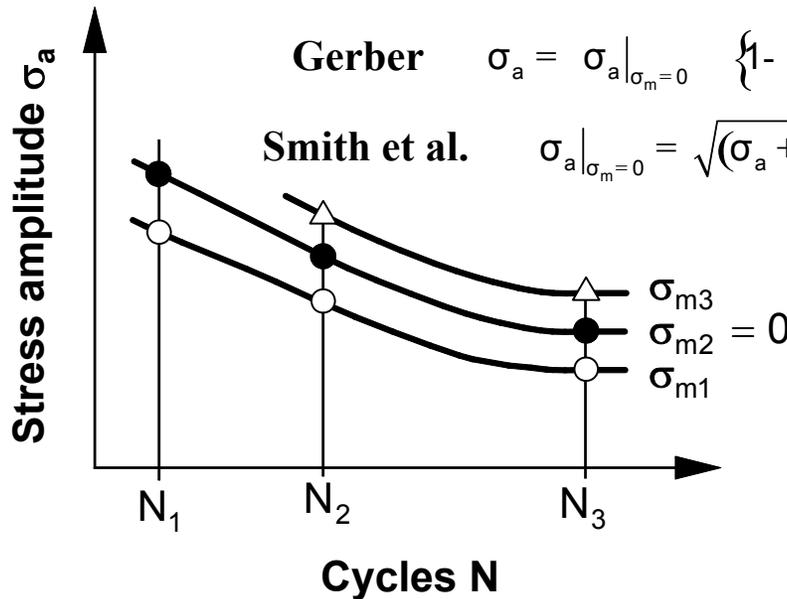
CRACK INITIATION LIFE ESTIMATION

Tensile mean stresses decrease, compressive increase fatigue life. Often used approximation formulas are proposed by:

Goodman $\sigma_a = \sigma_a|_{\sigma_m=0} \{1 - \sigma_m/\sigma_{UTS}\}$

Gerber $\sigma_a = \sigma_a|_{\sigma_m=0} \{1 - (\sigma_m/\sigma_{TS})^2\}$

Smith et al. $\sigma_a|_{\sigma_m=0} = \sqrt{(\sigma_a + \sigma_m) \times e_a \times E}$





FATIGUE

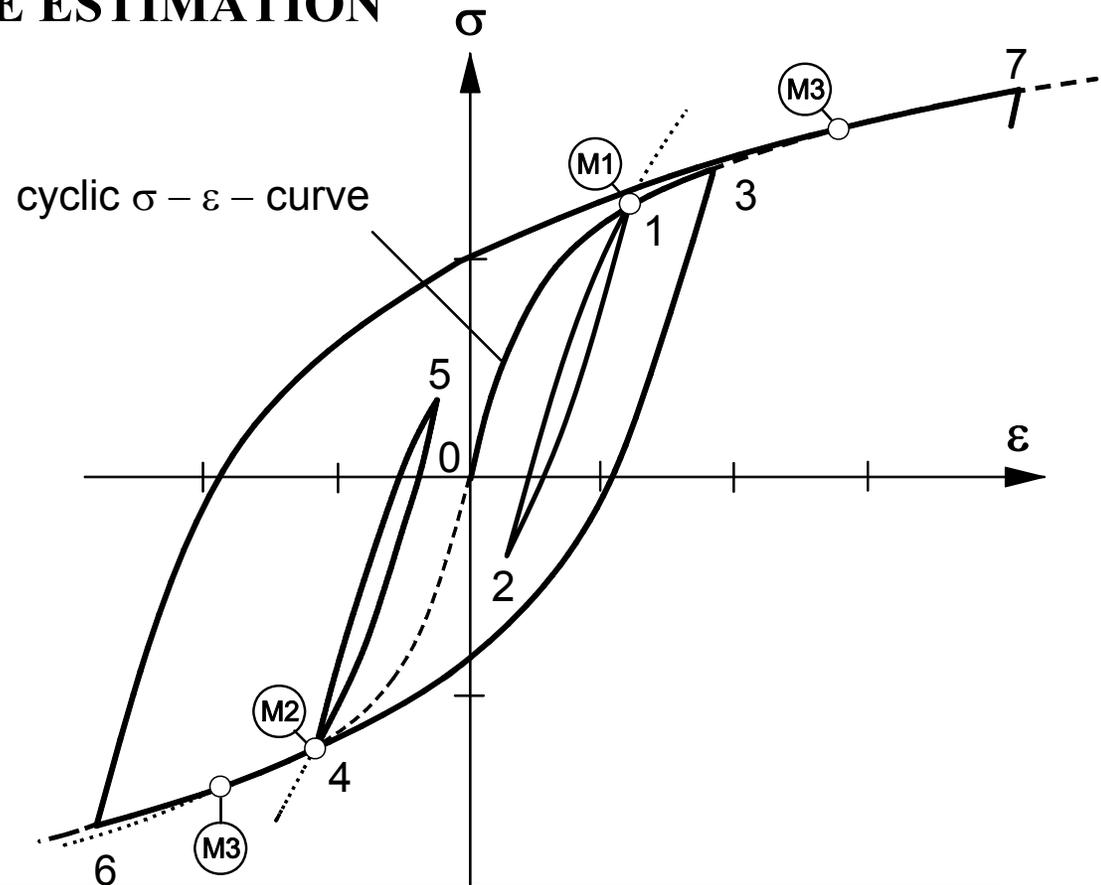
CRACK INITIATION LIFE ESTIMATION

Under variable amplitude loading closed hysteresis loops can be identified.

Doubling the cyclic σ - ε -curve describes the loop branches. The σ - ε -path of a branch kinks into a higher order path branch when both meet each other (Material Memory).

Counting closed loops is named **Rainflow Counting**.

The damage of individual cycles is summed according to [Miner's rule](#).



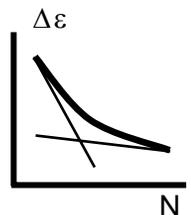
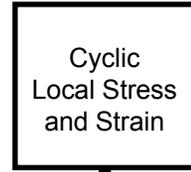
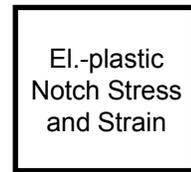


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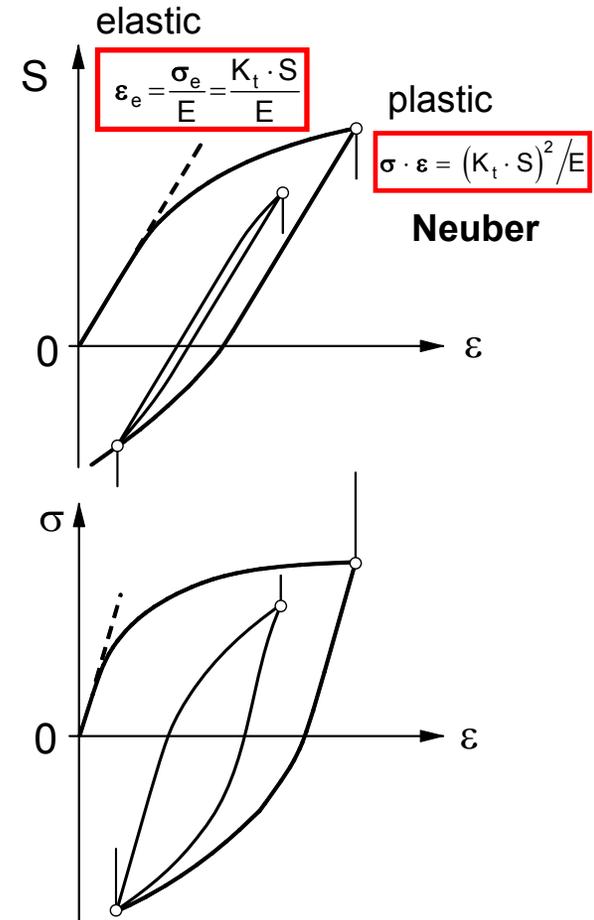
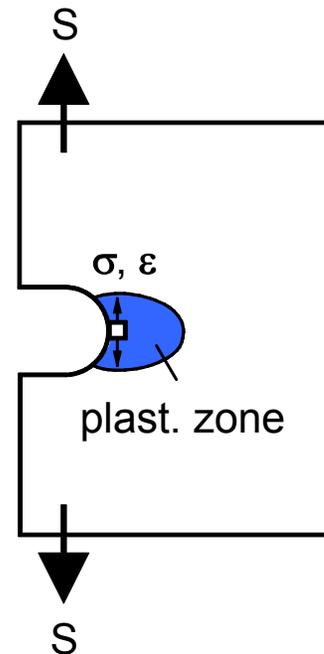
LOCAL STRAIN APPROACH

For notched components the σ - ε path is calculated at the critical locations (notch roots). The elastic stress concentration factor K_t must be known.

Notch stresses and strains can be approximated using **Neuber's rule**.



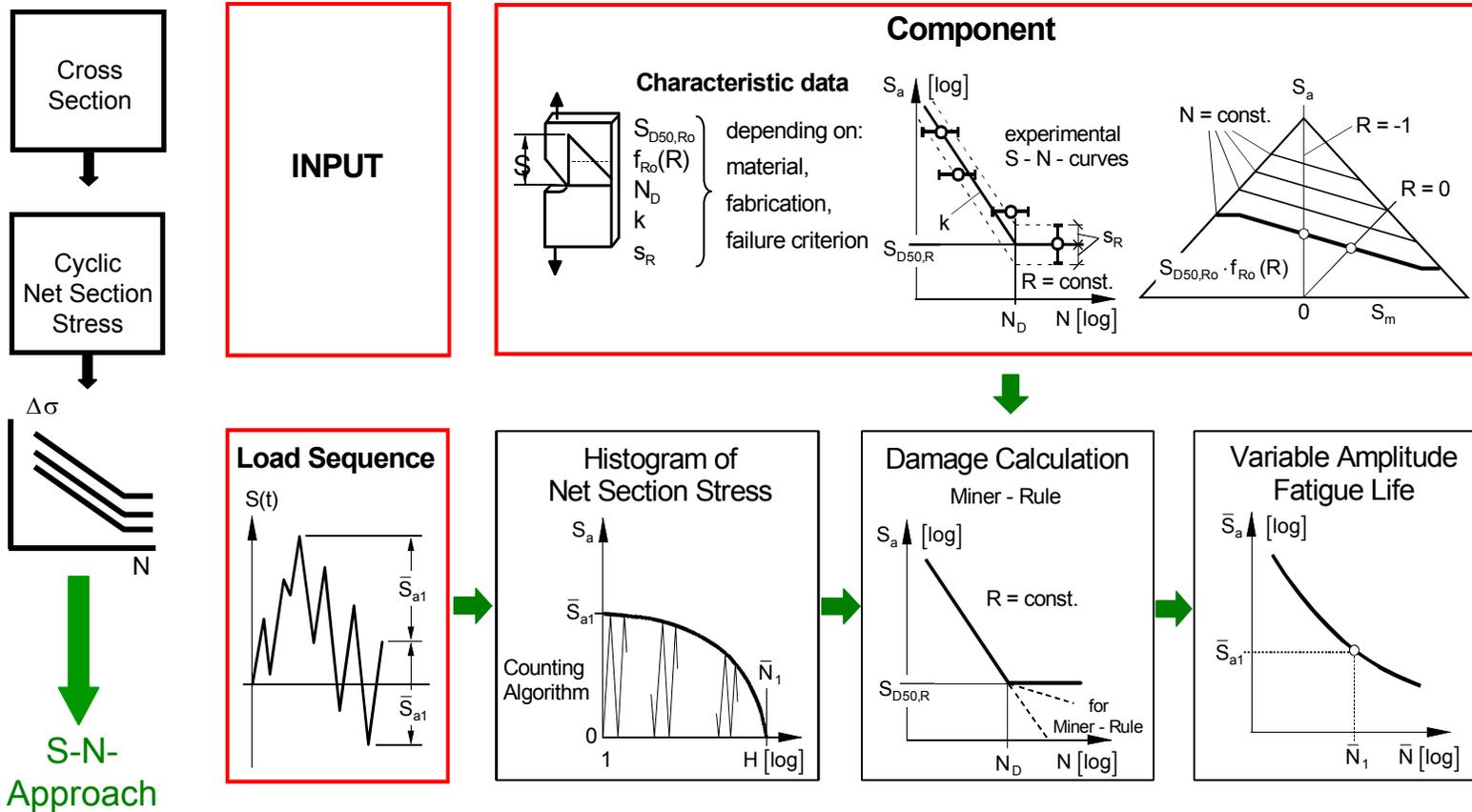
Local Strain Approach





FATIGUE

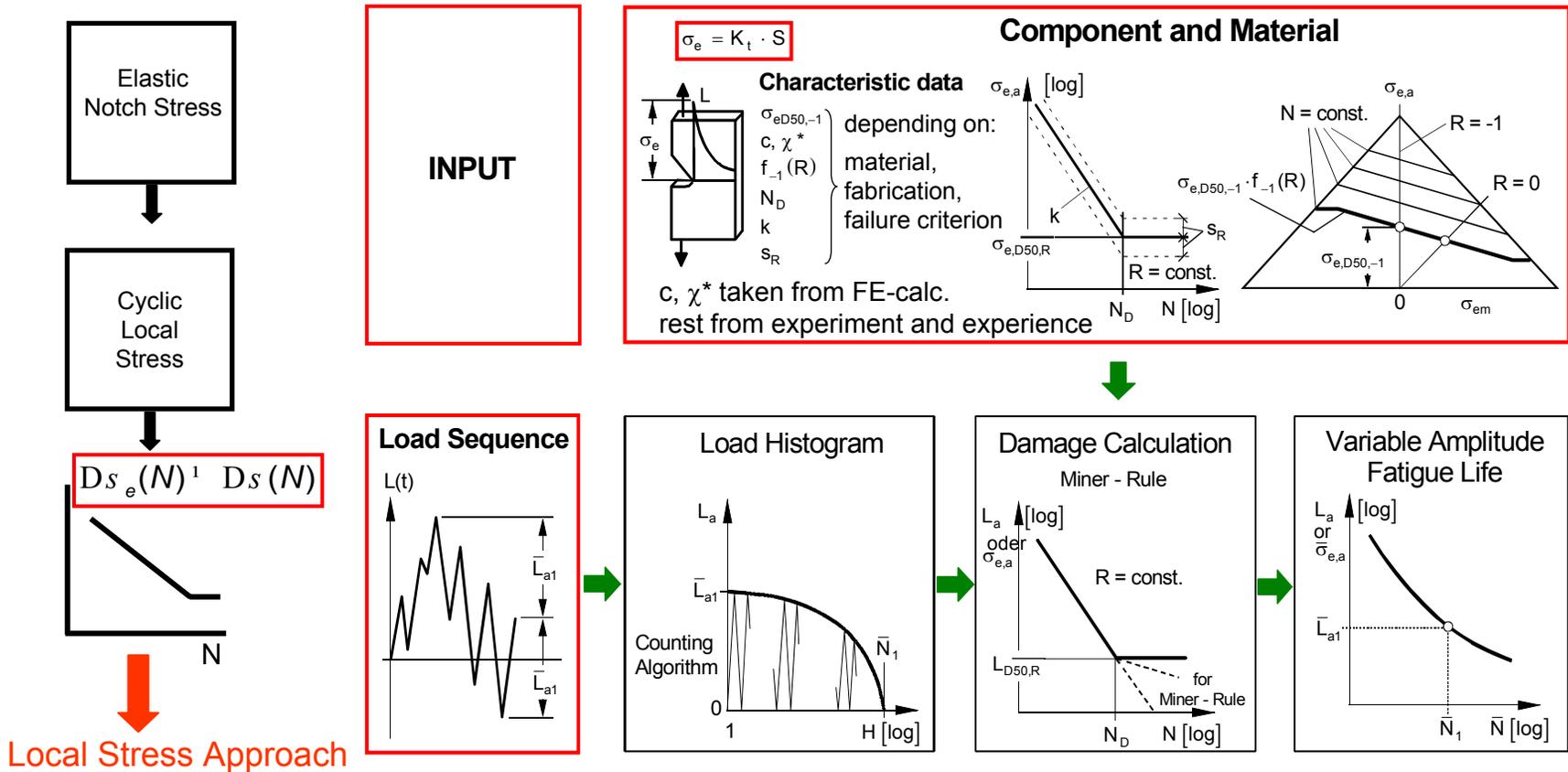
S - N APPROACH





FATIGUE

LOCAL STRESS APPROACH





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- Anderson T.L., “*Fracture Mechanics. Fundamentals and Applications*”, 2nd Edition, CRC Press, Boca Raton (1995).