D. EXAMPLES

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WORKED EXAMPLE I

Infinite Plate under fatigue

- **Introduction and Objectives**
- **Data**
- **Analysis**

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INTRODUCTION AND OBJECTIVES

One structural component of big dimensions is subjected to variable loading conditions everyday: 200 MPa during 12 hours and 20 MPa the rest of the day. During the maximum loading conditions other variable stresses appear, with a variation of 30 MPa (because of vibrations with a frequency 50 Hz).

Some NDT are performed, with equipment whose sensitivity is 0.2 mm and no cracks are detected.

Considering the component as an infinite plate:

a) Determine the crack length which is necessary to crack propagation because of vibrations

b) Critical crack length for final failure

c) Life time for the component

d) Evolution of the crack length with time in order to determine inspection periods

DATA

Material properties:

$K_{IC} = 100\ MPa \cdot m^{1/2}$

ΔK_{th} (or ΔK_0) $= 3\ MPa \cdot m^{1/2}$ if $R = P_{min}/P_{max} = 0.1$

$\Delta K_{th} = 1.5\ MPa \cdot m^{1/2}$ if $R = P_{min}/P_{max} = 0.85$

Paris Law:

$$\frac{da}{dN} = 1\cdot10^{\;8}\cdot(\Delta K)^2$$

da/dN in m/cycle when ΔK in $MPa \cdot m^{1/2}$

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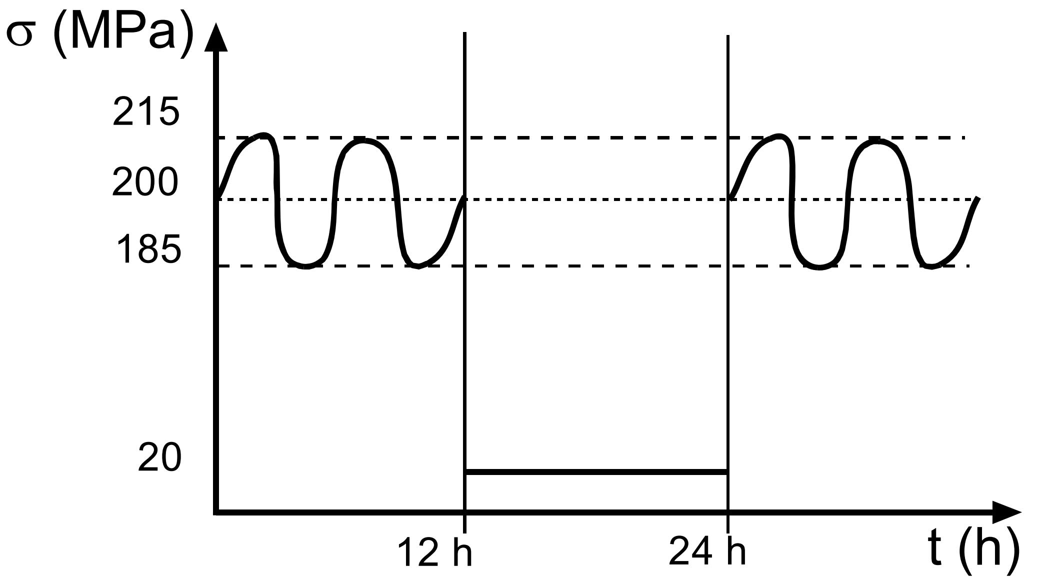
Working conditions are plotted in the next figure:



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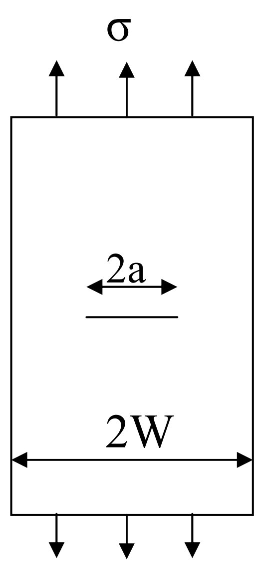
The component geometry can be simplified as:



The equipment sensitivity is 0.2 mm and no crack has been detected.
So, in the worst possible situation 2a = 0.2 mm.

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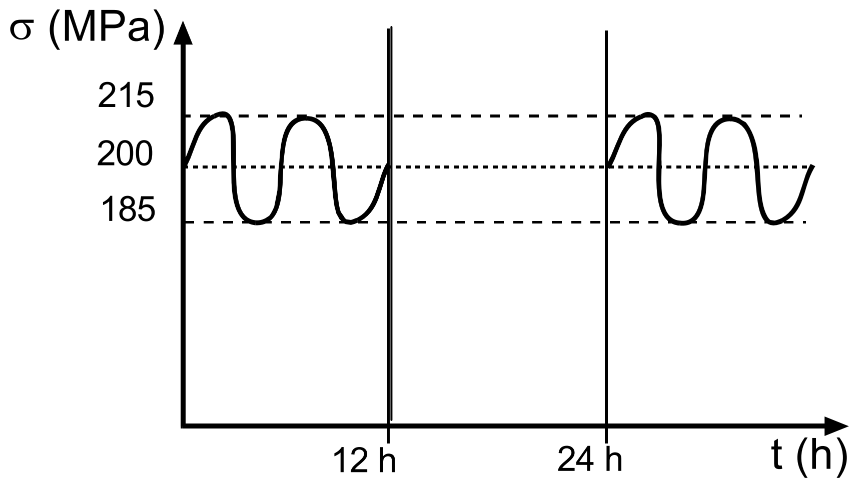
State I: VIBRATIONS



$R = P_{max}/P_{min} = 0.8$ $\Delta K_{th} = 1.5\ MPa{\cdot}m^{1/2}$

$\Delta K = \Delta\sigma{\cdot}(\pi{\cdot}a)^{1/2} = 30{\cdot}(\pi{\cdot}0.0001)^{1/2} = 0.53\ MPa{\cdot}m^{1/2} < \Delta K_{th}$

Existing cracks don´t propagate

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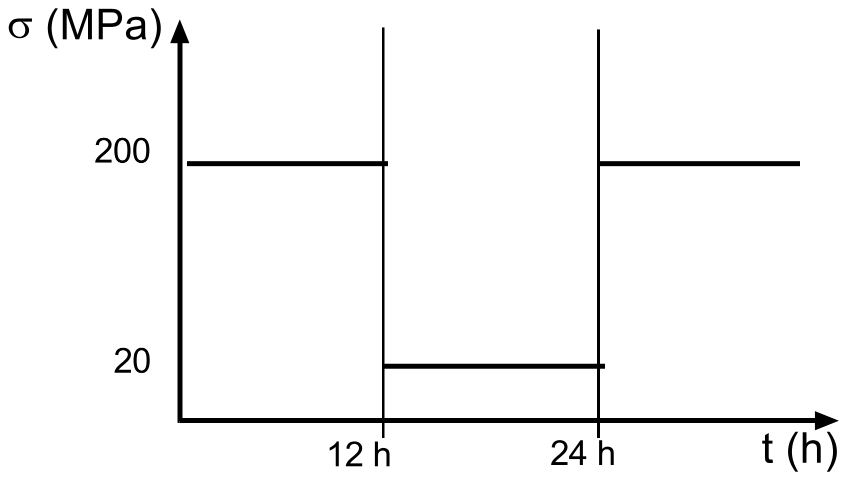
State II: MAIN LOADING CONDITIONS



As R = 0.10 $\Delta K_{th} = 3.0\ MPa \cdot m^{1/2}$

$\Delta K = \Delta\sigma \cdot (\pi \cdot a)^{1/2} = 180 \cdot (\pi \cdot 0.0001)^{1/2} = 3.19\ MPa \cdot m^{1/2} > \Delta K_{th}$

Existing cracks could propagate

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As existing cracks could propagate, it is necessary to determine the crack length for crack propagation because of vibrations:

$$\Delta K_{th} = 1.5 MPa \cdot m^{1/2} = 30 \cdot \sqrt{\pi \cdot a_v} \rightarrow a_v = 0.80\ mm$$

For shorter cracks propagation is only due to main loading variation ($\Delta\sigma$ =180 MPa, f = 1/86400 Hz)

The critical crack length determined at failure is:

$$K_{I\,max} = \sigma_{max} \cdot \sqrt{\pi \cdot a_c} = 215 \cdot \sqrt{\pi \cdot a_f} = 100 MPa \cdot m^{1/2}$$

$$a_f = 68\ mm$$

For crack length over 0.80 mm propagation is due to both main loading variation and vibrations ($\Delta\sigma$ = 30 MPa, f = 5 Hz)

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LIFE TIME:

The time necessary to initiate the effects of vibrations to cause crack propagation is determined through the Paris law:

$$N = \frac{1}{C \cdot Y^2 \cdot (\Delta\sigma)^2 \cdot \pi} \cdot \mathrm{Ln}\frac{a_f}{a_0} = \frac{1}{1 \cdot 10^{-8} \cdot (180)^2 \cdot \pi} \cdot Ln\frac{0.0008}{0.0001} = 2042 \text{ cycles}$$

2042 cycles is equivalent to 2042 days or 5.59 years

Once this crack length is reached, propagation is due to vibrations (mainly):

$a_0 = 0.8$ mm $\quad$ $a_f = 68$ mm

$$N = \frac{1}{C \cdot Y^2 \cdot (\Delta\sigma)^2 \cdot \pi} \cdot \mathrm{Ln}\frac{a_f}{a_0} = \frac{1}{1 \cdot 10^{-8} \cdot (30)^2 \cdot \pi} \cdot Ln\frac{68}{0.8} = 157126 \text{ cycles}$$

157126 cycles is equivalent to 0.73 days

The same day that cracks achive length to propagate due to vibration amplitude, the component fails. SO, LIFE TIME IS 5.59 YEARS.

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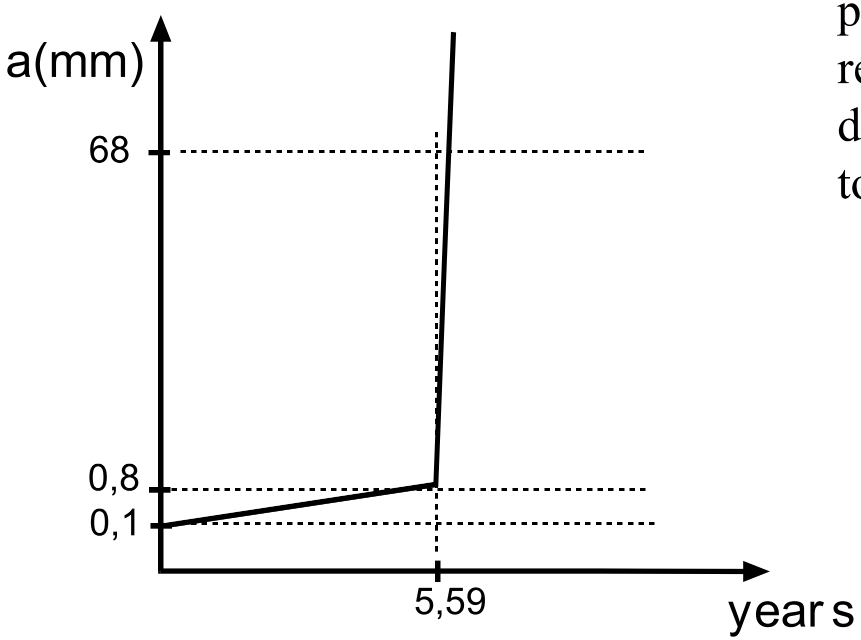
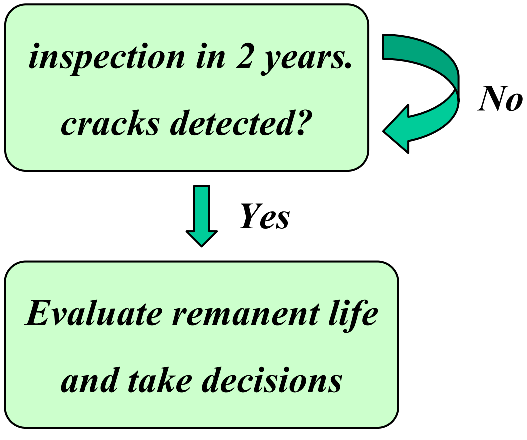
The evolution of semicrack length with time is (question d)):



It is reasonable to define periodical inspections to check if really cracks propagate over the detectable value of 0.2 mm in total length (i.e. every two years)



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WORKED EXAMPLE II

Fatigue test

- **Introduction and data**
- **Objectives**
- **Analysis**

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INTRODUCTION AND DATA

A fatigue test on a seven wire strand was performed. The maximum applied stress is $0.8 \cdot \sigma_u$ and the amplitude is 390 MPa.

The strand is one meter long and the diameter of the wires is 5 mm.

The test finished with three broken wires, the central one and two external (which were together), after 320.000 fatigue cycles (f = 8 Hz).

The SEM observation of the failure surfaces gave the following information about crack lengths:

- *Central wire*: 0.25 mm (depth) elliptical crack from non propagated initial defect.
- *External wire A*: 1.32 mm from a non differentiated initial defect.
- *External wire B*: 1.20 mm proceeding from a 0.30 mm in depth initial defect.

From a previous tension test, the mechanical behaviour of the strand was obtained:

- E = 195 GPa
- Failure Load = 256.2 kN   Strain for Failure Load = 4.7 % (gauge base 500 mm)

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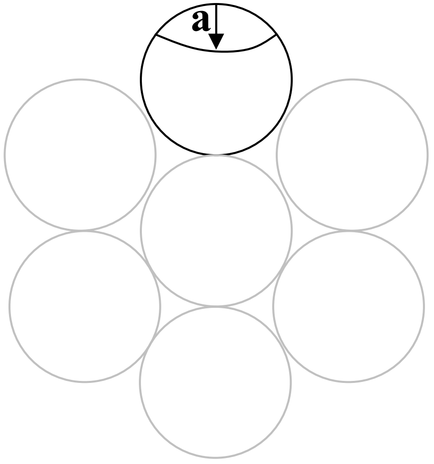
OBJECTIVES

From the testing results and the behaviour of the material, determine:

a) The failure sequence of the wires as well as their form of failure and the fracture toughness of the material.

b) Fatigue behaviour of the material considering a Paris exponent of 2.4 and the depth of the initial defect of wire A.



Consider for the wire geometry that $K_I = 2.12 \cdot \sigma \cdot a^{0.5}$

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The area of each wire is:

$$A_w = \pi \cdot r^2 = \pi \cdot 2.5^2 = 19.635 \text{ mm}^2$$

Therefore, the area of the strand is

$$A = 7 \cdot A_w = 7 \cdot 19.635 = 137.44 \text{ mm}^2$$

The failure load is 256.2 kN, so the failure stress can be calculated:

$$\sigma_f = 256200/137.44 = 1864 \text{ MPa}$$

The maximum and minimum stresses are:

$$\sigma_{max} = 0.8 \cdot \sigma_f = 0.8 \cdot 1864 = 1491.2 \text{ MPa}$$

$$\sigma_{min} = 0.8 \cdot \sigma_f - 390 = 1491.2 - 390 = 1101.2 \text{ MPa}$$

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Under these conditions, the first failure occurs in the external wire A because it has the bigger propagated defect. After that, the external wire B breaks and finally, the central one.

The external wire A breaks because of a fracture failure as a consequence of a fatigue process. Then:

$K_I = 2.12 \cdot \sigma \cdot a_c^{0.5} = 2.12 \cdot 1491.2 \cdot (0.00132)^{0.5} = 114.9 \ MPa \cdot m^{1/2}$

$\underline{K_{IC} = 114.9 \ MPa \cdot m^{1/2}}$

ANALYSIS

The external wire B also fails because of fracture, but with a smaller defect because the decrease of the section once the external wire A is broken. The new supported σ_{max} is:

$$\sigma_{max} = 7/6 \cdot 1491.2 = 1739.7 \text{ MPa}$$

This stress is smaller than the failure strength of the strand (1864 MPa). Therefore, failure happens as a consequence of sudden fracture or plastic collapse. This later as the applied stress (1739.7 MPa) is close to yield stress (even non considering a possible strain hardening effect, then $\sigma_{ymax} = \sigma_u = 1864$ MPa)

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We can calculate the stress intensity factor:

$$K_I = 2.12 \cdot \sigma \cdot a_c^{\ 0.5} = 2.12 \cdot 1739.7 \cdot (0.0012)^{0.5} \longrightarrow K_I = 127.76 \text{ MPa} \cdot \text{m}^{1/2}$$

This value is bigger than K_{IC} and it justifies the sudden failure of the external wire B. Now, there are only five wires in the section of the strand, so:

$$\sigma_{max} = 6/5 \cdot 1739.7 = 2087.6 \text{ MPa}$$

This stress is bigger than the failure stress of the cord (1864 MPa). Therefore, it is possible to affirm that the latest is the maximum stress in the strand, and the central wire fails because of tension. In effect, the necessary stress for fracture to occur would be:

$$114.9 = 2.12 \cdot \sigma \cdot 0.00025^{\ 0.5} \longrightarrow \sigma = 3428 \text{ MPa}$$

Such a value is not reached at any time.

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Let´s now determine the fatigue properties of the material. We know that the central wire had a defect of 0.25 mm which did not produce crack propagation under $\Delta\sigma = 390$ MPa. So:

$$\Delta K_{th} > 2.12 \cdot \Delta\sigma \cdot a^{0.5} = 2.12 \cdot 390 \cdot 0.00025^{0.5} = 13.07 \text{ MPa} \cdot \text{m}^{1/2}$$

We also know that an initial defect of 0.3 mm propagates in the external wire B and that the unknown initial defect of wire A should be higher than 0.3 because it reached a bigger final crack. So:

$$\Delta K_{th} < 2.12 \cdot \Delta\sigma \cdot a^{0.5} = 2.12 \cdot 390 \cdot 0.0003^{0.5} = 14.32 \text{ MPa} \cdot \text{m}^{1/2}$$

From both expressions the threshold SIF is limited from the following values:

$$\underline{13.07 < \Delta K_{th} < 14.32 \text{ MPa} \cdot \text{m}^{1/2}}$$

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The Paris law is:

$$\frac{da}{dN} = C\cdot(\Delta K)^{2.4} = C\cdot(2.12\cdot\Delta\sigma\cdot\sqrt{a})^{2.4}$$ where C has to be defined

We know: $\Delta\sigma = 390$ MPa

So: $$\frac{da}{dN} = C\cdot(\Delta K)^{2.4} = C\cdot(2.12\cdot 390\cdot\sqrt{a})^{2.4}$$

$$\frac{da}{a^{1.2}} = C\cdot(2.12\cdot 390)^{2.4}\cdot dN$$

$$a_i^{-0.2} - a_f^{-0.2} = C\cdot 2.008\cdot 10^6\cdot N \quad \left\{\begin{array}{l} a_i = 0.3\ \text{mm} \\ a_f = 1.2\ \text{mm} \\ N = 320000\ \text{cycles} \end{array}\right\} \text{conditions at wire B}$$

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$$C = \frac{0.0003^{-0.2} - 0.0012^{-0.2}}{2.008 \cdot 10^{6} \cdot 320000} = 1.909 \cdot 10^{-12}$$

Therefore, the Paris law is:

$$\frac{da}{dN} = 1.909 \cdot 10^{6} \cdot (\Delta K)^{2.4}$$

To calculate the initial defect in the external wire A, we will integrate the Paris law:

$$a_i^{-0.2} - a_f^{-0.2} = C \cdot 2.008 \cdot 10^{6} \cdot N \quad \left\{ \begin{array}{l} C = 1.909 \cdot 10^{\ 12} \\ a_f = 1.32 \text{ mm} \\ N = 320000 \text{ cycles} \end{array} \right.$$

$$a_i^{-0.2} - 0.00132^{-0.2} = C \cdot 2.008 \cdot 10^{6} \cdot N = 1.22665 \longrightarrow \underline{a_i = 0.3223 \text{ mm}}$$

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