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#### **D. EXAMPLES**

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#### WORKED EXAMPLE I

Flat Plate Under Constant Load

• Introduction and objectives

• Data

• Analysis

•Bibliography/References

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### **INTRODUCTION AND OBJECTIVES**

During a visual inspection of a C-Mn flat plate of width 100mm, a single edge notch of depth 20 mm is detected.

The plate operates at 380 °C under constant tension, P, corresponding to a nominal stress P/Bw = 100 MPa and the defect is assumed to have been present from the start of high temperature operation.

The objective is to assess the response of the component to the described conditions.



W = 100 mmA = 20 mm







- Material properties (I):

Young's Modulus = 185000 MPa

Some tests have been performed in order to obtain data to develop the assessment. The results are given in the next pages:









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- Material properties (III):

 $da/dt = 0.006 \cdot (C^*)^{0.85}$ 

((da/dt) in mh<sup>-1</sup>, C\* in MPa·mh<sup>-1</sup>)

Incubation COD (mm) = 0.06

- *Limit load for the geometry of the example:* 

 $P_L = 1.155\sigma_y Bw \{1-a/w-1.232(a/w)^2+(a/w)^3\}$ 

- Stress Intensity Factor:  $K = \sigma(\pi a)^{0.5} F(a/w)$ 

$$F = \left\{\frac{\tan\Theta}{\Theta}\right\}^{0.5} \frac{0.752 + 2.02 \cdot \left(\frac{a}{w}\right) + 0.37 \cdot (1 - \sin\Theta)^3}{\cos\Theta}$$
$$\Theta = \frac{\pi a}{2w}$$







• BASIC STRESS ANALYSIS:

The reference stress is calculated according to the limit load for this geometry:

 $\sigma_{ref} = (P/P_L)\sigma_y = 0.866(P/Bw) / \{1-a/w-1.232(a/w)^2+(a/w)^3\} = 114 \text{ MPa}$ a/w = 0.2 $\sigma = P/Bw = 100 \text{ MPa}$ 

And the stress intensity factor is:

 $K_{I} = F(a/w) \cdot \sigma(\pi a)^{0.5} = 34.3 \text{ MPa} \cdot m^{0.5}$ 

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• RUPTURE LIFE:

 $t_{CD} = t_r [\sigma_{ref}^p(a)] = 2.17 \cdot 10^6 h$ 









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## • INCUBATION TIME:

The creep strain that produces the critical crack opening displacement is:

 $\varepsilon_{c} = 0.5(\delta_{i}/R')^{n/n+1} = 0.5(0.06/90)^{n/n+1} = 0.001$ R'= (K<sup>2</sup>/ $\sigma_{ref}$ )<sup>2</sup> m = 90 mm

A value of n is not available and hence n/(n+1) is set equal to the exponent q in the crack growth law (q = 0.85), as suggested in Section A2.6 of the R5 procedure.

As depicted in the figure,  $t_i = 20000$  h



It may be noted that the elastic strain at the reference stress is  $\sigma_{ref}/E = 0.0006$ , which is less than the creep strain at incubation. Thus, the incubation time exceeds the redistribution time and the conservative expression used for  $\varepsilon_c$  is valid.







• CRACK SIZE AFTER GROWTH (I):

$$\mathbf{C}^* = \boldsymbol{\sigma}_{\mathrm{ref}} \cdot \overset{\circ}{\boldsymbol{\varepsilon}} \overset{c}{}_{\mathrm{ref}} \cdot \mathbf{R}'$$

The reference stress and the length parameter R' have already been calculated. From the figure on the previous page, the creep strain rate at the incubation time is:

$$\overset{\circ}{\epsilon}_{\rm ref}^{\rm c} = 3.10^{-8} \,\mathrm{h}^{-1}$$

Thus:

$$C^* = 3 \cdot 10^{-7} MPa m h^{-1}$$

at the incubation time







• CRACK SIZE AFTER GROWTH (II):

The corresponding crack growth rate growth rate using the crack growth law is

 $da/dt = 0.006 \cdot (3 \cdot 10^{-7})^{0.85} = 1.8 \cdot 10^{-5} \text{ mm h}^{-1}$ 

By assuming that the crack growth and creep strain rates are constant for a short time,  $\Delta t$ , the crack size and accumulated creep strain can be updated, and new values for reference stress and creep strain rate can be obtained. The value of C\* can then be obtained with R' evaluated for the new crack size, leading to a new value for da/dt.

The process is explained in the next three pages.







• CRACK SIZE AFTER GROWTH (III):

The crack growth process is divided into different steps with a crack length increment. For the initial crack length on each step, the reference stress and the stress intensity factor are calculated. Then, we can obtain the figure  $\varepsilon_c$ -t from the formulas:

$$\varepsilon_{c}(\sigma_{ref}, t) = A' \left\{ \frac{\sigma_{ref}}{\sigma_{R} + B'} \right\}^{C}$$
  

$$Log_{10}t_{r} = 10.68 + 153.2 \cdot (-1.26 + 2.62x - 2.06x^{2} + 0.72x^{3} - 0.094x^{4})$$
  

$$x = log_{10}\sigma$$

It is possible to consider different  $\sigma_R$  in the second formula and then, to obtain its  $t_r$ . Therefore,  $\sigma_{ref}$ ,  $\sigma_R$  and t are known and  $\varepsilon_c$  can be obtained from the first formula. Finally, it is possible to plot the  $\varepsilon_c$ -t figure for the different  $\sigma_{ref}$ .









- CRACK SIZE AFTER GROWTH (IV):
  - So, for each step, the pocess is:
  - 1)  $\sigma_{ref}$  and K
- 2)  $\varepsilon_c$ -t figure
- 3) R'=  $(K^2/\sigma_{ref})^2$
- 4)  $\varepsilon_c = 0.5 (\delta_i/R')^{n/n+1}$  (creep strain that produces the critical crack opening displacement)

$$\begin{array}{c}
5) t_{i} \\
6) \stackrel{\circ}{\epsilon} \stackrel{c}{}_{ref}^{c}
\end{array} \quad \text{from the } \epsilon_{c}\text{-t figure} \\
7) C^{*}$$







- CRACK SIZE AFTER GROWTH (V):
  - 8) da/dt
  - 9)  $\Delta t$  for each  $\Delta a$

This process is easily developed with computer programs. The crack size as a function of time is shown in the next figure:







#### • RECALCULATE RUPTURE LIFE AFTER GROWTH

As the reference stress is calculated at each stage of the crack growth calculations, it is straightforward to recalculate  $t_{CD}$  from equation:

 $t_{CD}(a_g) = Min\{t_r[\sigma_{ref}(a(t))] + t\} \text{ for } t \le t_i + t_g$ 

Even when the crack has grown to a depth of 35 mm, the reference stress is only 160 MPa and this corresponds to a remaining life of 650000 hours. It is clear from the timescale in the previous figure that in this example creep crack growth rather than creep rupture is the dominant failure mechanism.







• ASSESS SIGNIFICANCE OF RESULTS

The following conclusions can be drawn for this example:

- The remaining creep rupture life was found to be high at all stages of the assessment, showing that creep crack growth, rather than creep rupture, is the dominant failure mechanism.

- Widespread creep conditions are achieved prior to the incubation time.
- An incubation time of ti = 20000 h is predicted.
- The crack is predicted to grow by 15 mm over 380000 h.





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#### WORKED EXAMPLE II

#### **Cyclindrical Pipe Under Cyclic Loading**

#### • Introduction

• Data

• Analysis

•Bibliography/References

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#### **INTRODUCTION**

This example studies a cylindrical pipe with an internal, part-penetrating, fully circumferential defect under cyclic loading.

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The idealised structural geometry is shown in the figure. It comprises a homogeneous Type 316 Stainless Steel pipe of internal radius,  $R_i = 300$  mm and wall thickness, w = 100 mm. A defect is assumed to be present at the start of high temperature operation so that the life to date is taken as zero. The defect is assumed to be fully circumferential on the inside of the pipe with the initial depth,  $a_0$ , taken as 3 mm.



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The pipe is subjected to repeated cyclic loading from an initially unstressed shutdown condition at ambient temperature (20°C) to an operating condition at 600°C, comprising an internal pressure of 16 MPa together with through wall axial and hoop thermal bending stresses of 200 MPa. The bending stresses are such that tensile stresses arise on the inside surface of the pipe as shown in the figure. 500 equal cycles, with 3000 hour dwells at operating conditions, are assumed to occur during the desired future service life of  $1.5 \cdot 10^6$  hour.









Creep strain data are described by the following parametric expression proposed by White (see references):

 $\varepsilon = \varepsilon_{p} [1 - \exp(-rt^{\mu})] + (d\varepsilon/dt)t$ With the maximum primary strain,  $\varepsilon_{p}$ , given by  $\varepsilon_{p} = A'\sigma^{m(\Phi)} \exp[-P/(\Phi+273)]$ where  $m(\Phi) = \alpha - \gamma \Phi$  and the secondary creep strain rate is given by  $(d\varepsilon/dt)_{s} = B\sigma^{n} \exp[P/(\Phi+273)]$ Where  $\Phi$  is the term contains and z the reference strain z

Where  $\Phi$  is the temperature and  $\sigma$  the reference stress.

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The creep strain rate may be obtained by differentiating the equation for the creep strain with respect to time as:

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 $d\epsilon/dt = \epsilon_p r \mu t^{\mu-1} exp(-rt^{\mu}) + (d\epsilon/dt)$ 

However, as  $\mu < 1$ , the creep strain rate given by the above analytical expression becomes infinite at time zero. For short times and low strains ( <10<sup>-4</sup>), the creep strain rate is approximated by dividing the strain of 10<sup>-4</sup> by the time to reach this strain (obtained from the equation for  $\epsilon$ ).

The values of the coefficients A and q of the creep crack growth rate law (m/h) are: A = 0.0197 and q = 0.89

The values of the coefficients C and l of the cyclic crack growth rate law (m/cycle) are:

 $C = 2.0 \ 10^{-9}$  and l = 3







• BASIC STRESS ANALYSIS:

For cyclic loading, the following are required:

- A shakedown analysis
- The depth of the cyclic plastic zone on the surface of the defective section.
- The elastic follow-up factor.
- The stress intensity factors, Kmin and Kmax and the associated R ratio, which permit the effective stress intensity factor range,  $\Delta$ Keff, to be calculated.
- The reference stress for the creep dwell.





#### • SHAKEDOWN ANALYSIS (I):

Uncracked body elastic stresses are required as the starting point for the analysis.

In this example, the pressure stresses are given by de Lamé thick cylinder equations with the thermal stresses taken as through wall bending stresses of equal magnitude in the hoop and axial directions (see the figure). The initial total operating elastic stresses are then the sum of the pressure and thermal contributions.

In order to determine whether the structure is operating within shakedown it is necessary to generate a residual stress field. For this example, it is convenient to select a residual stress field which is a factor,  $\alpha$ , times the thermal stress field (i.e. axial and hoop bending stresses of 200 $\alpha$  MPa). The shakedown stress field, $\sigma_{s}^{*}$ , is then obtained by adding the residual stress field,  $\rho^{*}$ , to the elastically calculated stress field,  $\sigma_{el}^{*}$ . Thus:

$$\sigma_{s}^{*} = \sigma_{el}^{*} + \rho^{*}$$







• SHAKEDOWN ANALYSIS (II):

Shakedown stress fields are thereby determined for the cold (non-creep) and hot (creep) extremes of the loading cycle, denoted  $(\sigma_s)_{nc}$  and  $(\sigma_s)_c$  for shutdown and operating conditions, respectively.

For the structure to attain strict shakedown, the shakedown stress fields at the cold and hot extremes of the loading cycle must satisfy the following criteria:

$$(\sigma_{s})_{nc} \leq (K_{s}S_{y})_{nc}$$
$$(\sigma_{s})_{c} \leq (K_{s}S_{y})_{c}$$

where  $S_y$  is the minimum 0.2% proof stress and  $(\sigma_s)_{nc}$  and  $(\sigma_s)_c$  are the shakedown equivalent stresses at shutdown and operating conditions respectively. The shakedown factor  $K_s$  is an experimentally derived factor which can be applied to  $S_y$  to give a level,  $K_s S_y$ , which is the largest semi-stress range for which the material has stable cyclic stress-strain behaviour.

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#### **ANALYSIS**

• SHAKEDOWN ANALYSIS (III):

The variation of  $K_s$  with temperature for Type 316 steel is given in the next figure:









• SHAKEDOWN ANALYSIS (IV):

For the current example, which involves shutdown at 20°C, values of  $(K_s)_{nc} = 0.752$  and  $(S_y)_{nc} = 245$  MPa are assumed for the Type 316 Stainless Steel, leading to a shakedown criterion at shutdown of:

$$(\sigma_s)_{nc} \leq 184.2 \text{ MPa}$$

For operation at 600°C, assumed values of  $(K_s)_c=1.15$  and  $(S_y)_c=109.6$  MPa give a shakedown criterion at operation of:

$$(\sigma_s)_c \leq 126.8 \text{ MPa}$$

For this example, strict shakedown can be demonstrated for the pipe.







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## • SHAKEDOWN ANALYSIS (V):

Creep relaxation during early loading cycles reduces the stress at the hot extreme of the cycle until the cold extreme of the cycle reaches the limit of the shakedown criterion at shutdown. This situation is achieved using a residual stress field obtained by scaling the thermal stress field by  $\alpha = -0.921$ . Resulting steady cyclic stress profiles for the uncracked pipe are shown in the next figures for shutdown and operating conditions, respectively:









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#### ANALYSIS

#### • SHAKEDOWN ANALYSIS (VI):

In order to take account of early cycles prior to attainment of the steady cyclic state, it is also necessary to determine the initial stress state. For this example, the initial stress state is obtained using a Neuber construction (see R5, Vol 2/3) for the most highly stressed inside surface point. The initial elastic operating stress profiles are shown in the figure and give an initial elastic equivalent stress at the inner surface of 256.8 MPa.

This elastic equivalent stress has then been used, together with isochronous data for Type 316 Stainless Steel at 600°C, to estimate the initial equivalent stress at the inner surface as shown on the next page.







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# ANALYSIS

#### • SHAKEDOWN ANALYSIS (VII):



This initial equivalent stress at the inner surface (141.8 MPa) has then been used to infer an initial residual stress field, which when combined with the initial elastic stresses, gives the correct value of inner surface equivalent stress. The required initial residual stress field is obtained by scaling the thermal stress field by  $\alpha = -0.583$ .





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• SHAKEDOWN ANALYSIS (VIII):

Resulting initial stress profiles are shown in the next figures for shutdown and operating conditions, respectively.



Strict shakedown has been demonstrated for this example. There is therefore no cyclic plastic deformation at the inner surface of the defective pipe section and the cyclic plastic zone,  $r_p$ , is set equal to zero.







• STRESS INTENSITY FACTORS (I):

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 $\begin{array}{ll} \Delta K_{eff} = K_{max} - K_{min} & K_{max} \rightarrow \text{operation} \\ K_{min} \rightarrow \text{shutdown} \\ K = (F_m \sigma_m + F_b \sigma_b) \cdot (\pi a)^{1/2} & \sigma_m \rightarrow \text{membrane stress} \\ \sigma_b \rightarrow \text{bending stress} \\ F_m \rightarrow \text{membrane compliance function} \\ F_b \rightarrow \text{bending compliance function} \end{array}$ 

From the handbook of Tada, Paris and Irwin, (see references) and for  $R_i/w = 3$ :

 $F_{m} = 1.123 - 0.103 \cdot (a/w) + 2.030 \cdot (a/w)^{2} - 1.373 \cdot (a/w)^{3} + 0.790 \cdot (a/w)^{4}$  for 0 < a/w < 0.6The corresponding bending compliance function has been derived using the computer program R-Code:

 $F_{b} = 1.126 - 1.543 \cdot (a/w) + 2.613 \cdot (a/w)^{2} - 3.986 \cdot (a/w)^{3} + 2.123 \cdot (a/w)^{4}$  for 0 < a/w < 0.6







#### • STRESS INTENSITY FACTORS (II):

The effective stress intensity factor range,  $\Delta K_{eff}$ , has been evaluated as a function of crack depth from equations:

$$\Delta K_{eff} = q_0 \Delta K$$
  
 $q_0 = 1$   
 $q_0 = (1-0.5R)/(1-R)$ 

$$\Delta \mathbf{K} = \mathbf{K}_{\max} - \mathbf{K}_{\min}$$
$$\mathbf{R} \ge 0$$
$$\mathbf{R} < 0$$
$$\mathbf{R} = \mathbf{K}_{\min} / \mathbf{K}_{\max}$$

from both initial and shakedown conditions using the compliance functions given previously together with the axial stresses given in the next table.

	Oper	ation	Shutdown			
Loading Conditions	Membrane Stress (MPa)	Bending Stress <sup>#</sup> (MPa)	Membrane Stress (MPa)	Bending Stress <sup>#</sup> (MPa)		
Initial	20.6	83.4	0	-116.6		
(Start of first cycle)						
Shakedown	20.6	15.8	0	-184.2		
(Steady cyclic state)						

<sup>#</sup> Positive values indicate tensile stress on the inside surface of the pipe





#### • STRESS INTENSITY FACTORS (III):

The effective stress intensity factor ranges (together with associated values of  $K_{max}$ ) are shown as functions of crack depth in the figure for both the initial and shakedown conditions. Note that for the current example R < 0 and hence  $q_0 < 1$  for both initial and shakedown conditions (for all crack depths).

For the period prior to the attainment of the steady cyclic state (i.e.  $t < t_{cyc}$ ), the effective stress intensity factor range has been taken as the mean of the initial and shakedown values.



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• REFERENCE STRESSES (I):

 $\sigma_{\rm ref} = (F/F_L)\sigma_v$ 

If proportional loading is assumed, the limit loads can be determined from:

 $F_L/M_L = F/M$ 

The next table gives axial and hoop stresses appropriate to initial and shakedown conditions and associated forces and moments (per unit thickness) evaluated using:

$F = \sigma w$		Axial				Ноор			
$M = (\sigma_b w^2)/6$	Loading Conditions	Membrane Stress σ <sup>a</sup> <sub>m</sub> (MPa)	Bending Stress $\sigma^{a}_{b}$ (MPa)	Force per Unit Thickness F <sup>a</sup> (N/m)	Moment per Unit Thickness M <sup>a</sup> (Nm/m)	Membrane Stress σ <sup>h</sup> <sub>m</sub> (MPa)	Bending Stress $\sigma^{h}_{b}$ (MPa)	Force Per Unit Thickness F <sup>h</sup> (N/m)	Moment Per Unit Thickness M <sup>h</sup> (Nm/m)
	Initial (Start of First Cycle)	20.6	83.4	2.06x10 <sup>6</sup>	1.39x10 <sup>5</sup>	49.1	91.4	4.91x10 <sup>6</sup>	1.52x10 <sup>5</sup>
	Shakedown (Steady cyclic state)	20.6	15.8	2.06x10 <sup>6</sup>	2.63x10 <sup>4</sup>	49.1	23.8	4.91x10 <sup>6</sup>	3.97x10 <sup>4</sup>





#### • REFERENCE STRESSES (II):

Axial and hoop stresses have been evaluated for both steady cyclic and initial conditions (based on Neuber) using the stress profiles shown in the next figures:

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In both cases, the axial and hoop stresses can be well represented by membrane and bending stresses,  $\sigma_m$  and  $\sigma_b$ , respectively.







• REFERENCE STRESSES (III):

The limit loads for axially dominated collapse have the form:

$$F_{L}^{a} = (2y-a)\sigma_{y}$$
  

$$M_{L}^{a} = \{(w^{2}/4) + (a^{2}/4) - (at/2) - x^{2}\}\sigma_{y}$$

where w is the pipe wall thickness and y is the distance between the plastic neutral axis and the mid-wall thickness. The value of y is found from the equation  $F_L/M_L = F/M$  based on the values of F and M previously calculated and the expressions involving y for  $F_L^a$  an  $M_L^a$ . The resulting quadratic equation can then be easily solved.

For the hoop dominated collapse, the limit loads are:

$$\begin{split} F^h_{\ L} &= 2y\sigma_y \\ M^h_{\ L} &= \{(w^{2/4}) - y^2\}\sigma_y \end{split}$$

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• REFERENCE STRESSES (IV):

The maximum of the axial and hoop reference stress is then chosen. For both the initial and shakedown conditions, the reference stress is hoop dominated, and is therefore independent of crack depth.

For initial conditions the reference stress is

$$\sigma_{\rm ref}^{\rm cyc=1} = 88.1 \,\rm MPa$$

while for steady cyclic conditions

$$\sigma_{ref} = 57.6 \text{ MPa}$$

is obtained.







• CALCULATE CRACK SIZE AFTER GROWTH (I):

For the prupose of this example, it is assumed that both creep and fatigue are significant. The calculation of the incubation time is not considered in this example, although a conservative incubation time of zero is often assumed when creep and fatigue are significant.

Strict shakedown of the uncracked structure has been demonstrated for this example and so a Method I crack growth calculation is appropriate. The creep and fatigue crack growth contributions are separately calculated and added for each cycle.

The creep crack growth rate law takes the form:

 $da/dt = 0.0197 \cdot (C^*)^{0.89}$ 

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#### • CALCULATE CRACK SIZE AFTER GROWTH (II):

In general, the parameter  $C^*$  is calculated by the reference stress approach. It is also necessary to calculate a mean value of  $C^*$  for use in calculating creep crack growth occurring in the dwell periods prior to the attainment of the steady state (t < t<sub>cyc</sub>).

$$\mathbf{C}^* = \frac{(\sigma_{ref}^{cyc=1} + \sigma_{ref})}{2} \cdot \overset{\circ}{\mathcal{E}} \cdot \mathbf{R}' \qquad \text{where } d\epsilon/dt \text{ is evaluated for } \frac{(\sigma_{ref}^{cyc=1} + \sigma_{ref})}{2}$$

An estimate of  $t_{cyc}$  can be expressed in terms of the reference stress for the first cycle,  $\sigma_{ref(cyc=1)}$ , and the reference stress under steady cyclic conditions,  $\sigma_{ref}$ , as:

$$\varepsilon_{c} \left[ \left( \sigma_{ref}^{cyc=1} + \sigma_{ref} \right) / 2, t_{cyc} \right] = Z \cdot \left( \sigma_{ref}^{cyc=1} - \sigma_{ref} \right) / E$$

where Z is the elastic follow-up factor defined in Appendix A3 of the R5 procedure.

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### • CALCULATE CRACK SIZE AFTER GROWTH (III):

For the current example, the stresses acting during the dwell periods after the steady cyclic is reached are predominantly primary. Therefore, the small amount of stress relaxation that could occur during the dwell has been neglected and load-controlled loading has been assumed in calculating creep strain accumulation and crack growth during the dwell.

The stress intensity factor used for the calculation of R' is evaluated using the stresses at the beginning of the dwell and is therefore equal to  $K_{max}$ . Prior to attainment of the steady cyclic state, a mean value of  $K_{max}$  has been used in the calculation of R'. This is given by:

$$\overline{K}_{\max} = \frac{K_{\max}^{\text{cyc=1}} + K_{\max}}{2}$$

where  $K_{max}^{cyc=1}$  and  $K_{max}$  are the maximum stress intensity factors at the start of the first cycle (using a Neuber construction) and the cycle in the steady cyclic state, respectively.

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## • CALCULATE CRACK SIZE AFTER GROWTH (IV):

The cyclic crack growth rate law takes the form:

 $(da/dN) = 2 \ 10^{-9} \cdot (\Delta K_{eff})^3$ 

The total crack growth per cycle is obtained by adding the cyclic and creep contributions.

The crack extension over a desired future life of  $1.5 \cdot 10^6$  hours is then calculated iteratively using a computer program. The main features of the iterative procedure are as follows:

i) Calculate creep crack growth for the dwell period in the first cycle. It should be noted that this itself involves an iterative procedure in which the creep crack growth and strain rates are assumed constant for a short time,  $\Delta t$ . The crack depth and accumulated creep strain are then updated and new values of reference stress and creep strain rate obtained assuming a strain hardening rule. The value of C\*

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• CALCULATE CRACK SIZE AFTER GROWTH (V):

can then be obtained with R' evaluated for the new crack depth, leading to a new value of creep crack growth rate. For the current example, these calculations have actually been implemented by incrementing crack depth.

ii) Calculate cyclic crack growth for the first cycle and increment crack depth.

iii) Repeat calculations for subsequent cycles.

For the current example, it is also necessary to determine  $t_{cyc}$ , the time to redistribute to the steady cyclic state. A value of elastic follow up of Z=3 is arbitrarily assumed. With this assumption, the steady cyclic state is achieved after 1 cycle. Prior to attainment of the steady cyclic state, mean values of  $\Delta K_{eff}$  and C<sup>\*</sup> are used to calculate cyclic and creep components of crack growth as described above. After steady cyclic state has been established, values of  $\Delta K_{eff}$  and C<sup>\*</sup> appropriate to steady state conditions are used in the crack growth calculations.







#### • CALCULATE CRACK SIZE AFTER GROWTH (VI):

The results of these iterative calculations lead to the crack depth as a function of time shown in the figure:









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