



D. EXAMPLES



VALIDATION EXAMPLE

Planar Wide Plates

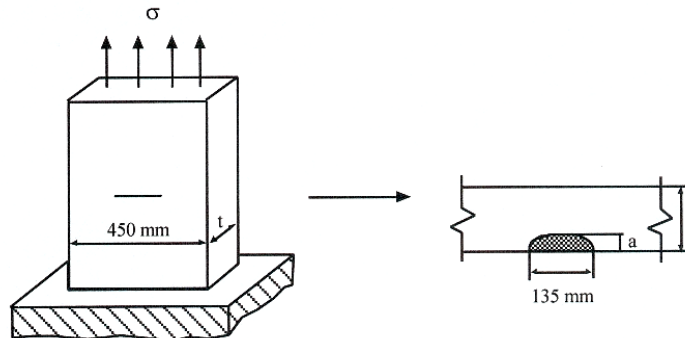
- **Introduction**
- **Geometry and Input Data**
- **Materials**
- **Toughness**
- **Formulation and Calculus**
- **Diagrams**
- **Results**
- **Analysis**
- **Bibliography/References**



INTRODUCTION

- **Description: 7 wide plates with different Y/T ratio**
- **Defect: Semi-elliptical Finite Surface Crack**
- **Different Quality in Tensile Data**
- **Different Toughness Data: Charpy and CTOD**
- **Different Crack Sizes (Nominal and Real Values)**
- **Calculation of Critical Stress for a given Crack**
- **Total: 63 calculations**
- **Experimental Values Available. Evaluation of Reserve Factors**

GEOMETRY AND INPUT DATA

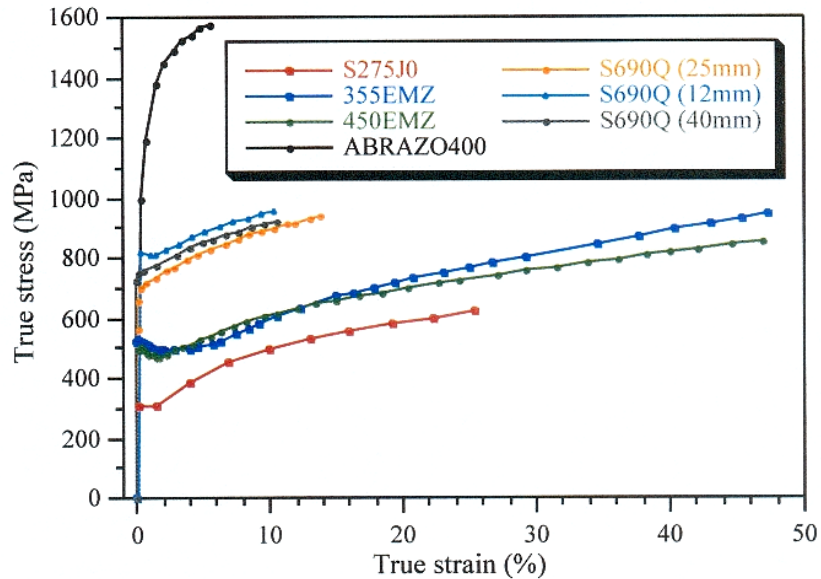


Geometry

Plate	1	2	3	4	5	6	7
Material	S275J0	355EMZ	450EMZ	S690Q	ABR.400	S690Q	S690Q
Thickness (mm)	25	25	25	25	25	12	40
R_{el} or $R_{p0.2}$ (MPa)	303	436	471	713	991	820	746
R_m (MPa)	467	548	565	792	1408	864	859
$LYS/UTS \equiv Y/T$	0.649	0.796	0.834	0.900	0.704	0.949	0.868
Type of curve	Discont.	Discont.	Discont.	Contin.	Contin.	Discont.	Contin.
N (measured)	0.231	0.282	0.151	0.092	0.157	0.071	0.068
Charpy T27J (°C)	-65	-115	-115	-50	-45	-85	-85
Charpy at -20°C (J)	70	220	>250	100	35	180	170
CTOD at -20°C (mm)	0.974	0.765	1.450	0.083	0.022	0.140	0.235
CTOD R-curve (mm)	$1.03\Delta a^{0.50}$	$1.04\Delta a^{0.63}$	$1.31\Delta a^{0.71}$	-	-	-	$0.44\Delta a^{0.62}$

Input Data

MATERIALS



Stress-Strain Curves

Plate	1	2	3	4	5	6	7
R_F (MPa)	385	492	518	752.5	1199.5	842	802.5
$L_{r \max}$	1.271	1.128	1.100	1.055	1.210	1.027	1.076
$\Delta\epsilon$	0.0261	0.0212	0.0198	-	-	0.0068	-
μ	-	-	-	0.295	0.212	-	0.282
λ	19.12	11.19	9.84	-	-	2.73	-
N	0.105	0.061	0.050	0.030	0.089	0.015	0.040
$L_{r \max}^{\text{est}}$	-	-	-	1.020	1.009	-	1.018

**Failure Assessment Diagram Parameters
Derived from Tensile Data**

TOUGHNESS

Plate	1	2	3	4	5	6	7
$l=2c$ (mm)	135	135	135	135	135	135	135
t (mm)	25	25	25	25	25	12	40
K_{mat25} (MPa·m ^{1/2})	-	-	-	-	71.0	-	-
P_f	-	-	-	-	0.05	-	-
$K_{mat} 1^*$ (MPa·m ^{1/2}) ⁽¹⁾	-	-	-	-	53.5	-	-
$K_{mat} 1^{**}$ (MPa·m ^{1/2}) ⁽²⁾	-	-	-	-	59.9	-	-
$K_{mat} 1$ (MPa·m ^{1/2}) ⁽³⁾	106.5	189.8	201.9	128.1	-	172.1	167.3
$K_{mat} 2$ (MPa·m ^{1/2}) ⁽⁴⁾	319.6	339.8	486.2	143.1	86.9	199.3	246.3

Calculated Toughness Values

- (1) Estimated from SINTAP lower bound, lower shelf correlation.
 (2) Estimated from Master Curve with failure probability = 0.05.
 (3) Estimated from upper shelf Charpy correlation.
 (4) Estimated from relationship between K_{mat} and CTOD.

FORMULATION AND CALCULUS

Plate	1	2	3	4	5	6	7
a real (mm)	5	6.5	6.8	5.5	5	3.1	8.4
a nominal (mm)	-	5	5	5	-	-	-

Crack Dimensions Considered

L_r (AII.42)

$$L_r = \frac{\sigma}{(1-\zeta)\sigma_Y}$$

where

$$\zeta = \frac{al}{t(l+2t)}$$

K_I (AI.3) -deepest point of the crack-

$$K_I = \sigma f_0 \sqrt{\pi a}$$

(Linear interpolation has been used for the determination of f_0)

Plate	1	2	3	4	5	6	7
ζ (real a)	0.1459	0.1897	0.1985	0.1605	0.1459	0.2193	0.1319
f_0 (real a)	1.2151	1.2904	1.3026	1.2421	1.2151	1.3991	1.1753
ζ (nominal a)	-	0.1459	0.1459	0.1459	-	-	-
f_0 (nominal a)	-	1.2151	1.2151	1.2151	-	-	-

ζ and f_0 Values for each Plate and Crack Combination

DIAGRAMS

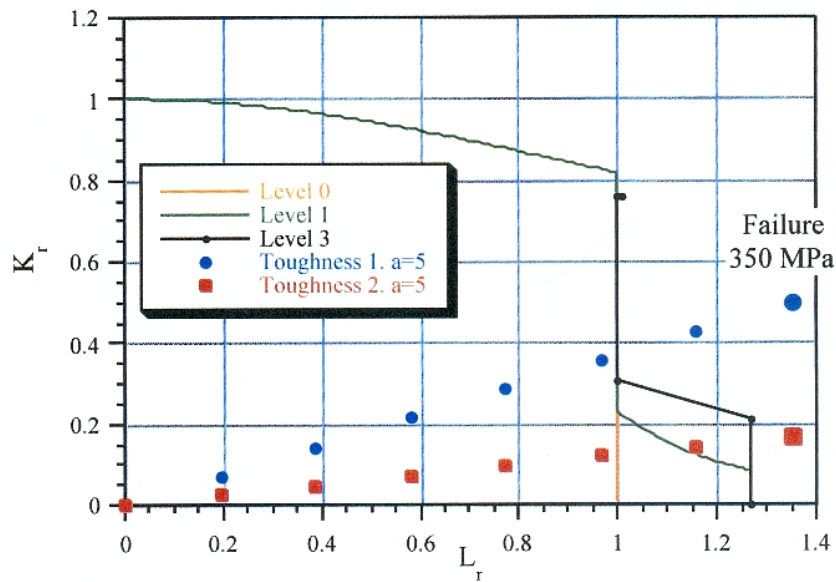


Plate 1

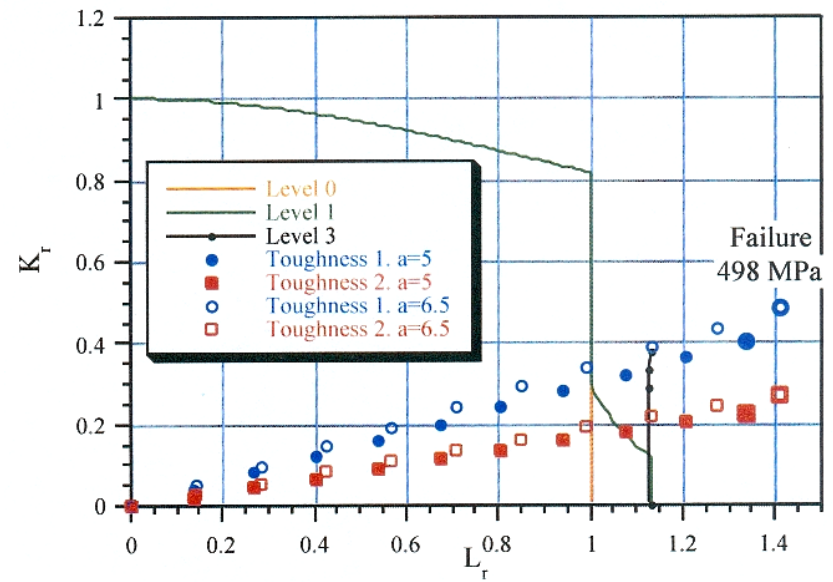


Plate 2

DIAGRAMS

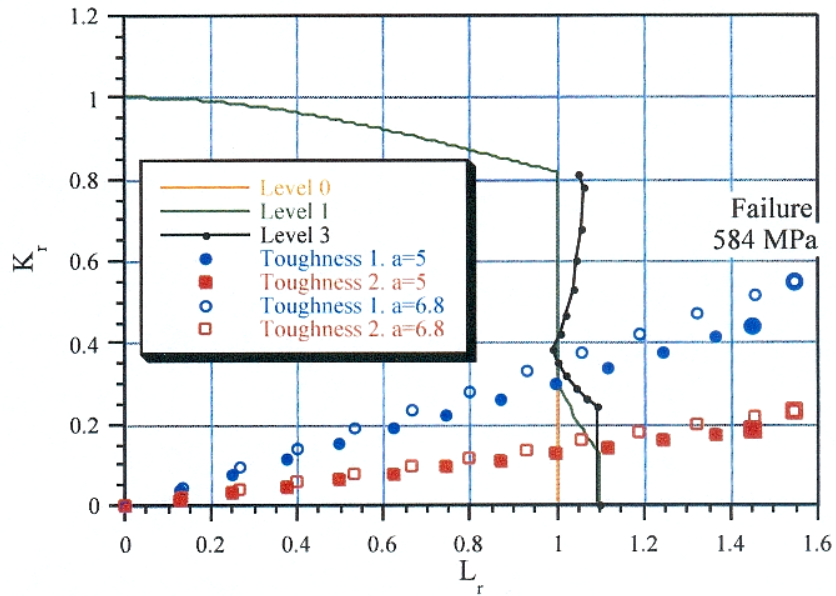


Plate 3

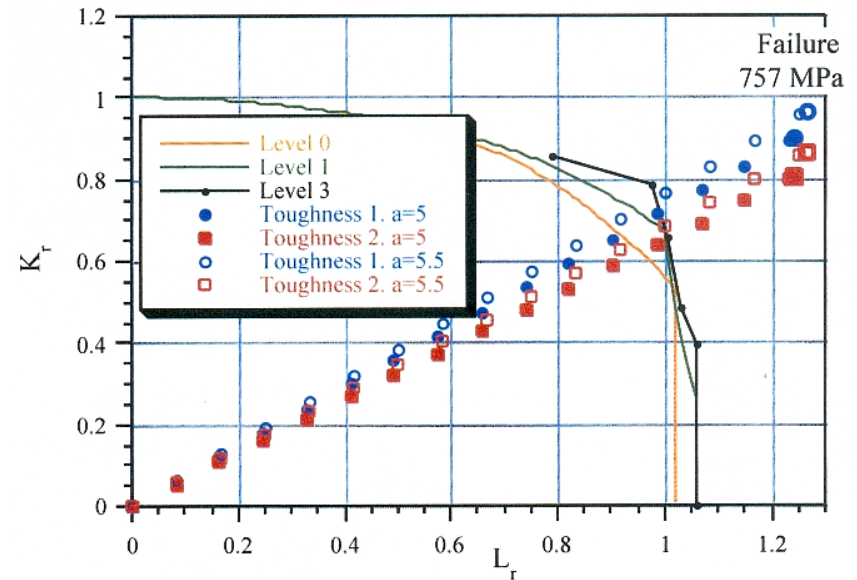


Plate 4

DIAGRAMS

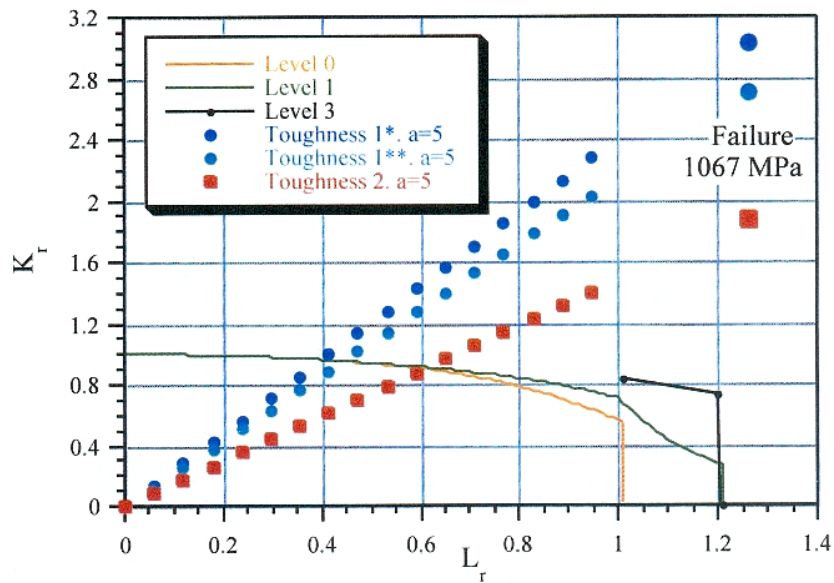


Plate 5

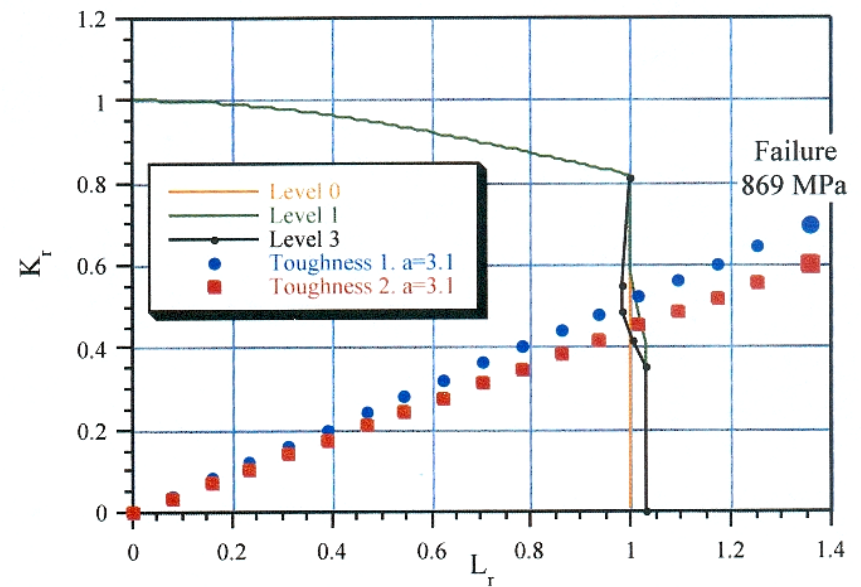


Plate 6

DIAGRAMS

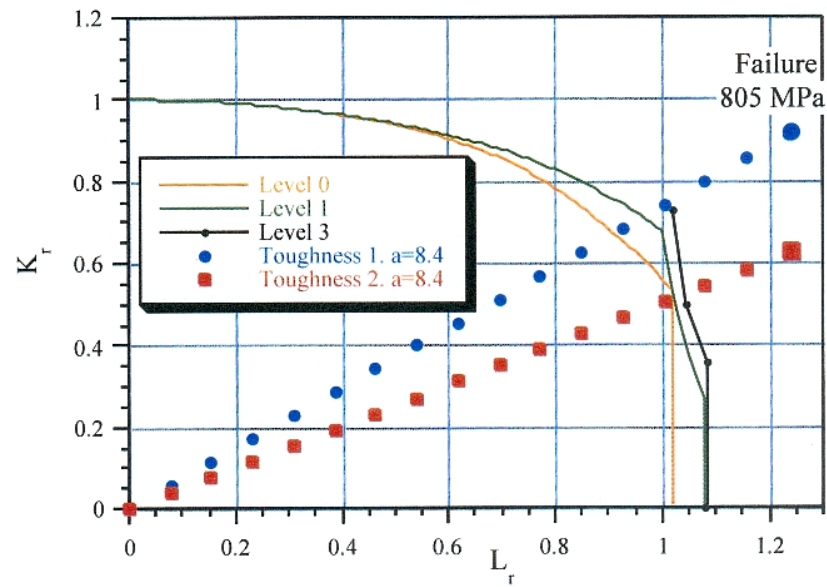


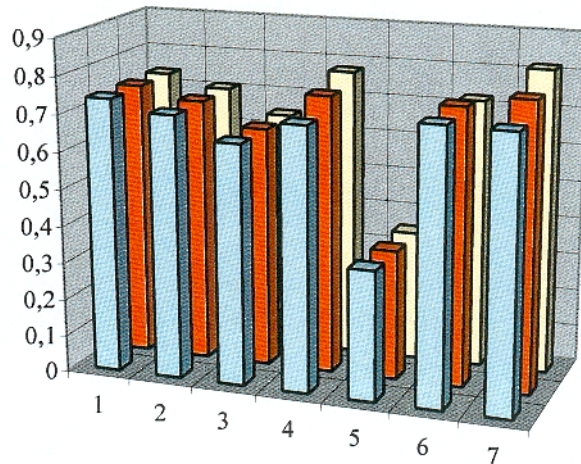
Plate 7

RESULTS

Plate	Crack size	Toughness	Failure Stress (MPa)			
			FAD 0	FAD 1	FAD 3	Experimental
1	5	1	259	259	259	350
		2	259	291	329	
2	5	1	372	372	419	498
		2	372	398	420	
	6.5	1	353	353	398	
		2	353	371	398	
3	5	1	402	404	412	584
		2	402	440	441	
	6.8	1	378	378	378	
		2	378	406	413	
4	5	1	558	589	604	757
		2	581	610	613	
	5.5	1	537	566	589	
		2	560	594	599	
5	5	1*	337	338	>338	1067
		1**	374	375	>375	
		2	513	519	>519	
6	3.1	1	640	647	629	869
		2	640	654	638	
7	8.4	1	589	623	≈660	805
		2	659	663	674	

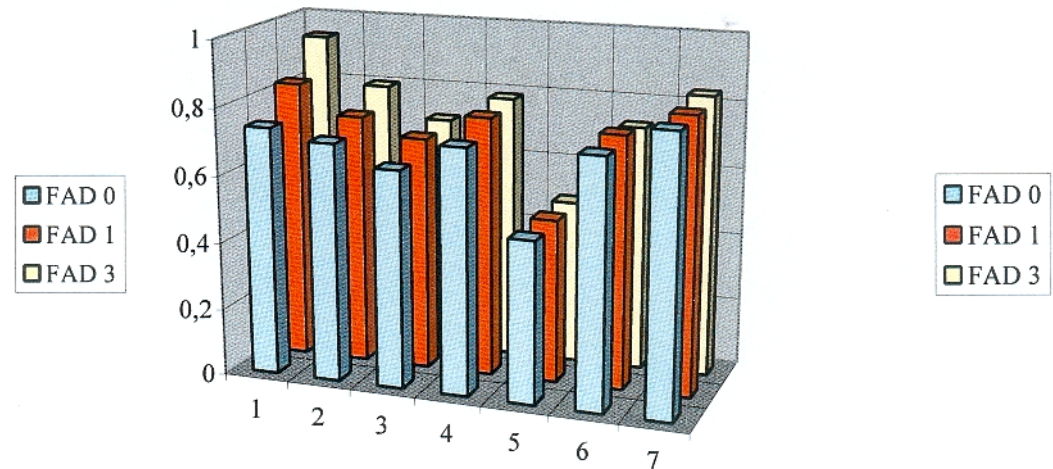
ANALYSIS

Predicted / Actual



Real Cracks, Charpy Toughness

Predicted / Actual



Real Cracks, CTOD Toughness



BIBLIOGRAPHY / REFERENCES

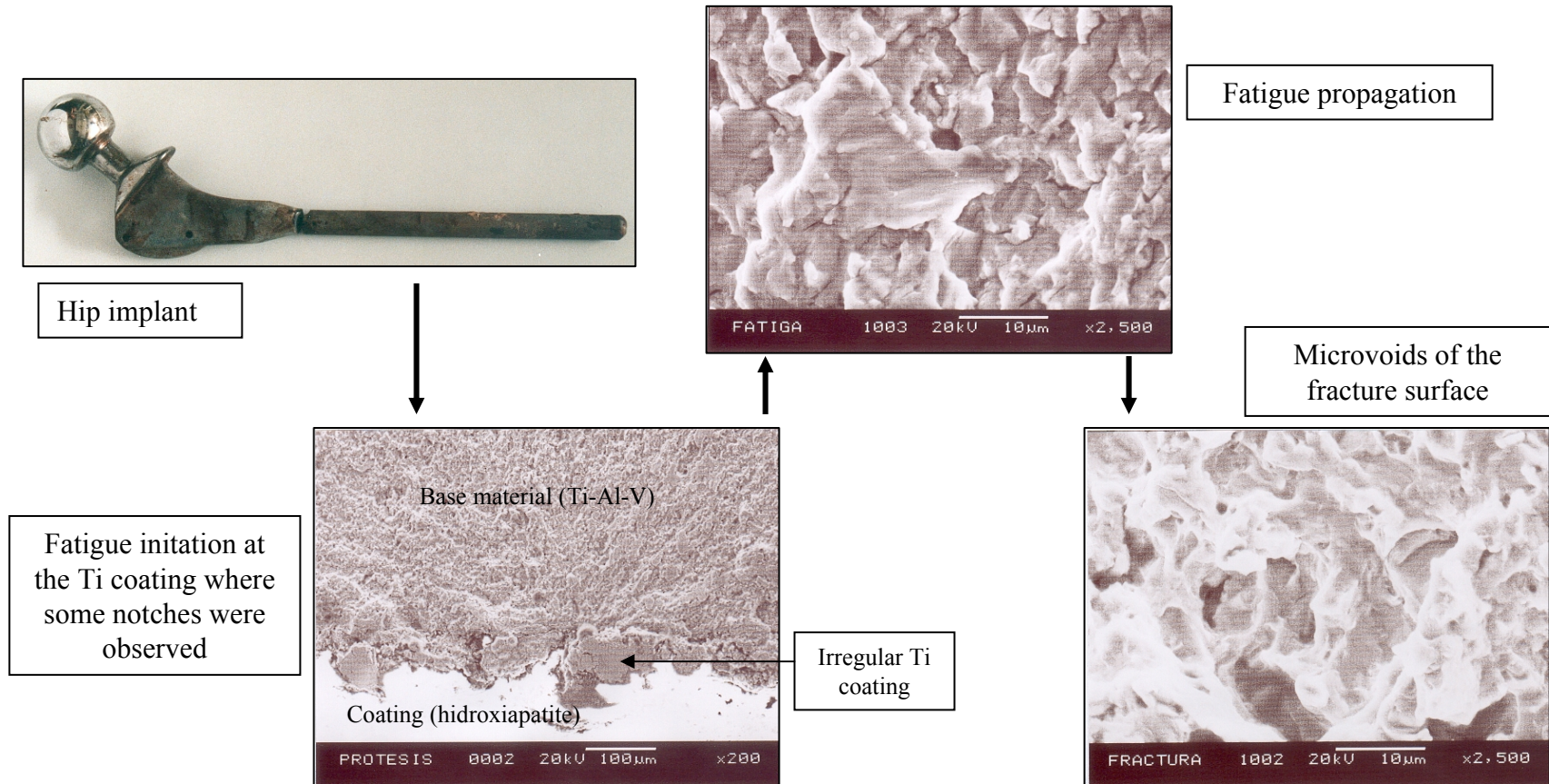
- Ruiz Ocejó J. and Gutiérrez-Solana F., “SINTAP Validation Report”, June 1999

CASE STUDY EXAMPLE

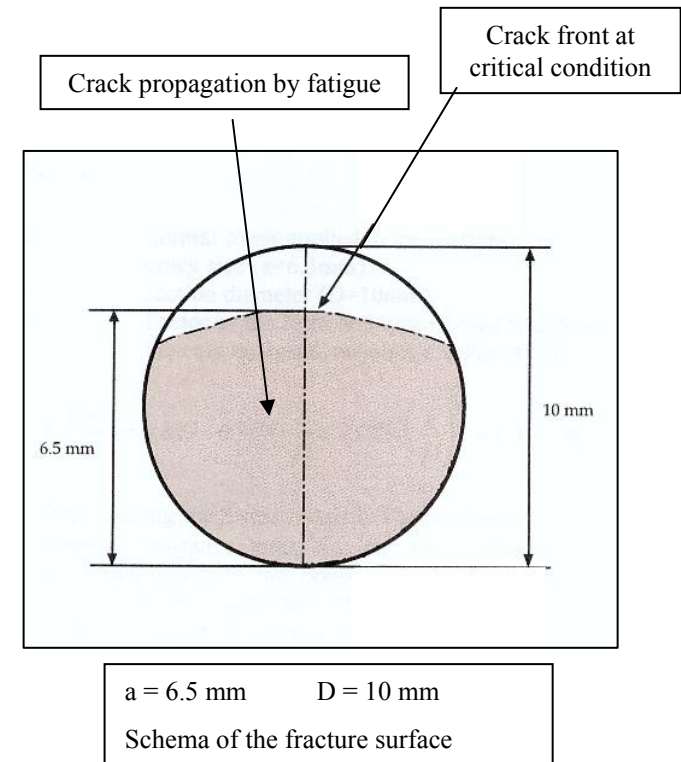
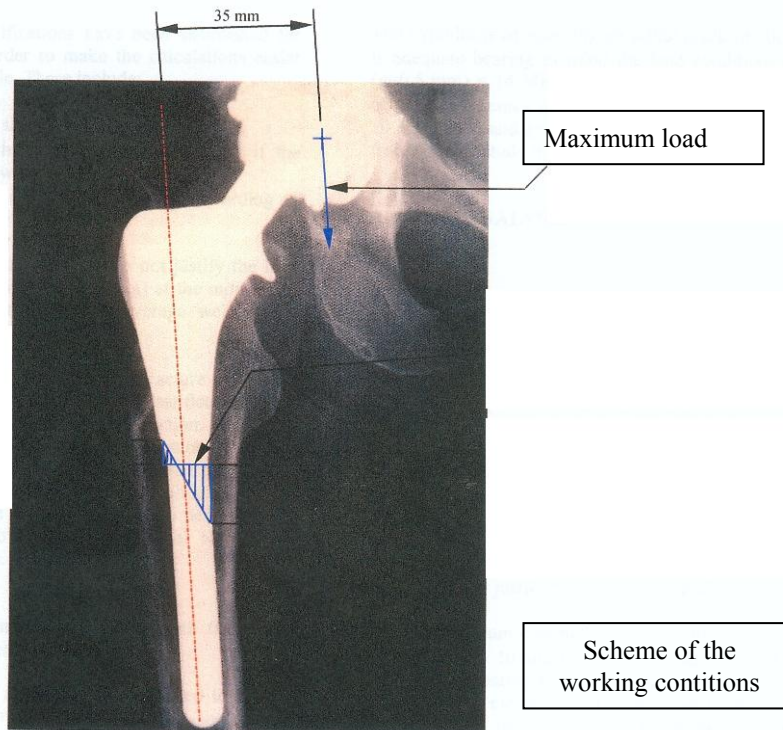
Hip Implant

- **Introduction: The Case Study**
- **Geometry**
- **Material Properties**
- **Objectives**
- **Failure Analysis**
- **Summary**

INTRODUCTION: THE CASE STUDY



GEOMETRY



MATERIAL PROPERTIES

$$K_{IC} = 110 \text{ MPa}\cdot\text{m}^{1/2}$$

$$\sigma_Y = 895 \text{ MPa}$$

$$\sigma_u = 1000 \text{ MPa}$$

$$E = 114 \text{ GPa}$$

$$da/dN = 3.54 \cdot 10^{-14} \cdot (\Delta K)^{4.19}$$

when ΔK is given in $\text{MPa}\cdot\text{m}^{0.5}$ and
 da/dN in m/cycles



OBJECTIVES:

- **FAILURE ANALYSIS**
- **NUMBER OF CYCLES BEFORE FAILURE CONSIDERING AN INITIAL DEFFECT OF 0.1 mm.**

FAILURE ANALYSIS:

DETERMINATION OF THE LOAD SUPPORTED AS A FUNCTION OF THE FRACTURE PARAMETERS:

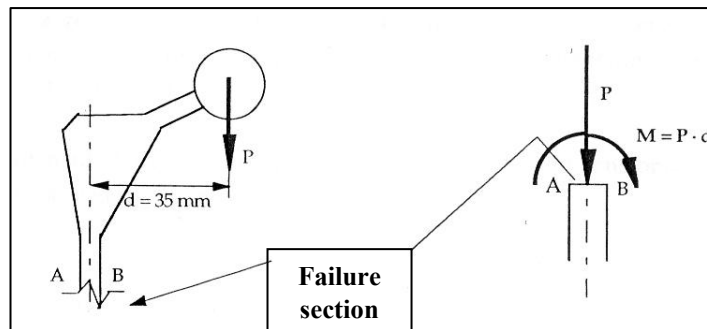
stress state = compression + pure bend

$$\sigma_{T,\max} = \sigma_F - \sigma_C$$

where:

$$\sigma_F = 32 \cdot M / \pi \cdot D^3$$

$$\sigma_C = 4 \cdot P / \pi \cdot D^2$$



FAILURE ANALYSIS:

Many studies have been developed in order to know the peak forces that appear in a hip implant when the patient is walking. A value of 2.5 BW (Body Weight) seems to be reasonable.

Three different steps are distinguished during the process that starts with the operation and finishes with the failure of the hip implant:

- ***Crack nucleation:*** It is considered very short, because there are defects at $t = 0$
- ***Quick propagation:*** We are going to consider that the patient has a “normal” activity. We will suppose that he/she walks 2 hours per day with 1 step per second (0.5 cycles/second). Peak forces are 2.5 BW.
- ***“Slow” propagation:*** After the propagation of the second step, the patient starts to suffer pain. Therefore, he/she reduces his/her activity (1 hour/day) and uses crutches. Peak forces are now 1.0 BW. Failure occurs in this step, so if we want to obtain the load that produces it, no dynamic effect has to be considered.

The whole process takes 9 months.

FAILURE ANALYSIS:

-The stress intensity factor, which characterises the stress state in the crack front, is defined by the expression:

$$K_I = \sigma \cdot Y_F(a/D) \cdot (\pi \cdot a)^{1/2}$$

where:

σ : normal stress applied to the section

a : crack size ($a = 6.5$ mm)

D : section diameter ($D = 10$ mm)

Y_F = geometric factor. In this case, this acquires a value of:

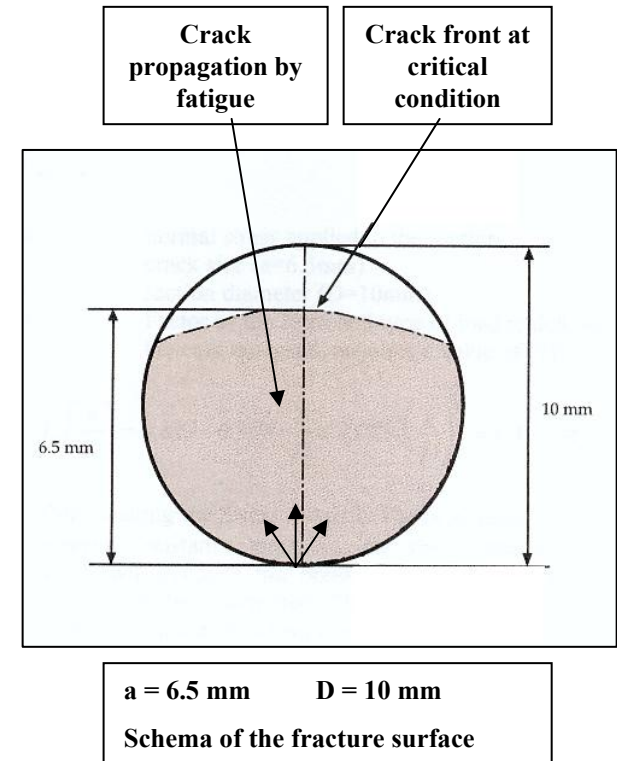
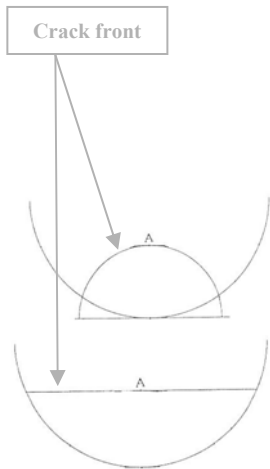
$$Y_F(a/D) = g \cdot (0.953 + 0.199 \cdot (1 - \sin(\psi))^4) = 1.41$$

$$g = 0.5857 \cdot (\tan \psi) / \psi^{0.5} / \cos \psi \quad (\text{API 579})$$

$$\psi = \pi \cdot a / 4 \cdot R$$

$$Y_F(a/D) = 1.04 - 3.64 \cdot (a/D) + 16.86 \cdot (a/D)^2 - 32.59 \cdot (a/D)^3 + 28.41 \cdot (a/D)^4 = 1.92$$

(API 579)



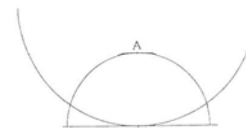
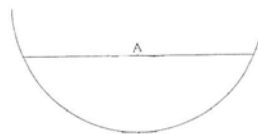
FAILURE ANALYSIS:

- CLASSIC LEFM.

Some simplifications have been established for this analysis in order to make the calculations easier and more accessible. These include:

- Working with the piece in projection
- Analysis of the stress intensity factor as if the element were working in pure bend
- Fracture toughness of the material according to reference value

$$K_I = K_{IC}$$



$$\sigma \cdot Y(a/D) \cdot (\pi \cdot 0.0065)^{1/2} = 110 \longrightarrow \sigma = 401 \text{ MPa}$$

$$\sigma = 546 \text{ MPa}$$

$$\sigma = 32 \cdot M / \pi \cdot D^3 - 4 \cdot P / \pi \cdot D^2 \longrightarrow P = 1.17 \text{ kN}$$

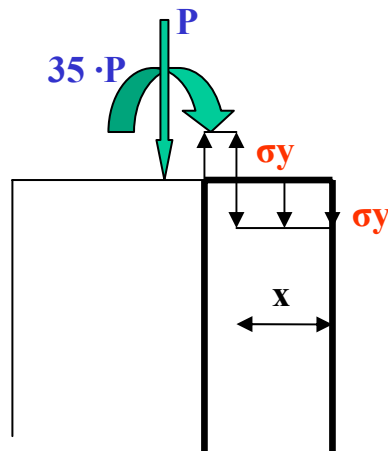
$$P = 1.66 \text{ kN}$$

However, these simplifications do not justify the high value resulting from load P (1.66 kN / 1.17 kN) at the moment of fracture, with reference to the average weight of a person (0.75 kN).

FAILURE ANALYSIS:

- LIMIT LOAD SOLUTION.

A second hypothesis of fracture has been considered: the generalised plastification of the remaining ligament in the cracked section. Therefore a FAD will be used. Considering the yield stress 895 MPa, it is obtained that the limit load is 0.56 kN for a straight front crack and 0.89 kN for a semicircular crack, much closer to the average weight of a person and in any case much lower than the critical size of the fracture hypothesis.



$$\left. \begin{aligned} \Sigma M_{\text{load}} &= \Sigma M_{\text{stress}} \\ \Sigma F_{\text{load}} &= \Sigma F_{\text{stress}} \end{aligned} \right\} \text{AT ANY POINT}$$

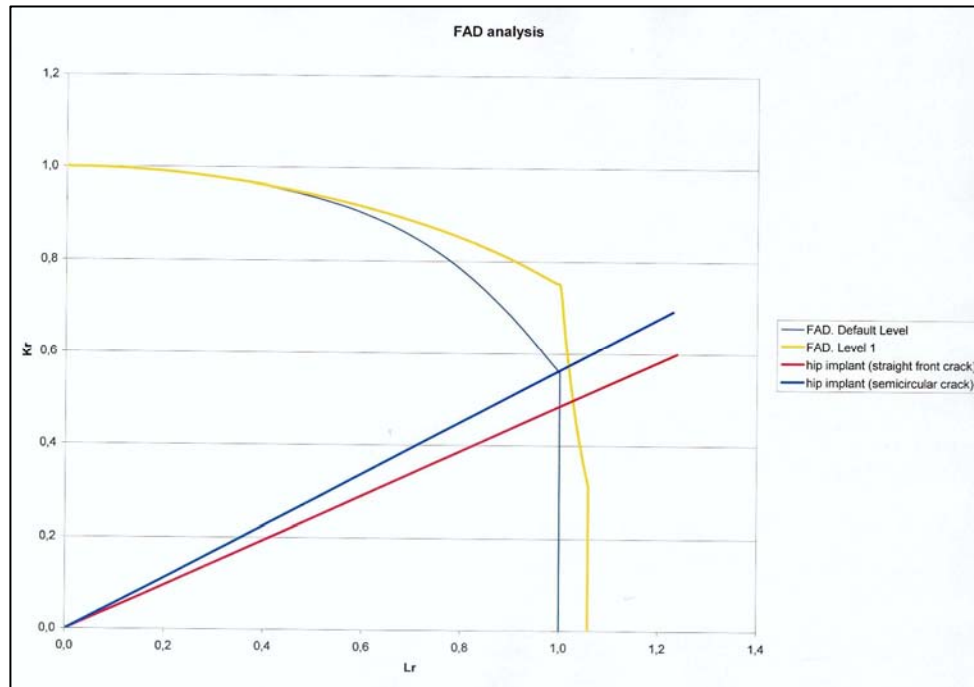
WE CAN OBTAIN **P** AND **x**.

P = 0.566 kN (straight front crack)

P = 0.895 kN (semicircular crack)

FAILURE ANALYSIS:

FAD:



Default level:	P = 0,566 kN	P = 0.895 kN
Level 1:	P = 0,582 kN	P = 0.915 kN

- 1) Loading critical conditions according to normal weight (real situation: 0.735 kN).
- 2) Final failure due to plastic collapse of residual ligament.
- 3) Good agreement with fractographic analysis and common sense.

FAILURE ANALYSIS:

DETERMINATION OF THE CRACK GROWTH TIME UNTIL CRITICAL SIZE IS REACHED:

-The fatigue crack growth time is adjusted to a Paris law, which has been taken from the bibliography and is given by equation:

$$da/dN = 3.54 \cdot 10^{-14} \cdot (\Delta K)^{4.19} \quad (1)$$

when ΔK is given in $\text{MPa}\cdot\text{m}^{0.5}$ and da/dN in m/cycle

- The load cycle to which the element is subjected varies from 0, support from the other leg or repose, up to 631.5 MPa, corresponding to the weight of 0.735 kN and peak forces of 2.5 BW. Thus the ΔK_I will have a value, depending on a , given by

$$\Delta K_I = Y_F(a/D) \cdot 631.5 \cdot (\pi \cdot a)^{1/2} \quad (2)$$

-Taking as the initial crack length $a_0 = 0.1$ mm, introducing expression (2) in (1) and integrating this, the number of cycles required for the crack to reach the critical size of 6.5 mm is obtained. The number is between 145.738 cycles (straight front crack) and 539.088 (semicircular crack).

FAILURE ANALYSIS:

DETERMINATION OF THE CRACK GROWTH TIME UNTIL CRITICAL SIZE IS REACHED:

a (mm)	a med (mm)	Y (straight)	Y (f.semic.)	ΔN (straight)	ΔN (semic.)	N (straight)	N (semic.)
0,1 - 0,5	0,30	0,945	0,660	108999	490750	108999	490750
0,5 - 1	0,75	0,849	0,644	18829	59855	127828	550605
1 - 1,5	1,25	0,792	0,635	7961	20040	135789	570645
1,5 - 2	1,75	0,771	0,635	4294	9709	140083	580354
2 - 2,5	2,25	0,776	0,643	2449	5377	142533	585731
2,5 - 3	2,75	0,799	0,661	1420	3139	143953	588870
3 - 3,5	3,25	0,836	0,689	824	1857	144777	590727
3,5 - 4	3,75	0,889	0,728	471	1089	145248	591816
4 - 4,5	4,25	0,963	0,781	259	623	145507	592438
4,5 - 5	4,75	1,069	0,852	133	343	145640	592781
5 - 5,5	5,25	1,218	0,945	62	180	145702	592961
5,5 - 6	5,75	1,431	1,071	26	88	145728	593049
6 - 6,5	6,25	1,729	1,242	10	40	145738	593088
BW = 2.5						N TOTAL	

FAILURE ANALYSIS:

DETERMINATION OF THE CRACK GROWTH TIME UNTIL CRITICAL SIZE IS REACHED:

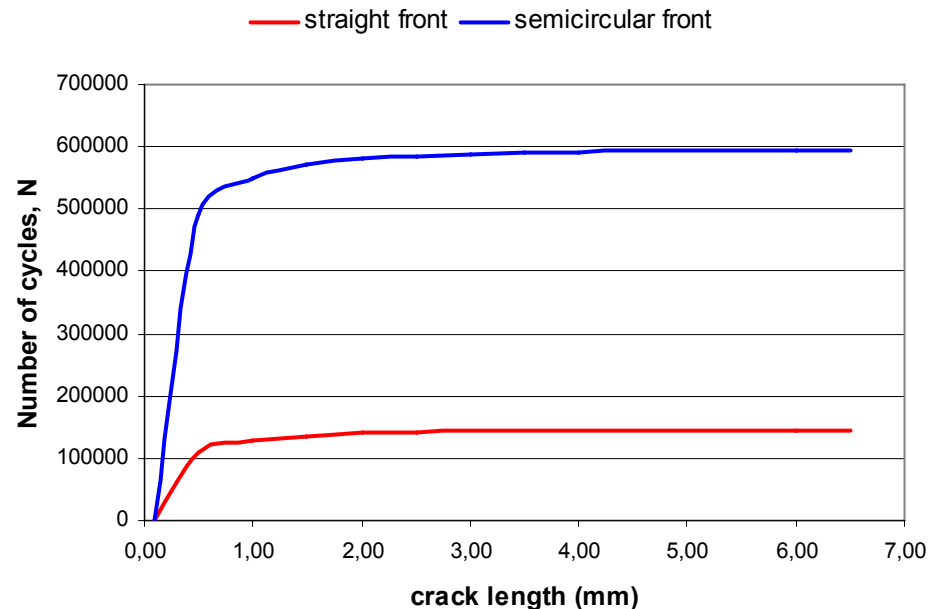
a (mm)	a med (mm)	Y (straight)	Y (f.semic.)	ΔN (straight)	Δ N (semic.)	N (straight)	N (semic.)
0,1 - 0,5	0,30	0,945	0,660	5067464	22815433	5067464	22815433
0,5 - 1	0,75	0,849	0,644	667745	2122708	5735208	24938140
1 - 1,5	1,25	0,792	0,635	291380	733428	6026589	25671568
1,5 - 2	1,75	0,771	0,635	158425	358205	6185014	26029773
2 - 2,5	2,25	0,776	0,643	90659	199035	6275672	26228808
2,5 - 3	2,75	0,799	0,661	52641	116360	6328313	26345168
3 - 3,5	3,25	0,836	0,689	30590	68903	6358903	26414071
3,5 - 4	3,75	0,889	0,728	17494	40417	6376397	26454488
4 - 4,5	4,25	0,963	0,781	9609	23124	6386006	26477612
4,5 - 5	4,75	1,069	0,852	4930	12740	6390936	26490352
5 - 5,5	5,25	1,218	0,945	2305	6673	6393241	26497025
5,5 - 6	5,75	1,431	1,071	970	3275	6394211	26500299
6 - 6,5	6,25	1,729	1,242	369	1478	6394580	26501777
BW = 1.0						N TOTAL	

FAILURE ANALYSIS:

DETERMINATION OF THE CRACK GROWTH TIME UNTIL CRITICAL SIZE IS REACHED.

INTERPRETATION OF RESULTS:

According with the conditions proposed for “normal” life (2.5 BW), the cycles obtained represent between 1.3 and 4.6 months of quick propagation before failure, depending on the crack front shape. However, the propagation under these conditions finished a few thousands of cycles before, when the patient starts to feel pain and, then, a new stage starts under new loading conditions (1.0 BW). The Figure shows that wherever the quick propagation finishes, it takes around 140000 cycles in case the crack front is straight or 500000 cycles in case the crack front is semicircular.



FAILURE ANALYSIS:

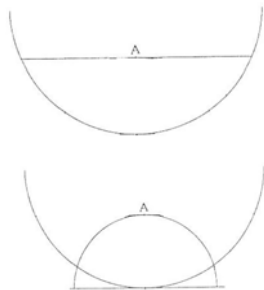
DETERMINATION OF THE CRACK GROWTH TIME UNTIL CRITICAL SIZE IS REACHED.

INTERPRETATION OF RESULTS:

Considering that there is no nucleation time due to the notch effect and adding a quick propagation step of 1.3 months (equivalent to near 140.000 cycles) for a straight front crack and 4.6 months (equivalent to near 500.000 cycles) for a semicircular crack, the duration of the final stage (BW=1.0) can be obtained. This is 7.7 months for a straight front and 4.4 months for a semicircular front. This is equivalent to 415.800 and 237.600 cycles respectively. If we start to count the cycles from the end to the beginning of the process, we obtain that such numbers are the amount of cycles that are necessary for a growth from 1.5 mm to 6.5 mm (straight) or from 2.0 mm to 6.5 mm (semicircular). As a summary, a fatigue process can be suggested as follows:

- No crack nucleation, as initial notches of 0.1 mm have been detected.
- STAGE 1: Propagation with dynamic effects, from 0.1 mm to a value between 1.5 and 2.0 mm. Taking mean values, this would take about 3 months (between 1.3 and 4.6).
- STAGE 2: Propagation without dynamic effects. This takes the rest of the implant life (an average of 6 months).

SUMMARY:



	Incubation	Quick propagation	Propagation without dynamic effect
STRAIGHT FRONT CRACK	0 months	1.3 months/ 1.5 mm	7.7 months/ 6.5 mm
SEMICIRCULAR CRACK	0 months	4.5 months/ 2.0 mm	4.5 months/ 6.5 mm



CASE STUDY EXAMPLE

Forklift

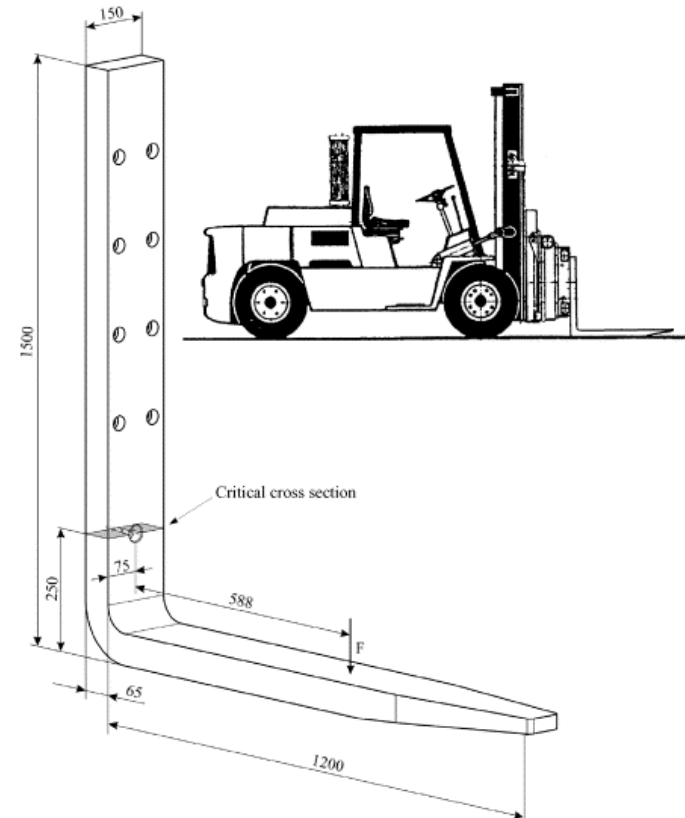
- **Introduction: The Case Study**
- **Geometry**
- **Material Properties**
- **Failure Analysis**
- **Conclusions**
- **Bibliography**

INTRODUCTION: THE CASE STUDY

A fork of a forklift broke in a brittle manner during transportation of an aluminium block of a weight of less than 3.5 tonnes, while the load carrying capacity the load was designed for is 3.5 tonnes.

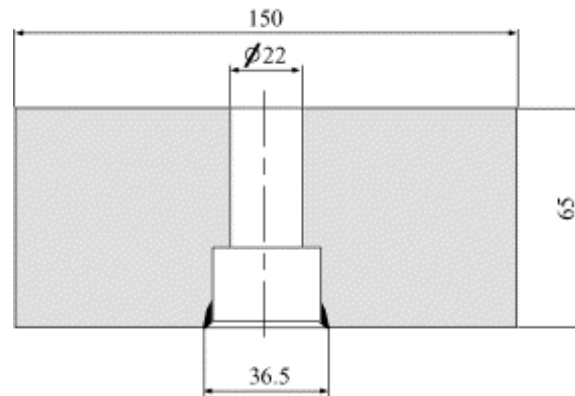
The failure happened at a temperature of 10°C

The aim of the present investigation is to figure out whether failure had to be expected for nominal loading and material conditions or if any other reason such as overloading or deficient material properties were the reason of failure.



GEOMETRY

The dimensions of the relevant cross section where fracture occurred are shown in the figure.



Failure analysis revealed that failure occurred at the bottom hole originating from small edge cracks at the front face at either side of the hole. The crack lengths at surface were 3 and 10 mm respectively.

MATERIAL PROPERTIES

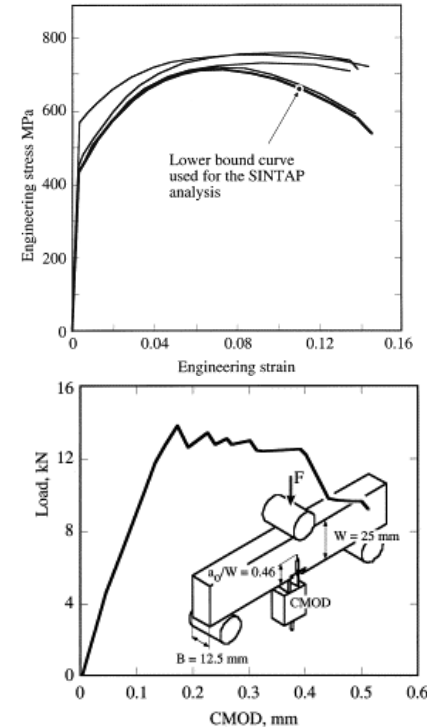
The engineering stress-strain curve of the material is shown in the figure. Five tests were carried out but only the lowest curve was used for the analysis. The true stress-strain curve are determined by:

$$\varepsilon_{\text{true}} = \ln (1 + \varepsilon) \text{ and } \sigma_{\text{true}} = \sigma (1 + \varepsilon).$$

The fracture toughness was determined in terms of the CTOD according to the BS 7448. The result was $\delta_c = 0.02$ mm, corresponding to $K_{\text{mat}} = 49.7 \text{ MPam}^{1/2}$.

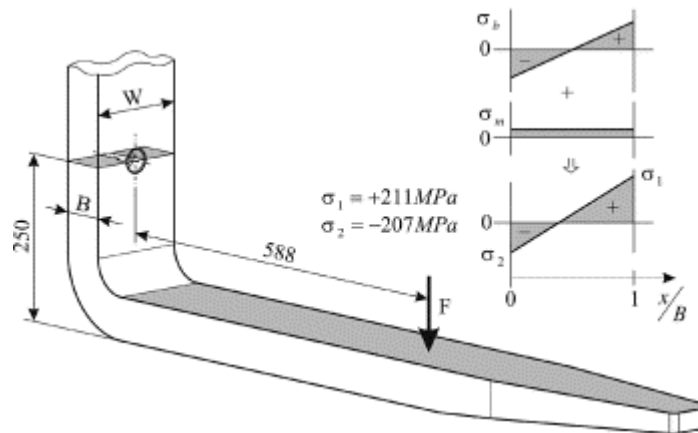
Charpy tests were performed as well. The results were:

Charpy impact toughness J/80 mm ²		
+10 °C	+20 °C	+50 °C
6, 6, 6,	7, 6, 7	9, 8, 9



FAILURE ANALYSIS (I)

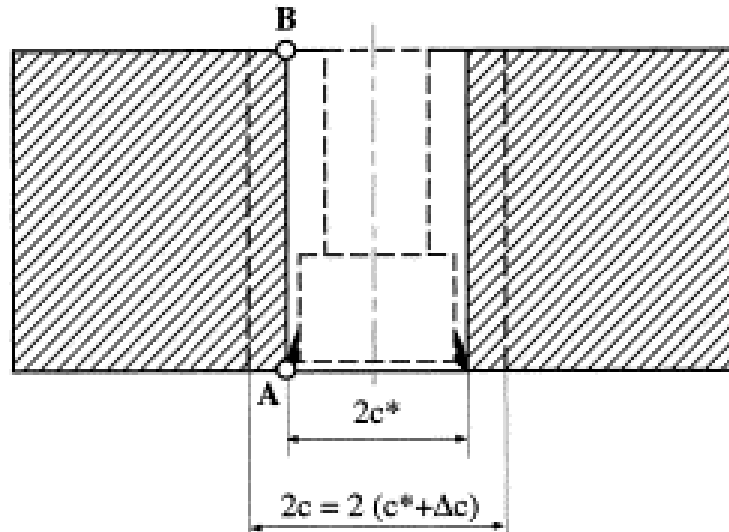
The loading type was predominantly bending, which would have allowed for the application of a simple analytical model for determining the bending stress. However, in order to consider also the membrane stress component, a finite element analysis was carried out, which gave the stress profile shown in the figure.



Based on this information $\sigma_b = 209 \text{ MPa}$ and $\sigma_m = 2 \text{ MPa}$ were determined. These values refer to one half of the nominal applied force of 35 kN , which the fork lift was designed for.

FAILURE ANALYSIS (II)

The two edge cracks are substituted by one through crack whose dimensions include the hole diameter as demonstrated in the figure. For simplicity the crack is assumed to be of constant length $2c$ over the wall thickness.



FAILURE ANALYSIS (III)

FAD analysis require the obtainment of parameters L_r and K_r . Here is the SINTAP formulation for the case studied:

$$L_r = F/F_Y = \sigma_{\text{ref}} / \sigma_Y$$

$$\sigma_{\text{ref}} = \frac{1}{1 - (2c/W)} \left\{ \frac{\sigma_b}{3} + \sqrt{\frac{\sigma_b^2}{9} + \sigma_m^2} \right\}$$

$$K_r = K_I / K_C$$

$$K_I(c, F) = \sqrt{\pi c} (\sigma_m \cdot f_m + \sigma_b \cdot f_b).$$

$f_m^A = 1$ and $f_b^A = 1$ for point A and $f_m^B = 1$ and $f_b^B = -1$ for point B

FAILURE ANALYSIS (IV)

Default, Basic and Advanced level can be performed.

DEFAULT level formulation:

$$f(L_r) = \left[1 + \frac{1}{2} L_r^2 \right]^{-1/2} \times \left[0.3 + 0.7 \exp \left(-0.6 L_r^6 \right) \right] \quad \text{for } 0 \leq L_r \leq L_{r \max},$$

$$L_{r \max} = 1 + \left[\frac{150}{R_{p0.2}} \right]^{2.5}, \quad R_{p0.2} \text{ in MPa.}$$

$$K_{\text{clst}} = \left[\left(12 \sqrt{KV} - 20 \right) \times \left(25/B \right)^{1/4} \right] + 20$$

FAILURE ANALYSIS (V)

BASIC level formulation:

$$f(L_r) = \left[1 + \frac{1}{2} L_r^2 \right]^{-1/2} \times \left[0.3 + 0.7 \exp \left(-\mu L_r^6 \right) \right] \quad \text{for } 0 \leq L_r \leq 1$$

$$\mu = \min \left[\begin{array}{l} 0.001 E/R_{p0.2} \\ 0.6 \end{array} \right]$$

$$f(L_r) = f(L_r = 1) \times L_r^{(N-1)/2N} \quad \text{for } 1 \leq L_r < L_{r \max}$$

$$N = 0.3 \left[1 - \frac{R_{p0.2}}{R_m} \right]$$

$$L_{r \max} = \frac{1}{2} \left[\frac{R_{p0.2} + R_m}{R_{p0.2}} \right]$$

FAILURE ANALYSIS (VI)

ADVANCED level formulation:

$$f'(L_t) = \left[\frac{E \varepsilon_{\text{ref}}}{\sigma_{\text{ref}}} + \frac{1}{2} \frac{L_t^2}{(E \varepsilon_{\text{ref}} / \sigma_{\text{ref}})} \right]^{-1/2} \quad \text{for } 0 \leq L_t \leq L_{t \max}$$

$$L_{t \max} = \frac{\sigma_f}{\sigma_Y} \text{ with } \sigma_f = \frac{1}{2} (\sigma_Y + R_m).$$

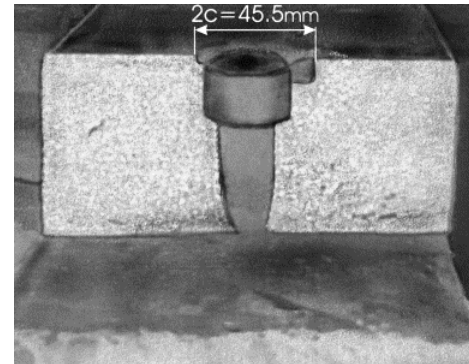
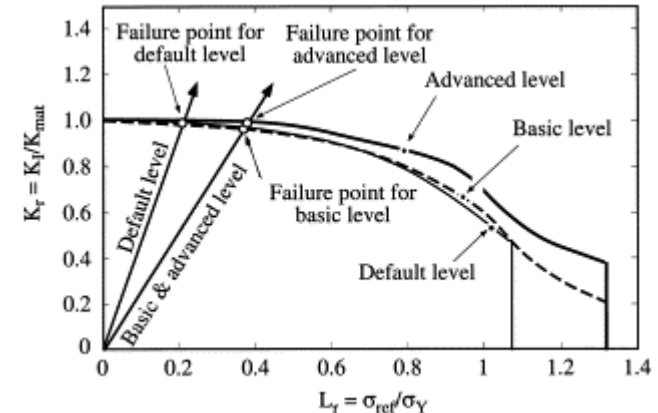
FAILURE ANALYSIS (VII)

As the final result the critical crack size was determined to be

- $2c = 10.35$ mm (default level analysis)
- $2c = 33.2$ mm (basic level analysis)
- $2c = 35.6$ mm (advanced level analysis).

Compared to the real overall surface dimension of the edge cracks at failure of 45.5 mm the predictions were conservative by

- 77.28% (default level analysis)
- 27.03% (basic level analysis)
- 21.75% (advanced level analysis)





CONCLUSIONS

In conclusion, it can be stated that the failure occurred as the consequence of inadequate design and not of inadmissible handling such as overloading. The failure could have been avoided by applying fracture mechanics in the design stage. The SINTAP algorithm was shown to be an easy but suitable tool for this purpose



BIBLIOGRAPHY

Gubeljak, N., Zerbst, U., Predan, J., and Oblak, M., “Application of the european SINTAP procedure to the failure analysis of a broken forklift”, Engineering Failure Analysis, Vol. 11, pp. 33-47, 2004