Annex A

Stress intensity factor (SIF) solutions

A.1 Introduction

The estimation of applied opening mode (mode I) stress intensity factor, K_I, for a given crack/component geometry is a critical input to a fitness-for-service calculation. Several handbooks of K-solutions have been published for a range of geometries and loading configurations, and may be used directly at the discretion of the user, eg [A.1][A.2][A.3][A.4]. The most widely-used solutions are also published in existing FFS procedures, eg [A.5][A.6][A.7][A.8][A.9][A.10][A.11][A.12], and the aim of this compendium is to bring a selection of them together in a single volume.

The major procedures use slightly different terminology and definitions. The underlying sources sometimes differ from one procedure to the other, and even when the sources are the same, the information may be presented in a different manner, eg as equations, graphs or lookup tables.

The stress intensity factor is generally expressed in the form $K = Y\sigma\sqrt{(\pi a)}$ where a is the crack length and σ the applied stress. The Y term allows the effects of finite width, crack shape, position along crack front, bulging, stress concentration factors, local stress concentration due to welds, etc. to be taken into account.

In order to ensure that the information presented in this compilation is given in a useful and consistent fashion, this report draws on information from several different procedures, principally BS 7910 and R6, but using BS 7910 terminology throughout wherever possible for equations and diagrams. The information is presented as follows:

Text: the K-solutions from the different procedures are given as equations or look-up tables with any relevant background information and associated references. The validity limits for the K-solutions are given using BS 7910 terminology where this is possible.

Plots: in cases where the stress intensity factor is presented as a closed-formed solution or a set of tables, curves of normalised stress intensity, $Y=K_I/\sigma\sqrt{(\pi a)}$ or a related parameter (eg M_m, M_b), are shown as a function of normalised crack size. Graphical presentation has a number of advantages:

- It allows the user to carry out preliminary calculations without specialist software, and is less error-prone than use of equations.
- It highlights the differences between the various FFS procedures for a given geometry.
- It shows trends within a given solution, for example the effects of crack aspect ratio or pipe radius.
- It shows the relationship between the simplest geometries (eg flat plates) and specific solutions for more complex geometries (eg cylinders and sphere).

In the plots, colour schemes are kept consistent (one colour per procedure) for ease of use as follows:

- blue for BS 7910,
- red for R6,
- green for API 579.

Magenta lines are also used in selected cases to illustrate comparisons between the geometry of interest and a simplified geometry, eg a flat plate.

The curves have been generated from various sources, for example:

- existing validated software (eg Crackwise®, FractureGraphic),
- validated spreadsheets,
- MathCad calculations;
- direct graphing of tabulated data.

In order to ensure traceability and maintainability of the compendium, the method of generating the graph is reported in each case examined.

Note that the user also has the option of deriving K-solutions from alternative approaches such as finite element analysis (FEA) or weight function methods, provided that the basis of the method and the results are fully documented.

A.1.1 General Notes on BS 7910 K Solutions

In BS 7910 Annex M, the general form of the stress intensity factor solutions is:

$$K_I = (Y\sigma)\sqrt{\pi a} \quad \text{(A.1)}$$

where σ is a general stress term.

For fatigue assessments, the corresponding stress intensity factor range is:

$$\Delta K_I = Y(\Delta\sigma)\sqrt{\pi a} \quad \text{(A.2)}$$

For fracture assessments, the following equation applies:

$$Y\sigma = (Y\sigma)_p + (Y\sigma)_s \quad \text{(A.3)}$$

where

$(Y_\sigma)_p$ and $(Y_\sigma)_s$ represent contributions from primary and secondary stresses, respectively. They are calculated as follows:

$$(Y\sigma)_p = Mf_w\left[k_{tm}M_{km}M_m P_m + k_{tb}M_{kb}M_b\left\{P_b + (k_m - 1)P_m\right\}\right] \quad \text{(A.4)}$$

$$(Y\sigma)_s = M_m Q_m + M_b Q_b \quad \text{(A.5)}$$

For fatigue assessments the following equation applies:

$$(Y\Delta\sigma)_p = Mf_w\left[k_{tm}M_{km}M_m\Delta\sigma_m + k_{tb}M_{kb}M_b\left\{\Delta\sigma_b + (k_m - 1)\Delta\sigma_m\right\}\right] \quad \text{(A.6)}$$

Expressions for M, f_W, M_m and M_b are given on a case-by-case basis in the following sections. The factors M_{km} and M_{kb} apply when the crack is in a region of local stress concentration such as close to the toe of a weld (see Section A.6). For k_t, k_{tm}, k_{tb} and k_m, reference should be made to Section 4 and Annex I.

Note that the K-solutions (Annex M of BS 7910) and the reference stress solutions (Annex P of BS 7910) do not always match each other in terms of validity ranges, since they may be derived from different sources.

A.1.2 General notes on R6 K Solutions

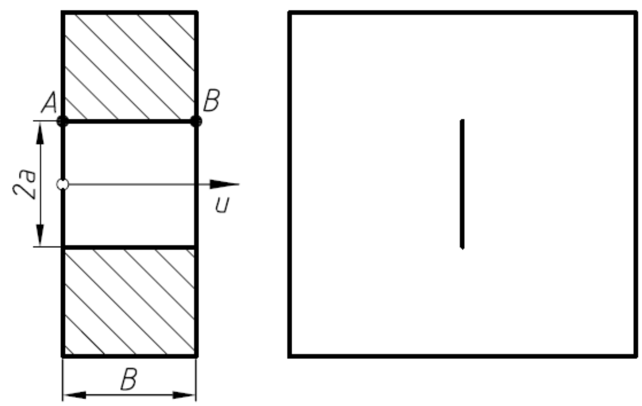
Section IV.3 of R6 contains a collection of stress intensity factor solutions for plates, cylinders and spheres. The equations are presented in terms of stress intensity (K_I) rather than normalised stress intensity (Y) or M_m, M_f and this style has been retained in the current compendium. The components are generally considered to be of infinite size, so that the influence of the remote boundary on solutions is not included. In contrast with BS 7910, where solutions are presented in terms of bending and membrane stress only, many of the R6 solutions are presented in terms of weight functions, allowing stress intensity factors to be evaluated for arbitrary stress fields. Smith [A.13] has compared R6 K-solutions for cylinders with those of other procedures; consequently the R6 K-solution compendium contains some comment on the accuracy of the postulated solutions. However, the original R6 equations have been adjusted where necessary so that the terminology is consistent with that used for the BS 7910 equations.

A.1.3 General Notes on API 579 K Solutions

In API 579 Appendix C, the K solutions are given for one or more of the following through-wall stress distributions: general (arbitrary) stress distribution, 4th order polynomial stress distribution and membrane plus through-wall bending stress. Some K-solutions were derived specifically for API 579 using finite element analysis, in which case matching reference stress solutions are often available (Annex D of API 579).

A.2 Flat Plates

A.2.1 Central through-thickness Crack



BS 7910 Solution

The solution for this geometry is given by Eq (A.1 to (A.6 where $M=M_m=M_b=1$.

For a finite width plate, the finite width correction factor, f_w, is:

$$f_w = \sqrt{\sec\frac{\pi a}{W}} \qquad \text{(A.7)}$$

For an infinite width plate, Y=1.

Validity limits

None stated

R6 Solution [A.4][A.14]

For a linearly varying stress distribution through the thickness, which does not vary with the in-plane co-ordinate x, the stress intensity factor K_I is given by:

$$K_I = \sqrt{\pi a}\left(P_m + P_b f_b\right) \qquad \text{(A.8)}$$

In equation (A.8, P_m and P_b are the membrane and bending stress components respectively, which define the stress distribution P according to

$$P = P(u) = P_m + P_b\left(1 - \frac{2u}{B}\right) \quad \text{for } 0 \le u \le B \qquad \text{(A.9)}$$

P is to be taken normal to the prospective crack plane in an uncracked plate. The co-ordinate u is defined in the sketch above.

The geometry function f_b is equal to 1.0 at the free surface at u = 0 (A) and f_b = -1.0 at u = B (B), see sketch above.

For a stress which is constant through the thickness but varies with the in-plane dimension as $P(x)$,

$$K_I = \frac{1}{\sqrt{\pi a}} \int_{-a}^{a} P(x) \left[\frac{(a+x)}{(a-x)} \right]^{1/2} dx \qquad \text{(A.10)}$$

Accuracy These infinite plate solutions are exact.

For a finite plate of width W, assuming a remote uniform stress, P, normal to the crack plane

$$K_I = P\sqrt{\pi a}\{1 - 0.01\,(a/W)^2 + 0.96\,(a/W)^4\}[\sec(\pi a/W)]^{1/2} \qquad \text{(A.11)}$$

Validity limits

For the finite width case ‘any a/W’

API 579 solution [A.15][A.16]

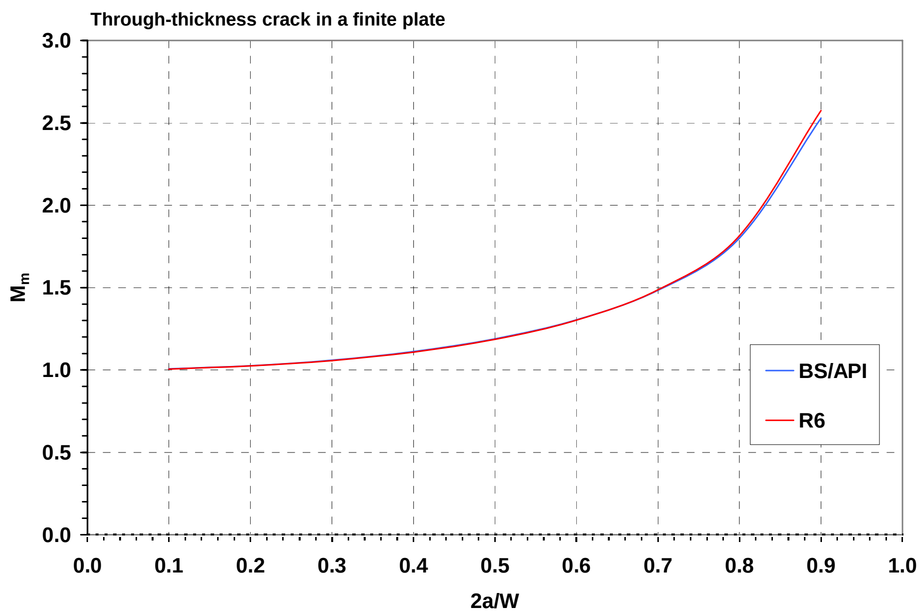
The finite width correction factor is as given above for BS 7910 (although the terminology is different). A thickness-dependent term is given for M_b.

Plots

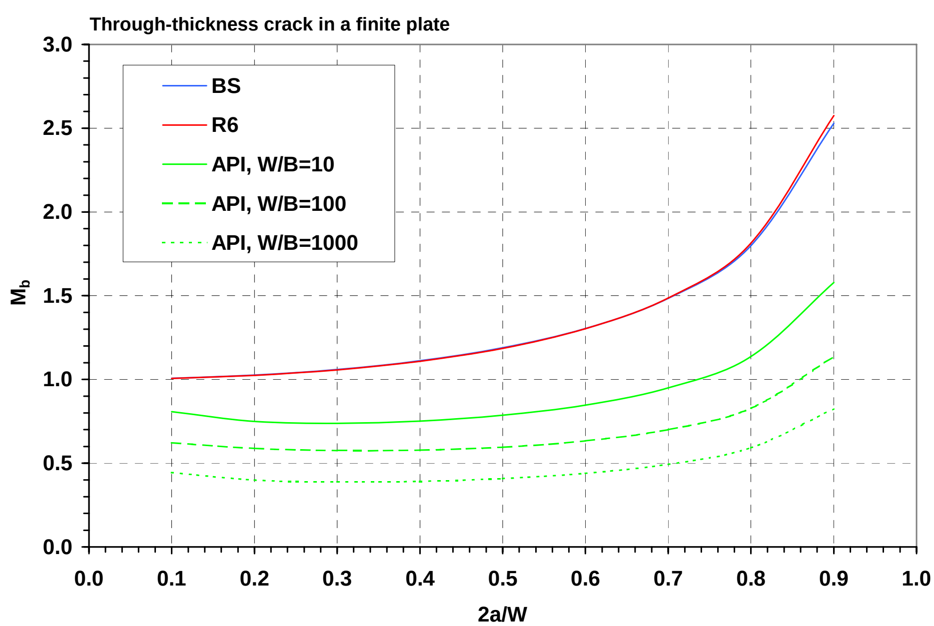
The functions recommended by various procedures are shown in as a function of relative crack size a/W. Note that the API solutions for M_b are not actually implemented in the API-based software FractureGraphic, which uses M_b=1 instead. BS 7910 and R6 solutions diverge slightly for large a/W because of differences in the definition of f_W.

Figure A.1 Normalised K-solution for a through-thickness crack in a finite plate

a) Membrane stress

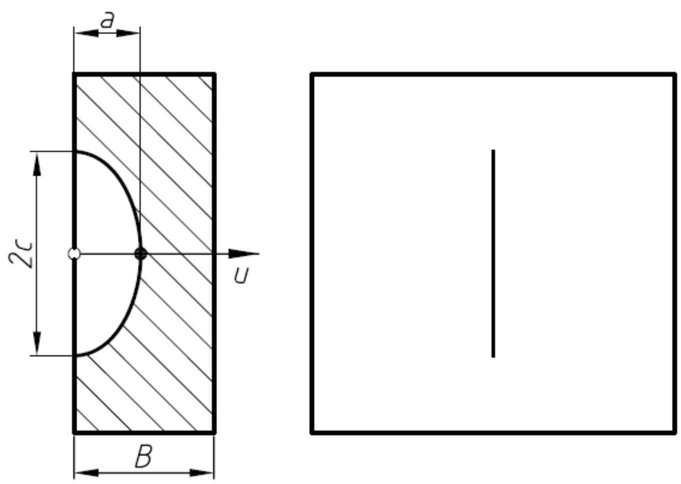


b) Bending stress



A.2.2 Surface Crack

A.2.2.1 Finite crack



BS 7910 Solution [A.15][A.17]

The stress intensity factor solution presented in this section is applicable to both normal restraint and pin-jointed boundary conditions (see Annex B). The stress intensity factor is given by equations (A.1 to (A.6 where M = 1 and:

$$f_w = \left\{ \sec\left[\left(\Pi c / W \right) \left(a / B \right)^{0.5} \right] \right\}^{0.5} \tag{A.12}$$

where f_W=1.0 if $a/2c$=0.

M_m and M_b are defined below.

For Membrane loading

Conditions

The following conditions apply.

$0 \le a/2c \le 1.0$

$0 \le \Theta \le \Pi$

where θ is the parametric angle around the crack front

and

$a/B < 1.25\,(a/c + 0.6)$ for $0 \le a/2c \le 0.1$
$a/B < 1.0$ for $0.1 \le a/2c \le 1.0$

Solution

$$M_m = \left\{M_1 + M_2\left(a/B\right)^2 + M_3\left(a/B\right)^4\right\} g f_\Theta / \Phi \quad \text{(A.13)}$$

where

M_1	=	$1.13 - 0.09(a/c)$	for $0 \le a/2c \le 0.5$
M_1	=	$(c/a)^{0.5}\{1 + 0.04(c/a)\}$	for $0.5 < a/2c \le 1.0$
M_2	=	$[0.89/\{0.2 + (a/c)\}] - 0.54$	for $0 \le a/2c \le 0.5$
M_2	=	$0.2(c/a)^4$	for $0.5 < a/2c \le 1.0$
M_3	=	$0.5 - 1/\{0.65 + (a/c)\} + 14\{1 - (a/c)\}^{24}$	for $a/2c \le 0.5$
M_3	=	$-0.11\,(c/a)^4$	for $0.5 < a/2c \le 1.0$
g	=	$1 + \{0.1 + 0.35(a/B)^2\}(1 - \sin\Theta)^2$	for $a/2c \le 0.5$
g	=	$1 + \{0.1 + 0.35(c/a)(a/B)^2\}(1 - \sin\Theta)^2$	for $0.5 < a/2c \le 1.0$
f_θ	=	$\{(a/c)^2 \cos^2\Theta + \sin^2\Theta\}^{0.25}$	for $0 \le a/2c \le 0.5$
f_θ	=	$\{(c/a)^2 \sin^2\Theta + \cos^2\Theta\}^{0.25}$	for $0.5 < a/2c \le 1.0$

Φ, the complete elliptic integral of the second kind, may be determined from standard tables or from the following solution, which is sufficiently accurate:

$$\Phi = \sqrt{1 + 1.464\left(\frac{a}{c}\right)^{1.65}} \quad \text{(A.14)}$$

for $0 \le a/2c \le 0.5$, and

$$\Phi = \sqrt{1 + 1.464\left(\frac{c}{a}\right)^{1.65}} \quad \text{(A.15)}$$

for $0.5 \le a/2c \le 1.0$

Simplifications

The following simplifications may be used as indicated.

a) At the deepest point on the crack front:

g	=	1	
f_θ	=	1	for $0 \le a/2c \le 0.5$
f_θ	=	$(c/a)^{0.5}$	for $0.5 < a/2c \ge 1$

b) At the ends of the crack, $\theta=0$, so that:

g	=	$1.1 + 0.35\,(a/B)^2$	For $0 \le a/2c \le 0.5$
g	=	$1.1 + 0.35\,(c/a)\,(a/B)^2$	For $0.5 < a/2c \le 1.0$
f_θ	=	$(a/c)^{0.5}$	For $0 \le a/2c \le 0.5$

f_θ	=	1.0	For $0.5 < a/2c \leq 1.0$

c) If $a/2c > 1.0$ use solution for $a/2c = 1.0$.

For Bending loading

Conditions

The conditions are as given for membrane loading

Solutions

$$M_b = HM_m \qquad \text{(A.16)}$$

where M_m is calculated from equation (A.13

$$H = H_1 + (H_2 - H_1)\sin^q \Theta \qquad \text{(A.17)}$$

where

q	=	$0.2 + (a/c) + 0.6(a/B)$	for $0 \leq a/2c \leq 0.5$
q	=	$0.2 + (c/a) + 0.6(a/B)$	for $0.5 < a/2c \leq 1.0$
H_1	=	$1 - 0.34(a/B) - 0.11(a/c)(a/B)$	for $0 \leq a/2c \leq 0.5$
H_1	=	$1 - \{0.04 + 0.41(c/a)\}(a/B) + \{0.55 - 1.93(c/a)^{0.75} + 1.38(c/a)^{1.5}\}(a/B)^2$	for $0.5 < a/2c \leq 1.0$
H_2	=	$1 + G_1\,(a/B) + G_2\,(a/B)^2$	

where

G_1	=	$-1.22 - 0.12(a/c)$	for $0 \leq a/2c \leq 0.5$
G_1	=	$-2.11 + 0.77(c/a)$	for $0.5 < a/2c \leq 1.0$
G_2	=	$0.55 - 1.05(a/c)^{0.75} + 0.47(a/c)^{1.5}$	for $0 \leq a/2c \leq 0.5$
G_2	=	$0.55 - 0.72(c/a)^{0.75} + 0.14\,(c/a)^{1.5}$	for $0.5 < a/2c \leq 1.0$

Simplifications

The following simplifications may be used as indicated.

a) At the deepest point on the crack front, $\theta=\pi/2$, so that $H = H_2$ and:

g	=	1	
f_θ	=	1	for $0 \leq a/2c \leq 0.5$
f_θ	=	$(c/a)^{0.5}$	for $0.5 < a/2c \leq 1$

b) At the ends of the crack, $\Theta = 0$, so that:

g	=	$1.1 + 0.35(a/B)^2$	for $0 \leq a/2c \leq 0.5$;
g	=	$1.1 + 0.35(c/a)(a/B)^2$	for $0.5 < a/2c \leq 1.0$;
f_θ	=	$(a/c)^{0.5}$	for $0 \leq a/2c \leq 0.5$;
f_θ	=	1.0	for $0.5 < a/2c \leq 1.0$;

and

$H = H_1$.

c) If $a/2c > 1.0$, use solution for $a/2c = 1.0$.

For a finite plate, the values should be multiplied by the finite width correction factor, f_W, in accordance with equations (A.1 to A.6.

Validity limits

The finite width correction factor is valid for 2c/W<0.8

0<=a/2c<=1.0

R6 Solution [A.14][A.18]

The stress intensity factor K_I is given by

$$K_I = \sqrt{\pi a}\sum_{i=0}^{5} P_i f_i\left(\frac{a}{B}, a/2c\right) \quad \text{(A.18)}$$

P_i (i = 0 to 5) are stress components which define the stress distribution P according to

$$P = P(u) = \sum_{i=0}^{5} P_i\left(\frac{u}{a}\right)^i \quad \text{for } 0 \le u \le a \quad \text{(A.19)}$$

where P is to be taken normal to the prospective crack plane in an uncracked plate. The co-ordinate u is the distance from the plate surface as shown above.

f_i (i = 0 to 5) are geometry functions which are given in Table **A.1** and Table **A.2** below for the deepest point of the crack (position A, θ=90º), and at the intersection of the crack with the free surface (position B, θ=0º), respectively.

Table A.1 R6 geometry functions for a finite surface crack in an infinite plate: deepest point of the crack (position A, θ=90º)

a/2c=0.5						
a/B	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.659	0.471	0.387	0.337	0.299	0.266
0.2	0.663	0.473	0.388	0.337	0.299	0.269
0.4	0.678	0.479	0.390	0.339	0.300	0.271
0.6	0.692	0.486	0.396	0.342	0.304	0.274
0.8	0.697	0.497	0.405	0.349	0.309	0.278
a/2c=0.4						
a/B	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.741	0.510	0.411	0.346	0.300	0.266
0.2	0.746	0.512	0.413	0.352	0.306	0.270
0.4	0.771	0.519	0.416	0.356	0.309	0.278
0.6	0.800	0.531	0.422	0.362	0.317	0.284
0.8	0.820	0.548	0.436	0.375	0.326	0.295
a/2c=0.3						
a/B	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.833	0.549	0.425	0.351	0.301	0.267
0.2	0.841	0.554	0.430	0.359	0.309	0.271
0.4	0.885	0.568	0.442	0.371	0.320	0.285
0.6	0.930	0.587	0.454	0.381	0.331	0.295
0.8	0.960	0.605	0.476	0.399	0.346	0.310
a/2c=0.2						
a/B	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	0.939	0.580	0.434	0.353	0.302	0.268
0.2	0.957	0.595	0.446	0.363	0.310	0.273
0.4	1.057	0.631	0.475	0.389	0.332	0.292
0.6	1.146	0.668	0.495	0.407	0.350	0.309
0.8	1.190	0.698	0.521	0.428	0.367	0.324
a/2c=0.1						
a/B	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	1.053	0.606	0.443	0.357	0.302	0.269
0.2	1.106	0.640	0.467	0.374	0.314	0.277
0.4	1.306	0.724	0.525	0.420	0.348	0.304
0.6	1.572	0.815	0.571	0.448	0.377	0.327
0.8	1.701	0.880	0.614	0.481	0.399	0.343
a/2c=0.05						
a/B	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	1.103	0.680	0.484	0.398	0.344	0.306
0.2	1.199	0.693	0.525	0.426	0.364	0.323

0.4	1.492	0.806	0.630	0.499	0.417	0.364
0.6	1.999	1.004	0.838	0.631	0.514	0.437
0.8	2.746	1.276	1.549	1.073	0.817	0.660
a/2c=0.025						
a/B	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	1.120	0.686	0.504	0.419	0.365	0.325
0.2	1.245	0.708	0.553	0.452	0.389	0.346
0.4	1.681	0.881	0.682	0.538	0.451	0.394
0.6	2.609	1.251	0.971	0.722	0.583	0.493
0.8	4.330	1.885	2.016	1.369	1.026	0.819
$a/2c \to 0$						
a/B	f_0^A	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	1.123	0.682	0.524	0.440	0.386	0.344
0.2	1.380	0.784	0.582	0.478	0.414	0.369
0.4	2.106	1.059	0.735	0.578	0.485	0.423
0.6	4.025	1.750	1.105	0.814	0.651	0.548
0.8	11.92	4.437	2.484	1.655	1.235	0.977

Table A.2 R6 geometry functions for a finite surface crack in an infinite plate: intersection of crack with free surface (position B, θ=0º)

a/2c=0.5						
a/B						
0	0.716	0.118	0.041	0.022	0.014	0.010
0.2	0.729	0.123	0.045	0.023	0.014	0.010
0.4	0.777	0.133	0.050	0.026	0.015	0.011
0.6	0.839	0.148	0.058	0.029	0.018	0.012
0.8	0.917	0.167	0.066	0.035	0.022	0.015
a/2c=0.4						
a/B						
0	0.730	0.124	0.041	0.021	0.013	0.010
0.2	0.749	0.126	0.046	0.023	0.014	0.010
0.4	0.795	0.144	0.054	0.028	0.017	0.012
0.6	0.901	0.167	0.066	0.033	0.021	0.015
0.8	0.995	0.193	0.076	0.042	0.026	0.017
a/2c=0.3						
a/B						
0	0.723	0.118	0.039	0.019	0.011	0.008
0.2	0.747	0.125	0.044	0.022	0.014	0.010
0.4	0.803	0.145	0.056	0.029	0.018	0.012
0.6	0.934	0.180	0.072	0.037	0.023	0.016
0.8	1.070	0.218	0.087	0.047	0.029	0.020
a/2c=0.2						
a/B						
0	0.673	0.104	0.032	0.015	0.009	0.006
0.2	0.704	0.114	0.038	0.018	0.011	0.007
0.4	0.792	0.139	0.053	0.027	0.016	0.011
0.6	0.921	0.183	0.074	0.038	0.024	0.017
0.8	1.147	0.244	0.097	0.052	0.032	0.021
a/2c=0.1						
a/B						
0	0.516	0.069	0.017	0.009	0.005	0.004
0.2	0.554	0.076	0.022	0.011	0.007	0.005
0.4	0.655	0.099	0.039	0.019	0.012	0.008
0.6	0.840	0.157	0.063	0.032	0.020	0.013
0.8	1.143	0.243	0.099	0.055	0.034	0.023
a/2c=0.05						
a/B						

0	0.384	0.067	0.009	0.004	0.003	0.002
0.2	0.422	0.074	0.011	0.006	0.004	0.003
0.4	0.546	0.096	0.020	0.010	0.006	0.004
0.6	0.775	0.136	0.031	0.016	0.010	0.007
0.8	1.150	0.202	0.050	0.028	0.017	0.011
a/2c=0.025						
a/B						
0	0.275	0.048	0.004	0.002	0.001	0.001
0.2	0.310	0.054	0.006	0.003	0.002	0.001
0.4	0.435	0.075	0.010	0.005	0.003	0.002
0.6	0.715	0.124	0.016	0.008	0.005	0.003
0.8	1.282	0.221	0.025	0.014	0.009	0.006
a/2c→0						
a/B						
0	0.000	0.000	0.000	0.000	0.000	0.000
0.2	0.000	0.000	0.000	0.000	0.000	0.000
0.4	0.000	0.000	0.000	0.000	0.000	0.000
0.6	0.000	0.000	0.000	0.000	0.000	0.000
0.8	0.000	0.000	0.000	0.000	0.000	0.000

API solution [A.15]

The solutions are based on those of Newman and Raju, and coincide with the BS 7910 solutions for pure membrane and bending stresses. Full equations are therefore not given here. Note that API 579 recommends an alternative finite width solution to that presented in equation (A.12. For the membrane component of stress, a new finite width factor, f_{wm}, is defined:

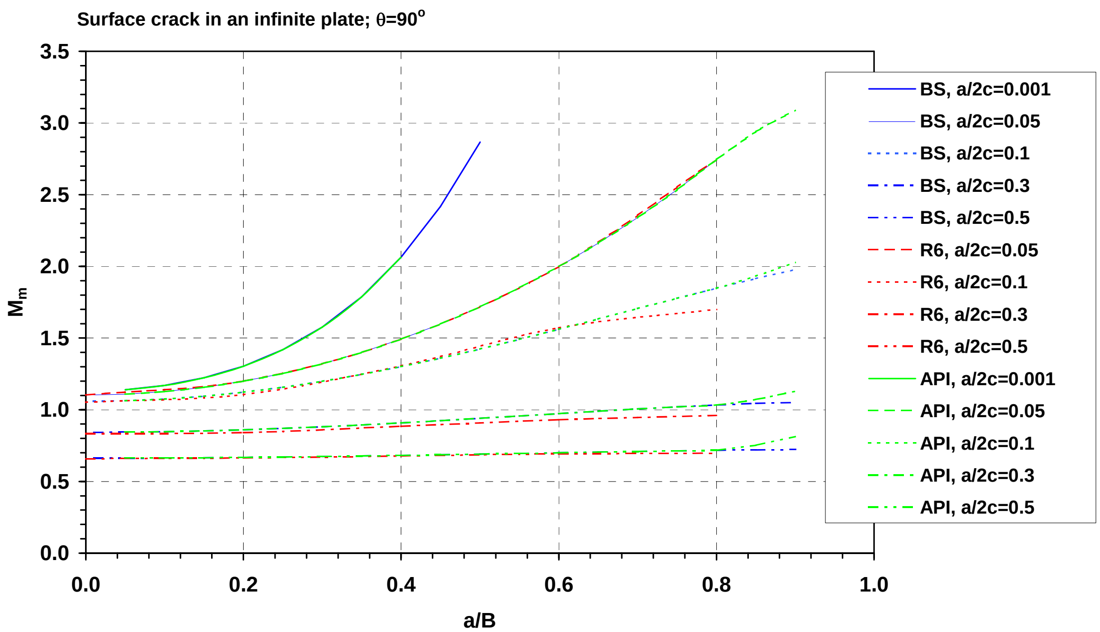
$$f_{wm} = f_W \left\{ \sec\left[\left(\frac{\pi c}{W} \right) \cdot \sqrt{\left(\frac{a}{t} \right) \left(1 - 0.6 \sin\varphi \right)} \right] \right\}^{0.5} \quad \text{(A.20)}$$

[Note that the original equation (equation [C.40] of the 2000 edition of API579 is incorrect, and the corrected version is given in equation (A.20.

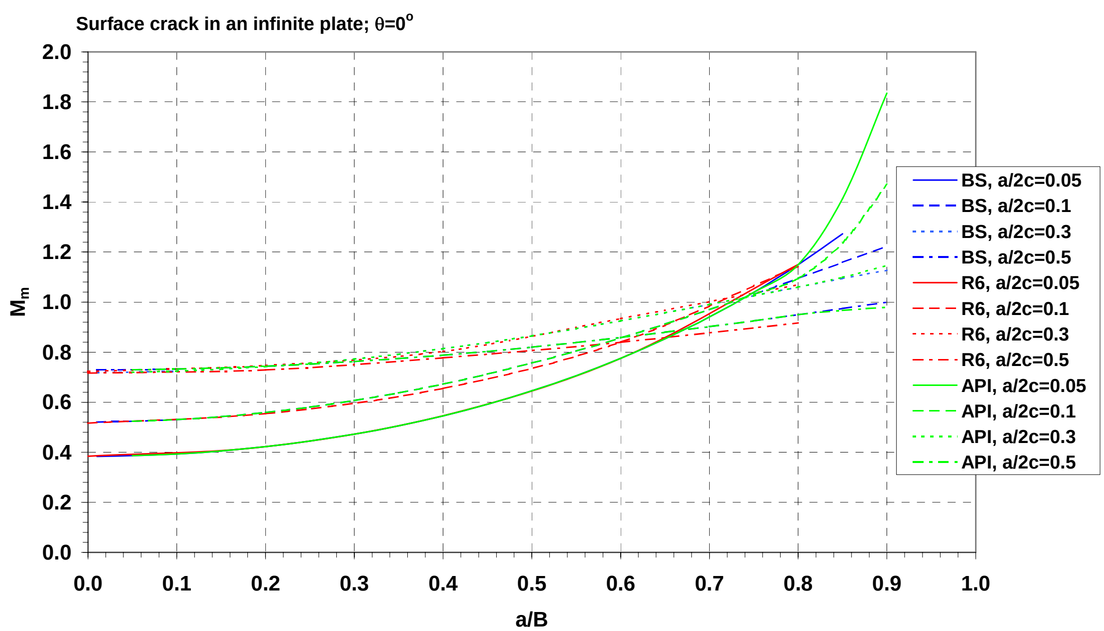
Plots

A comparison of the solutions is shown in Figure A.2 for various flaw aspect ratios, under pure membrane and pure bending loads, at positions θ=90º and θ=0º around the crack front. Figure A.2e shows the BS 7910 and API 579 finite width correction factors.

Figure A.2 Normalised K-solution for a surface-breaking crack in an infinite plate

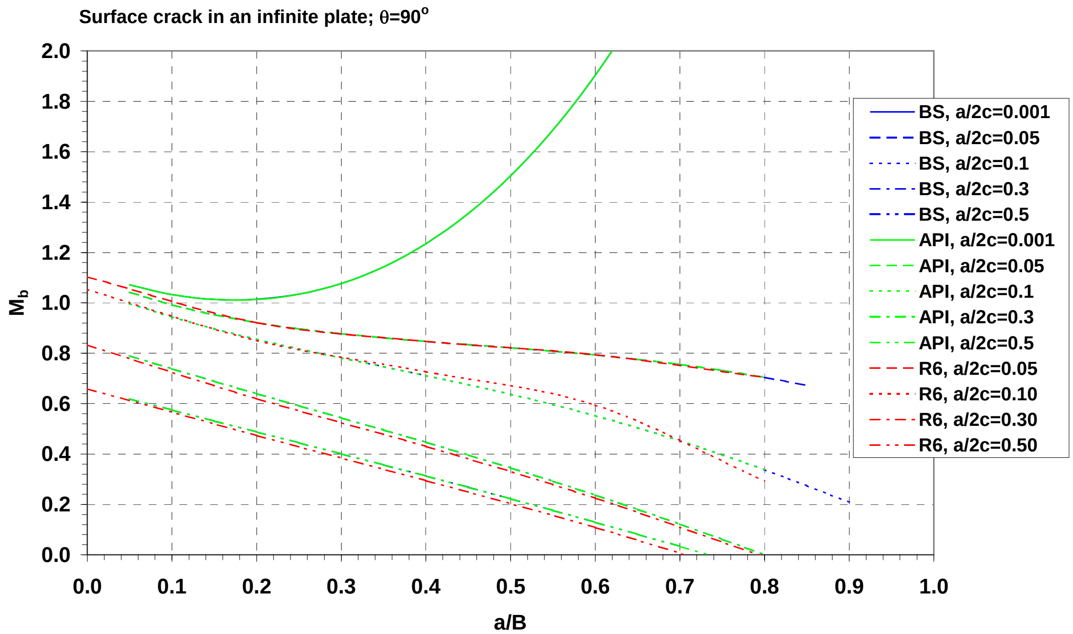


a) Membrane stress, θ=90º

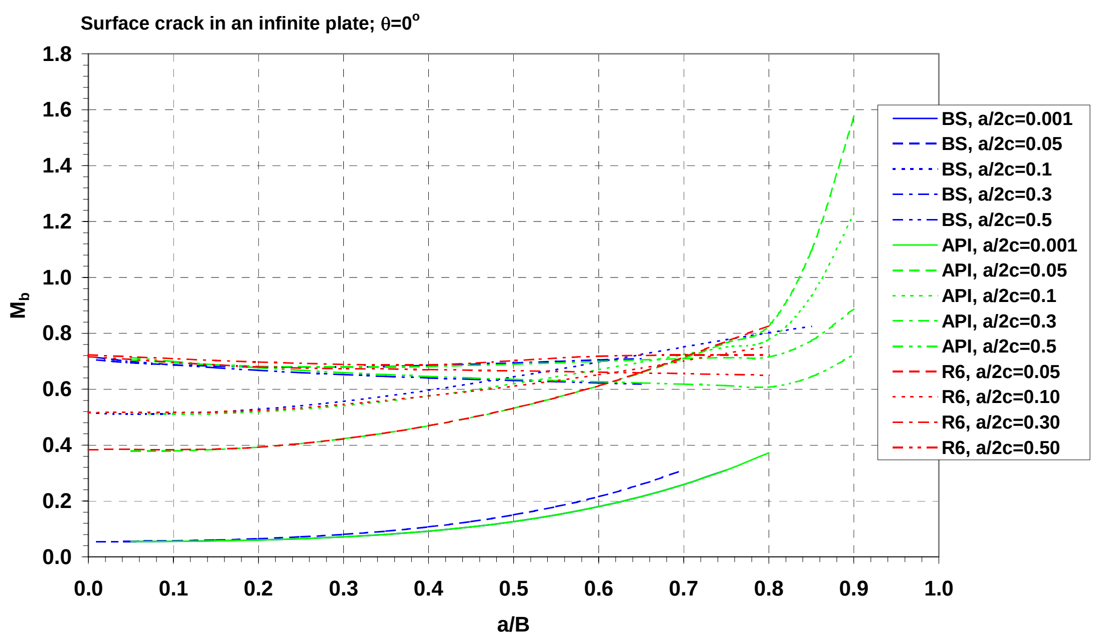


b) Membrane stress, θ=0º

Normalised K-solution for a surface-breaking crack in an infinite plate (cont'd)

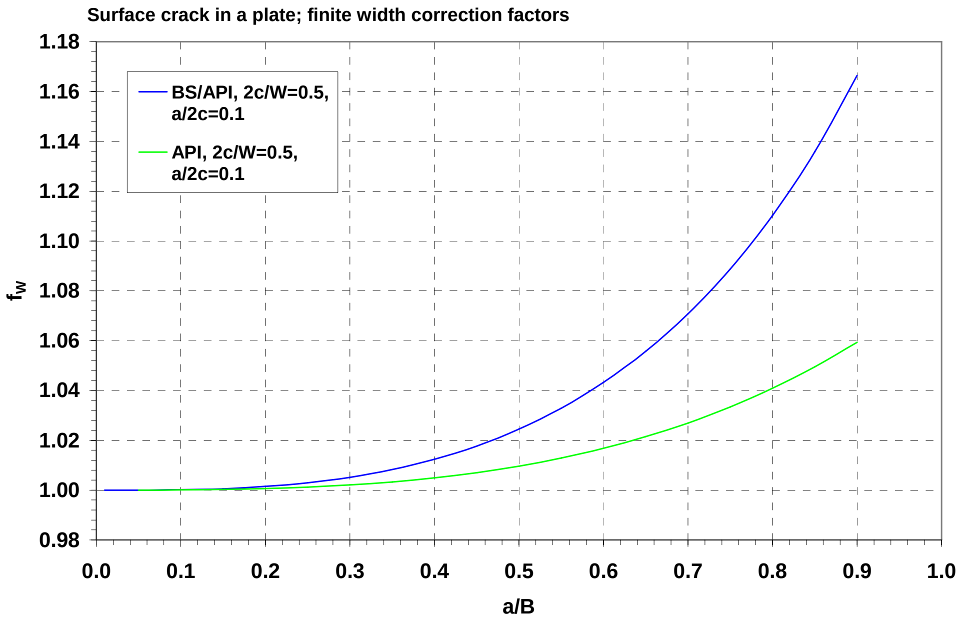


c) Bending stress, θ=90°



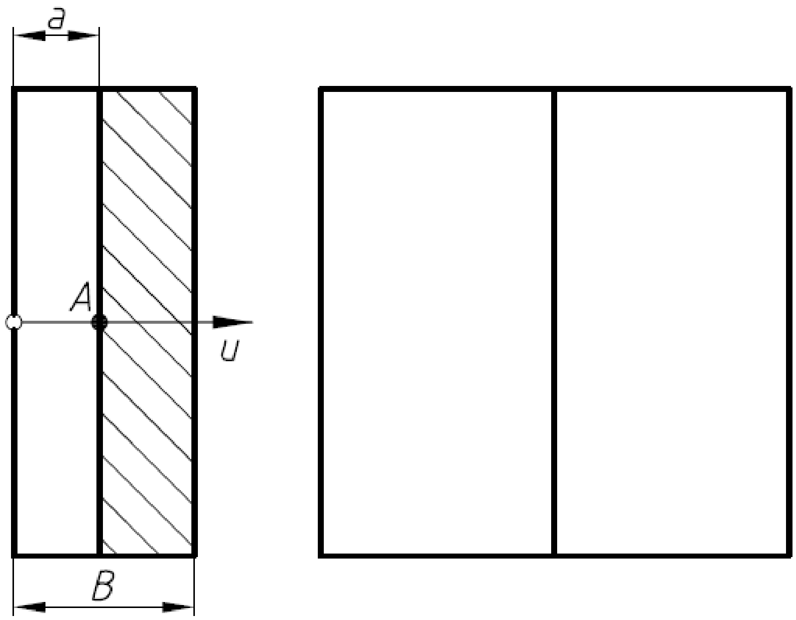
d) Bending stress, θ=0°

Normalised K-solution for a surface-breaking crack in an infinite plate (cont'd)



e) Finite width correction factor

A.2.2.2 Extended crack



BS 7910 solution [A.3]

The stress intensity factor is given by equations (A.1 to (A.6, where $f_w = 1$, and M_m and M_b are given below:

$$M_m = 1.12 - 0.23(a/B) + 10.6(a/B)^2 - 21.7(a/B)^3 + 30.4(a/B)^4 \quad \text{(A.21)}$$

$$M_b = 1.12 - 1.39(a/B) + 7.32(a/B)^2 - 13.1(a/B)^3 + 14(a/B)^4 \quad \text{(A.22)}$$

Validity limits

$a/B \leq 0.6$

R6 Solution [A.19] [Wu and Carlsson]

The stress intensity factor K_I is given by

$$K_I = \frac{1}{\sqrt{2\pi a}} \int_0^a P(u) \sum_{i-1}^{i=5} f_i(a/B) \left(1 - \frac{u}{a}\right)^{i-\frac{3}{2}} du \quad \text{(A.23)}$$

The stress state P = P(u) is to be taken normal to the prospective crack plane in an uncracked plate. The co-ordinate u is defined in the figure above.

The geometry functions f_i (i = 1 to 5) are given in Table A.3.

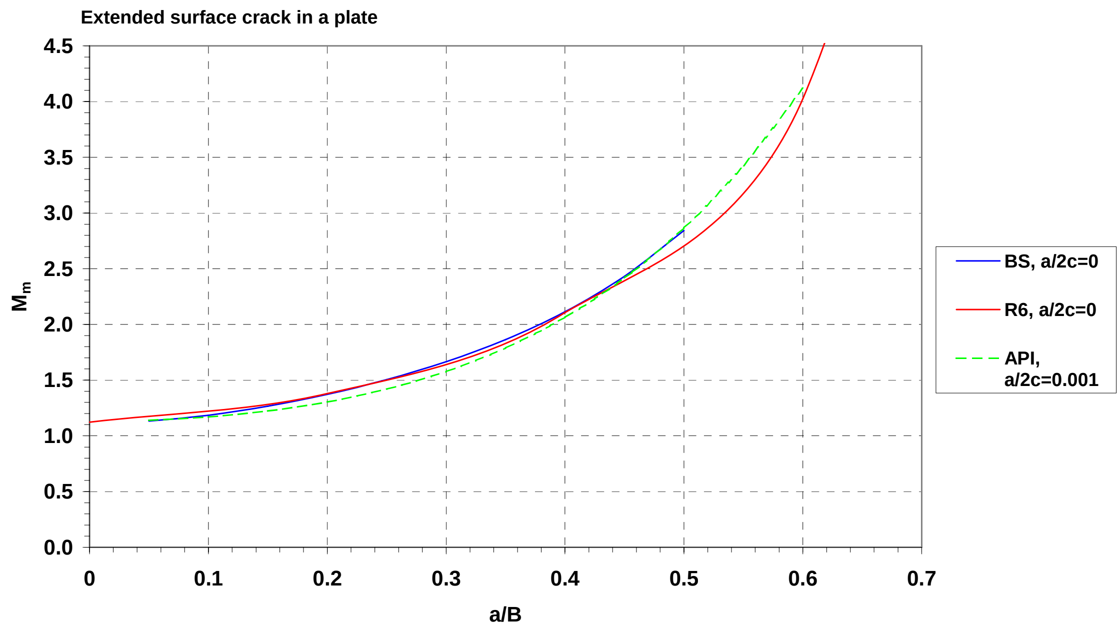
Table A.3 R6 geometry functions for an extended surface crack in an infinite width plate.

a/B	f_1^A	f_2^A	f_3^A	f_4^A	f_5^A
0	2.000	0.977	1.142	-0.350	-0.091
0.1	2.000	1.419	1.138	-0.355	-0.076
0.2	2.000	2.537	1.238	-0.347	-0.056
0.3	2.000	4.238	1.680	-0.410	-0.019
0.4	2.000	6.636	2.805	-0.611	0.039
0.5	2.000	10.02	5.500	-1.340	0.218
0.6	2.000	15.04	11.88	-3.607	0.786
0.7	2.000	23.18	28.03	-10.50	2.587
0.8	2.000	38.81	78.75	-36.60	9.871
0.9	2.000	82.70	351.0	-207.1	60.86

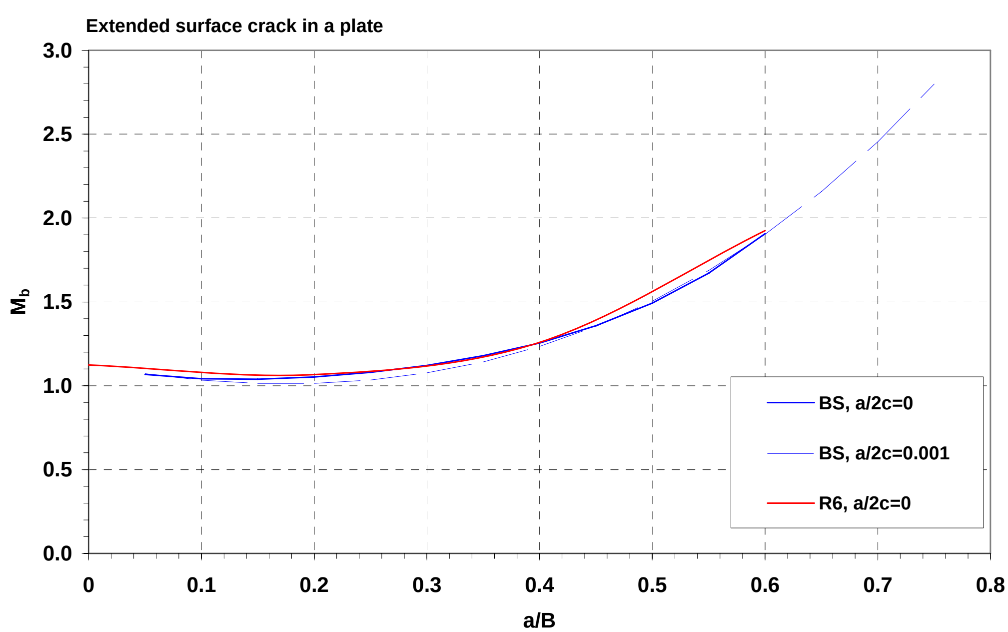
Plots

Solutions for pure membrane and bending stresses are shown in Figure A.3. Selected solutions from Section A.2.2.1 (low aspect ratio flaws) are shown for comparison.

Figure A.3 Normalised K-solution for an extended surface crack in a plate

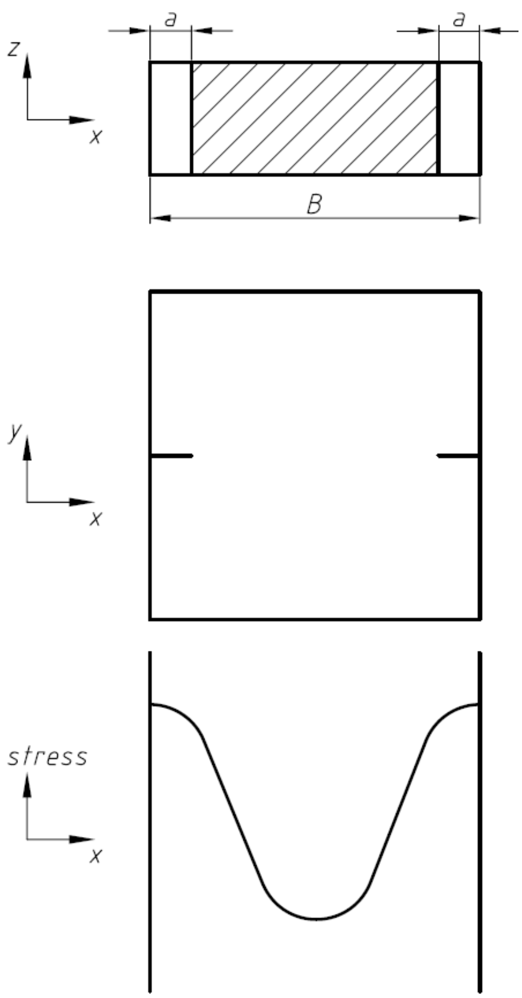


a) Membrane stress



b) Bending stress

A.2.2.3 Extended double crack

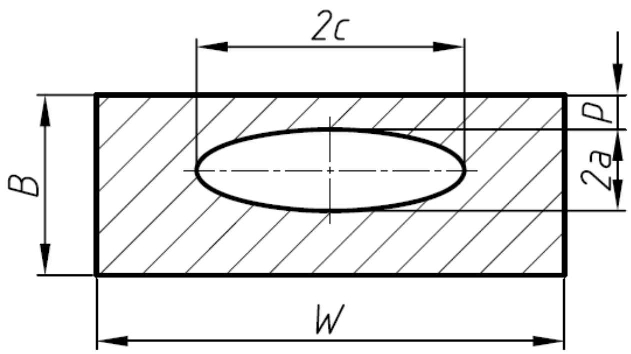


This case is considered as a double edge crack in an infinite plate (see Section A.2.5.2)

A.2.3 Embedded Crack

A.2.3.1 Finite crack

BS 7910 solution [A.17]



The BS 7910 solution uses slightly different terminology from the R6 solution; consequently two different sketches are shown here, one for each solution.

The stress intensity factor is given by equations (A.1 to (A.6, where *M* = 1,

$$f_w = \{\sec[(\pi c/W)(2a/B')^{0.5}]\}^{0.5} \quad \text{(A.24)}$$

and solutions for M_m and M_b are given in [(A.25] and [(A.29]; B' is the 'effective thickness', assuming the crack to lie at the centre of the plate of thickness B'. Hence B' = 2*a* + 2*p*.

Membrane loading

Conditions

The conditions for membrane loading are as follows:

$0 \leq a/2c \leq 1.0$
$2c/W < 0.5$
$-\Pi \leq \Theta \leq \Pi$

$a/B' < 0.625(a/c + 0.6)$ for $0 \leq a/2c \leq 0.1$

where

B' is the effective thickness, equal to 2*a* + 2*p*.

Solution

$$M_m = \left\{M_1 + M_2\left(2a/B'\right)^2 + M_3\left(2a/B'\right)^4\right\} g f_\Theta / \Phi \quad \text{(A.25)}$$

where

Φ is defined in equations (A.14 and (A.15

M_1 = 1 for $0 \leq a/2c \leq 0.5$
M_1 = $(c/a)^{0.5}$ for $0.5 < a/2c \leq 1.0$

$$M_2 = \frac{0.05}{0.11 + (a/c)^{1.5}} \tag{A.26}$$

$$M_3 = \frac{0.29}{0.23 + (a/c)^{1.5}} \tag{A.27}$$

$$g = 1 - \left[\frac{(2a/B')^4 \{2.6 - (4a/B')\}^{0.5}}{1 + 4(a/c)} \right] |\cos\theta| \tag{A.28}$$

f_Θ = $\{(a/c)^2 \cos^2\Theta + \sin^2\Theta\}^{0.25}$ for $0 \leq a/2c \leq 0.5$
f_Θ = $\{(c/a)^2 \sin^2\Theta + \cos^2\Theta\}^{0.25}$ for $0.5 < a/2c \leq 1.0$

Simplifications

The following simplifications may be used as indicated.

a) At the point on the crack front closest to the material surface, $\Theta = \Pi/2$ so that:

g = 1

f_Θ = 1 for $a/2c \leq 0.5$
f_Θ = $(c/a)^{0.5}$ for $0.5 < a/2c \leq 1$

b) At the ends of the crack, $\Theta = 0$ so that:

$$g = 1 - \left[\frac{(2a/B')^4 \{2.6 - (4a/B')\}^{0.5}}{1 + 4(a/c)} \right];$$

and

f_Θ = $(a/c)^{0.5}$ for $0 \leq a/2c \leq 0.5$
f_Θ = 1 for $0.5 < a/2c \leq 1.0$

c) If $a/2c > 1.0$, use solution for $a/2c = 1.0$.

Bending loading

Conditions

The conditions for bending loading are as follows:

$0 \leq a/2c \leq 0.5$
$\Theta = \Pi / 2$

(ie solution only refers to the ends of the minor axis of the elliptical crack).

Solution

$$M_b = \left[\lambda_1 + \lambda_2 \left(p/B\right) + \lambda_3 \left(a/B\right) + \lambda_4 \left(pa/B^2\right)\right] / \Phi \qquad \text{(A.29)}$$

where

— for $p/B \leq 0.184\,1$:

$\lambda_1 = 1.044$
$\lambda_2 = -2.44$
$\lambda_3 = 0$
$\lambda_4 = -3.166$

— for $p/B > 0.184\,1$ and $a/B \leq 0.125$:

$\lambda_1 = 0.94$
$\lambda_2 = -1.875$
$\lambda_3 = -0.114\,6$
$\lambda_4 = -1.844$

— for $p/B > 0.184\,1$ and $a/B > 0.125$:

$\lambda_1 = 1.06$
$\lambda_2 = -2.20$
$\lambda_3 = \lambda_4 = -0.666\,6$

Validity limits

Finite width equation is 'safe' up to 2c/W=0.8

For membrane loading:

$0 \leq a/2c \leq 1.0$

$2c/W < 0.5$

$-\pi \le \theta \le \pi$

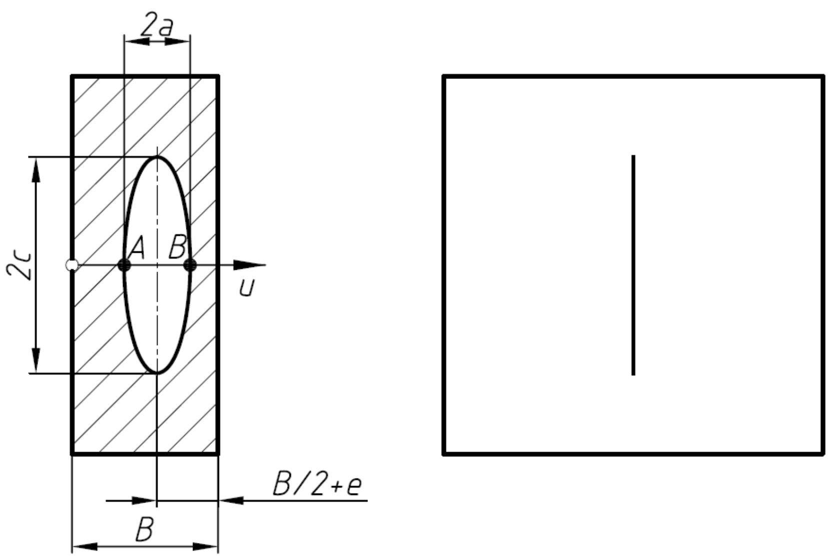
a/B' < 0.625(a/c+0.6) for $0 \le a/2c \le 0.1$, where B' = 2a + 2p

For bending loading:

$0 \le a/2c \le 0.5$

$\theta = \pi/2$

R6 Solution [A.20]



The stress intensity factor K_I is given by

$$K_I = \sqrt{\pi a}\left(P_m f_m\left(\frac{2a}{B},\frac{a}{c},\frac{e}{B}\right) + P_b f_b\left(\frac{2a}{B},\frac{a}{c},\frac{e}{B}\right)\right) \quad \text{(A.30)}$$

Here, the K-solution is given in terms of relative flaw depth (2a/B), flaw aspect ratio (a/c) and the displacement of the flaw from the centre of the plate (e/B), where e/B=0 denotes a centrally located flaw.

In equation (A.30, P_m and P_b are the membrane and bending stress components respectively, which define the stress state P according to

$$P = P(u) = P_m + P_b\left(1 - \frac{2u}{B}\right) \quad \text{(A.31)}$$

for $0 \le u \le B$

The stress P is to be taken normal to the prospective crack plane in an uncracked plate. P_m and P_b are determined by fitting P to equation (A.31. The co-ordinate u is defined in the figure above.

The geometry functions f_m and f_b are given in Table A.4 and Table A.5 for Points A (close to the smaller ligament) and B (close to the larger ligament) respectively.

Table A.4 R6 geometry functions for an embedded elliptical crack in an infinite width plate at Point A (closest to u = 0)

a/c=1						
2a/B	***e/B* = 0**		***e/B* = 0.15**		***e/B* = 0.3**	
0	0.638	0.000	0.638	0.191	0.638	0.383
0.2	0.649	0.087	0.659	0.286	0.694	0.509
0.4	0.681	0.182	0.725	0.411	-	-
0.6	0.739	0.296	0.870	0.609	-	-
a/c=0.5						
2a/B	***e/B* = 0**		***e/B* = 0.15**		***e/B* = 0.3**	
0	0.824	0.000	0.824	0.247	0.824	0.494
0.2	0.844	0.098	0.862	0.359	0.932	0.668
0.4	0.901	0.210	0.987	0.526	-	-
0.6	1.014	0.355	1.332	0.866	-	-
a/c=0.25						
2a/B	***e/B* = 0**		***e/B* = 0.15**		***e/B* = 0.3**	
0	0.917	0.000	0.917	0.275	0.917	0.550
0.2	0.942	0.102	0.966	0.394	1.058	0.749
0.4	1.016	0.220	1.129	0.584	-	-
0.6	1.166	0.379	1.655	1.034	-	-
a/c→0						
2a/B	***e/B* = 0**		***e/B* = 0.15**		***e/B* = 0.3**	
0	1.010	0.000	1.010	0.303	1.010	0.606
0.2	1.041	0.104	1.071	0.428	1.189	0.833
0.4	1.133	0.227	1.282	0.641	-	-
0.6	1.329	0.399	2.093	1.256	-	-

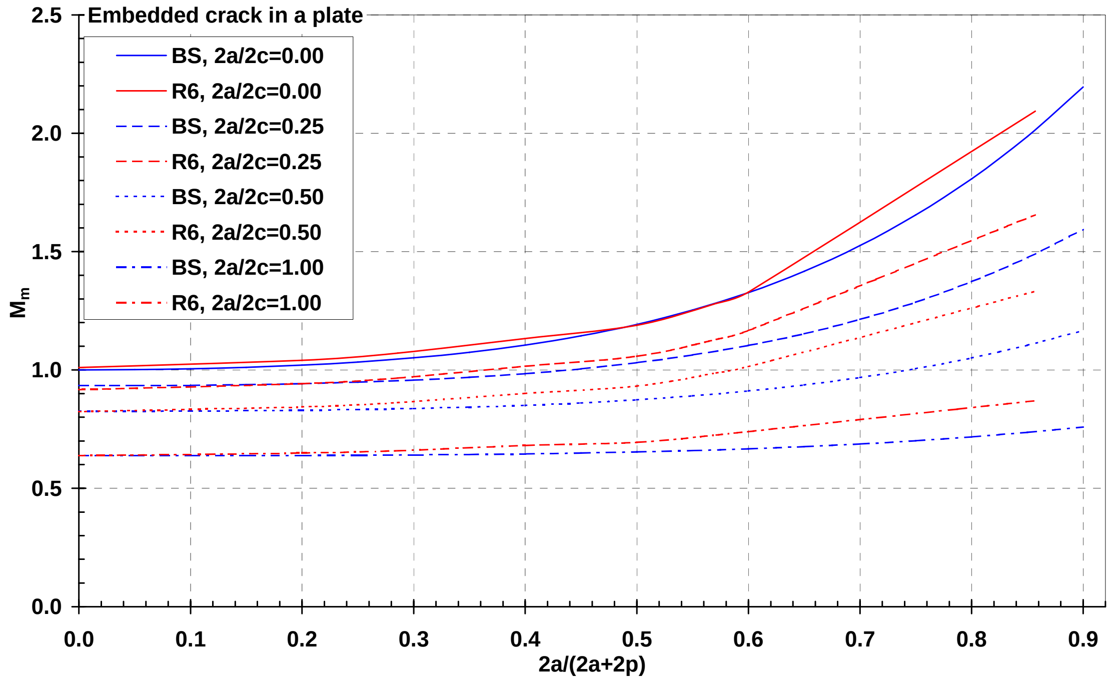
Table A.5 R6 geometry functions for an embedded crack in an infinite width plate at Point B (furthest from u = 0)

a/c=1						
2a/B	***e/B* = 0**		***e/B* = 0.15**		***e/B* = 0.3**	
0	0.638	0.000	0.638	0.191	0.638	0.383
0.2	0.649	-0.087	0.646	0.108	0.648	0.303
0.4	0.681	-0.182	0.668	0.022	-	-
0.6	0.739	-0.296	0.705	-0.071	-	-
a/c=0.5						
2a/B	***e/B* = 0**		***e/B* = 0.15**		***e/B* = 0.3**	
0	0.824	0.000	0.824	0.247	0.824	0.494
0.2	0.844	-0.098	0.844	0.155	0.866	0.418
0.4	0.901	-0.210	0.902	0.060	-	-
0.6	1.014	-0.355	1.016	-0.051	-	-
a/c=0.25						
2a/B	***e/B* = 0**		***e/B* = 0.15**		***e/B* = 0.3**	
0	0.917	0.000	0.917	0.275	0.917	0.550
0.2	0.942	-0.102	0.945	0.181	0.980	0.482
0.4	1.016	-0.220	1.029	0.086	-	-
0.6	1.166	-0.379	1.206	-0.030	-	-
***a/c*→0**						
2a/B	***e/B* = 0**		***e/B* = 0.15**		***e/B* = 0.3**	
0	1.010	0.000	1.010	0.303	1.010	0.606
0.2	1.041	-0.104	1.048	0.210	1.099	0.550
0.4	1.133	-0.227	1.162	0.116	-	-
0.6	1.329	-0.399	1.429	0.000	-	-

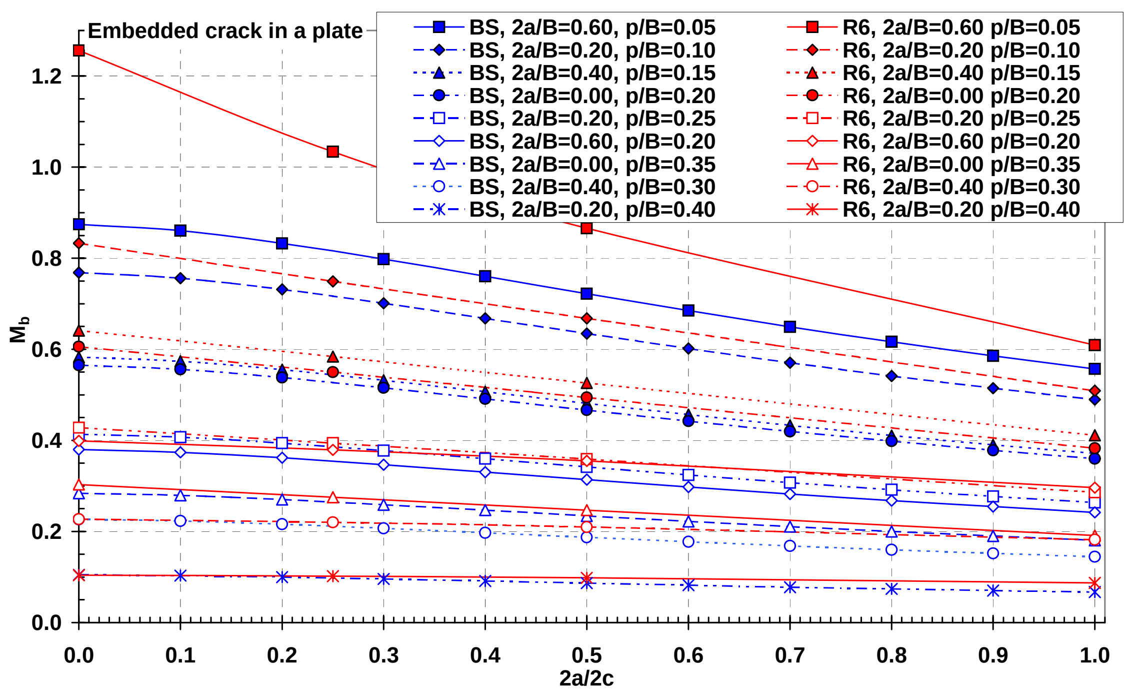
Plots

Graphical solutions for M_m and M_b at the point nearest the free surface (designated Point 'A' in R6) are given in Figure A.4. For membrane loading, the BS 7910 and R6 solutions are close for shallow flaws, diverging somewhat as the through-wall height of the flaw increases, with higher values of M_m from the R6 solution. Under bending, the R6 and BS 7910 solutions are coincident for relatively small, centrally located flaws (low values of 2a/B, high values of p/B), but diverge for larger flaws close to the surface, with the R6 solutions giving higher values of M_b.

Figure A.4 Normalised K-solution for an embedded elliptical crack in a plate



a) Membrane stress



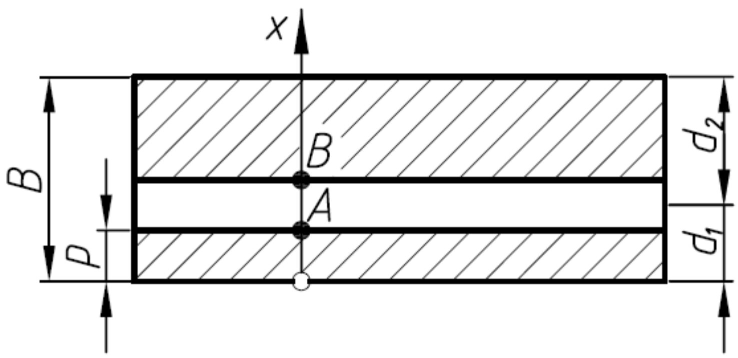
b) Bending stress

A.2.3.2 Extended crack

BS 7910, R6

No specific solution is given, although the solutions for finite cracks given in Section A.2.3.1 for a/c→0 can be used.

API 579 Solution [A.21]



API 579 gives a solution in terms of the distances, d_1 and d_2, between the free surface and the mid-point of the crack, and a, the half-height of the crack. The API equations have been re-written below in terms of p, the smaller of the two ligament heights, for consistency with Section A.2.3.2 and BS 7910 nomenclature. Hence d_1=p+a, and, for a through-wall 4th order polynomial stress distribution:

$$Y = M_0\left\{\sigma_0 + \sigma_1\left[\frac{a+p}{B}\right] + \sigma_2\left[\frac{a+p}{B}\right]^2 + \sigma_3\left[\frac{a+p}{B}\right]^3 + \sigma_4\left[\frac{a+p}{B}\right]^4\right\} + \tag{A.32}$$

$$M_1\left\{\sigma_1 + 2\sigma_2\left[\frac{a+p}{B}\right] + 3\sigma_3\left[\frac{a+p}{B}\right]^2 + 4\sigma_4\left[\frac{a+p}{B}\right]^3\right\}\left[\frac{a}{B}\right] +$$

$$M_2\left\{\sigma_2 + 3\sigma_3\left[\frac{a+p}{B}\right] + 6\sigma_4\left[\frac{a+p}{B}\right]^2\right\}\left[\frac{a}{B}\right]^2 +$$

$$M_3\left\{\sigma_3 + 4\sigma_4\left[\frac{a+p}{B}\right]\right\}\left[\frac{a}{B}\right]^3 + M_4\sigma_4\left[\frac{a}{B}\right]^4$$

where the coefficients M_0 to M_4 are given in Table A.6. Solutions are given for the crack front close to the surface (A) and the other crack front (B).

Table A.6 API coefficients for an extended embedded crack in a plate

$\frac{p+a}{B}$	$\frac{a}{\left(\frac{B}{2}-\left[(p+a)-\frac{B}{2}\right]\right)}$	Point A					Point B				
		M_0	M_1	M_2	M_3	M_4	M_0	M_1	M_2	M_3	M_4
0.25	0.20	1.0211	-0.4759	0.4601	-0.3141	0.3025	1.0180	0.4777	0.4600	0.3165	0.3034
	0.40	1.0923	-0.4804	0.4779	-0.3162	0.3113	1.0651	0.4757	0.4715	0.3155	0.3090
	0.60	1.2628	-0.5219	0.5423	-0.3521	0.3638	1.1505	0.4806	0.5122	0.3372	0.3491
	0.80	1.7105	-0.6027	0.6859	-0.4216	0.4634	1.3097	0.4740	0.5726	0.3551	0.4030
0.50	0.20	1.0259	-0.4758	0.4613	-0.3140	0.3031	1.0259	0.4784	0.4619	0.3168	0.3043
	0.40	1.1103	-0.4993	0.5121	-0.3550	0.3587	1.1103	0.4987	0.5110	0.3553	0.3585
	0.60	1.3028	-0.5299	0.5691	-0.3795	0.3974	1.3028	0.5292	0.5680	0.3790	0.3965
	0.80	1.8103	-0.6451	0.7104	-0.4432	0.4754	1.8013	0.6446	0.7094	0.4425	0.4744
0.75	0.20	1.0180	0.4777	0.4600	0.3165	0.3034	1.0211	-0.4759	0.4601	-0.3141	0.3025
	0.40	1.0651	0.4757	0.4715	0.3155	0.3090	1.0923	-0.4804	0.4779	-0.3162	0.3113
	0.60	1.1505	0.4806	0.5122	0.3372	0.3491	1.2628	-0.5129	0.5423	-0.3521	0.3638
	0.80	1.3097	0.4740	0.5726	0.3551	0.4030	1.7105	-0.6027	0.6859	-0.4216	0.4634

The solution can be used for cylinders and spheres when $B/r_i \leq 0.2$. In this case, the finite width correction factor should be set to 1.

Validity limits: (BS 7910 terminology)

$p/B \geq 0.2$ when $p+a \leq B/2$

$(B-(p+2a))/B \geq 0.2$ when $B-(p+2a) \leq B/2$

$0.25 \leq (p+a)/B \leq 0.75$

FKM Solution [A.22][A.2]

The FKM procedure contains a solution for a centrally located flaw only:

$$M_m = \frac{1-0.025\left(a/B\right)^2+0.06\left(a/B\right)^4}{\sqrt{\cos\left[\frac{\pi a}{2B}\right]}} \tag{A.33}$$

Validity limits:

None given

Plots

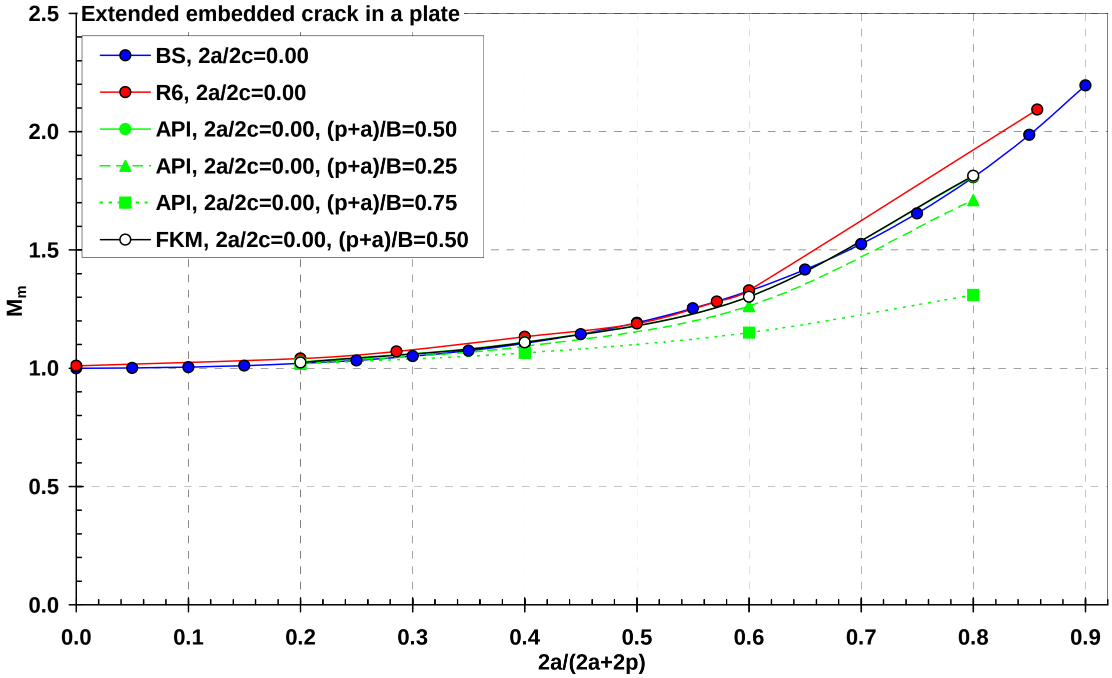
Figure A.5 compares the API and FKM functions with the R6 and BSI solutions for embedded flaws of low aspect ratio (see Section A.2.3.1).

The (p+a)/B variable shows crack position, with (p+a)/B=0.5 indicating a centrally located crack. Note that the API and FKM solutions are more or less coincident for the case of a centrally located crack.

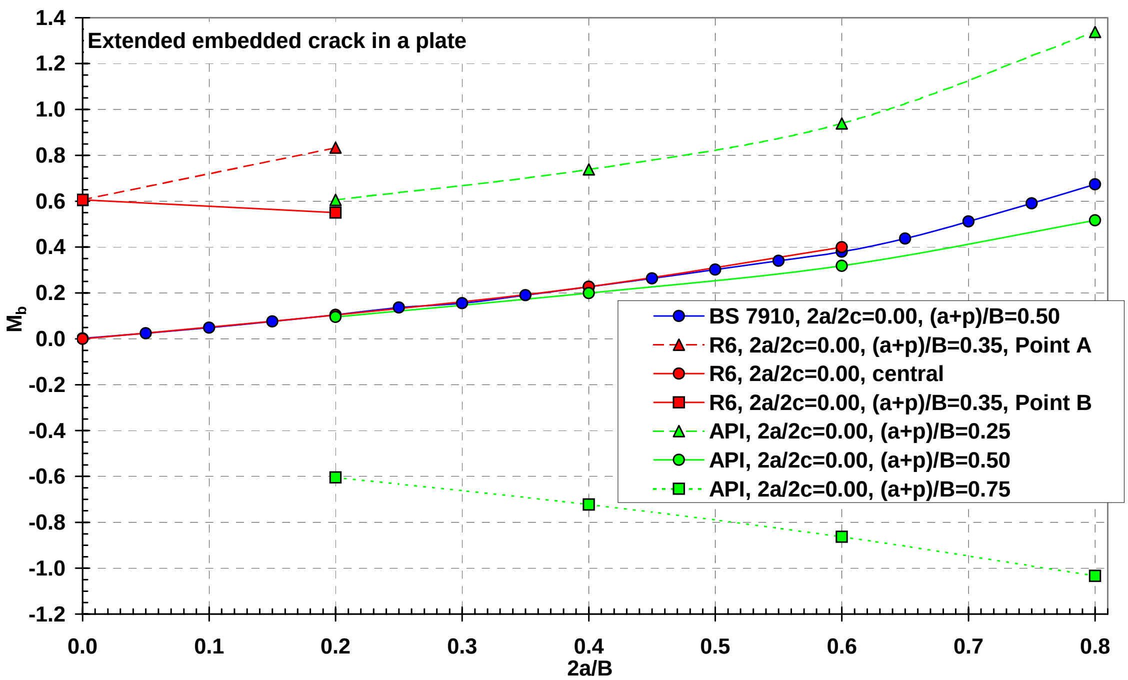
Note: The API curves in Figure A.5a are simply the coefficients (here designated M0) from Table C.4 of API 579, whilst the FKM curve is based on a spreadsheet.

In Figure A.5b, results for both centrally located and eccentric flaws under bending are shown. The R6 solutions for both Point A (adjacent to the smaller ligament) and B (larger ligament) are indicated, and both points are associated with a positive M_b as expected (the crack is relatively shallow so tensile stress acts throughout the ligament). The API results show M_b to be equal in magnitude and opposite, suggesting that K is evaluated at the same Point (A) but for a bending stress reversed in sign.

Figure A.5 Normalised K-solution for an extended embedded crack in a plate



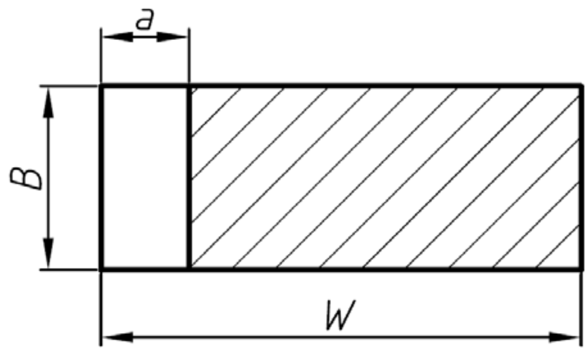
a) Membrane stress



b) Bending stress

A.2.4 Edge Crack

A.2.4.1 Single edge crack (tension)



BS 7910 Solution [A.17]

The stress intensity factor is given by equations (A.1 to (A.6, where, for $a/W \leq 0.6$, $M = 1$, $f_w = 1$ and:

$$M_m = 1.12 - 0.23\left(\frac{a}{W}\right) + 10.6\left(\frac{a}{W}\right)^2 - 21.7\left(\frac{a}{W}\right)^3 + 30.4\left(\frac{a}{W}\right)^4 \quad \text{(A.34)}$$

NOTE This solution has the same form as that for long surface cracks (equation (A.21) although the plate membrane and bending stresses have been superimposed. Equation (A.34 does not account for in-plane bending (eg a SENB specimen). In such cases, a modified form of the long surface crack solution may be used.

Validity limits

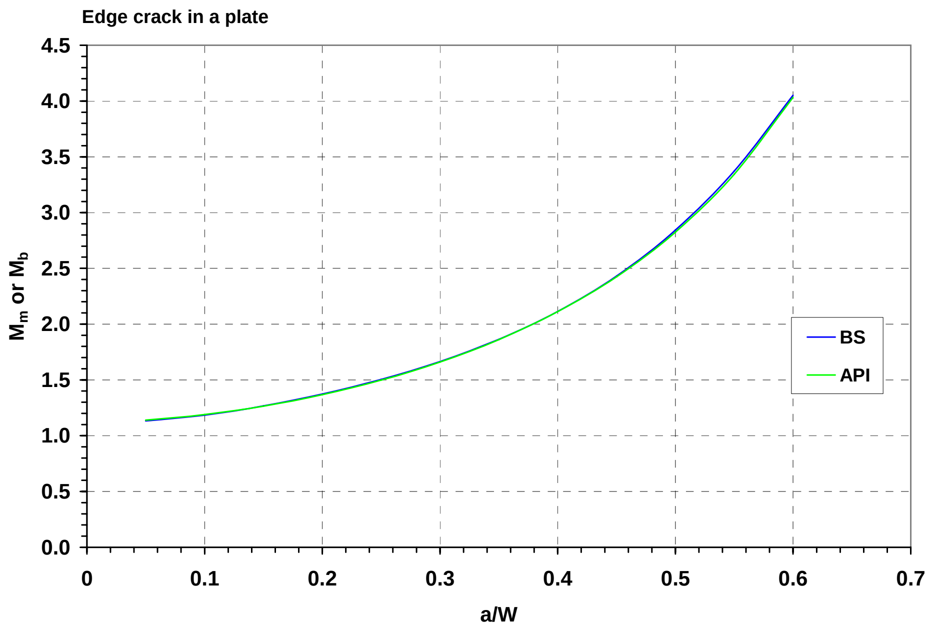
Solution does not account for in-plane bending (eg SENB specimen)

$a/W \leq 0.6$

R6, API 579

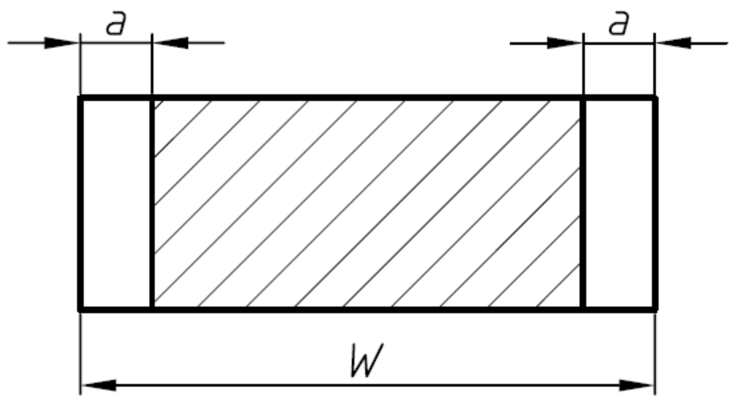
No solution available.

Figure A.6 Normalised K-solution for an edge crack in a plate



A.2.5 Double Edge Crack

A.2.5.1 Finite plate



BS 7910

No solution available

R6 solution [A.2]

The stress intensity factor K_I is given in terms of the remote uniform stress P by:

$$K_I = P\sqrt{\pi a}\, f(a/W) \tag{A.35}$$

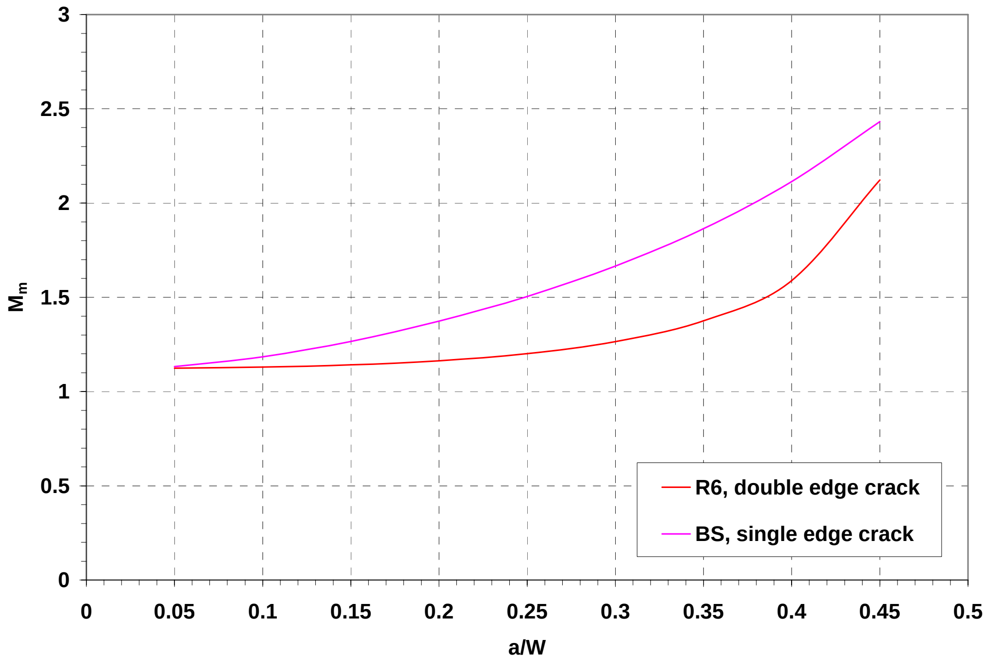
where

$$f(a/W) = \sqrt{\frac{1}{1-\left(\frac{2a}{W}\right)}}\left[1.122\left(1-\left(\frac{a}{W}\right)\right)-0.06\left(\frac{a}{W}\right)^2+0.728\left(\frac{a}{W}\right)^3\right] \tag{A.36}$$

Plot

Figure A.**7** shows the R6 solution for a double edge crack as a function of a/W, where W is the width of the whole plate, and compares it with the BS 7910 solution for a single edge crack (Section A.2.4).

Figure A.7 Normalised K-solution for a double edge crack in a plate; single-edge crack solution also shown for comparison



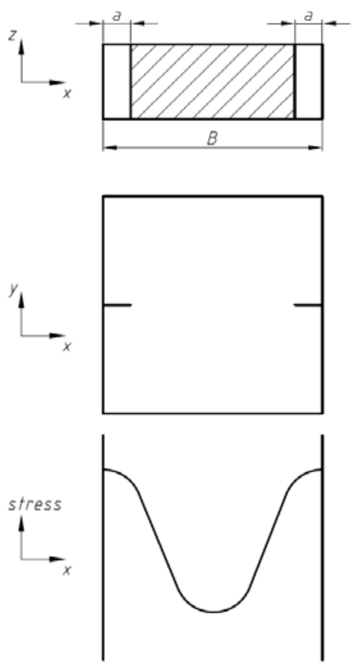
A.2.5.2 Infinite plate

BS 7910

No solution available

R6 Solution [A.23]

For a uniform stress, this is similar to the finite plate solution in Section A.2.5.1; here the plate thickness, 2t, replaces the plate width, W, of the earlier solution.



The stress intensity factor K_I for a stress distribution P(x) in the uncracked body which is symmetric about the centre-line of the plate with a value P_0 at the mouth of each crack (x=0) is

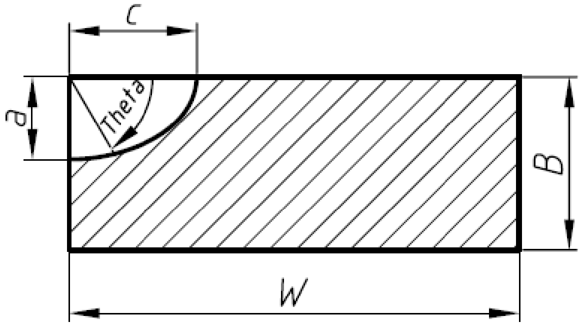
$$K_I = \sqrt{\frac{\pi a}{\left(1-\frac{2a}{B}\right)}}\left(P_0+\frac{2F}{B}\right)f\left(a/B\right) \tag{A.37}$$

$$f\left(a/B\right)=1.22\left(1-\frac{a}{B}\right)-0.06\frac{a}{B}+0.728\left(\frac{a}{B}\right)^3 \tag{A.38}$$

and

$$F=\int_0^a \frac{B/2-x}{\pi/2}\left(\frac{dP}{dx}\right)\cos^{-1}\left(\frac{x}{a}.\frac{B/2-a}{B/2x}\right)dx \tag{A.39}$$

A.2.6 Corner Crack



BS 7910 Solution [A.17]

The stress intensity factor is given by equations (A.1 to (A.6, where M = 1, with f_w given in equation (A.40

$$f_w = 1 - 0.2\lambda + 9.4\lambda^2 - 19.4\lambda^3 + 27.1\lambda^4 \qquad \text{for } c/W \le 0.5 \tag{A.40}$$

where:

$$\lambda = (c/W)\sqrt{(a/B)} \tag{A.41}$$

Solutions for M_m and M_b are given below

For Membrane loading

Conditions

$0.2 \le a/c \le 2$, $a/B < 1$, $0 \le \Theta \le \Pi/2$ and $c/W < 0.5$.

Solution

$$M_m = \left\{M_1 + M_2\left(a/B\right)^2 + M_3\left(a/B\right)^4\right\} g_1 g_2 f_\Theta / \Phi \tag{A.42}$$

Where:

Φ is defined in equations (A.14 (for 0≤a/c≤1) and (A.15 (for 1≤a/c≤2).

M_1	=	$1.08 - 0.03(a/c)$	for $0.2 \le a/c \le 1$
M_1	=	$\{1.08 - 0.03(c/a)\}(c/a)^{0.5}$	for $1 < a/c \le 2$
M_2	=	$\{1.06/(0.3 + a/c)\} - 0.44$	for $0.2 \le a/c \le 1$
M_2	=	$0.375(c/a)^2$	for $1 < a/c \le 2$
M_3	=	$-0.5 + 0.25(a/c) + 14.8(1 - a/c)^{15}$	for $0.2 \le a/c \le 1$
M_3	=	$-0.25(c/a)^2$	for $1 < a/c \le 2$
g_1	=	$1 + \{0.08 + 0.4(a/B)^2\}(1 - \sin\Theta)^3$	for $0.2 \le a/c \le 1$
g_1	=	$1 + \{0.08 + 0.4(c/B)^2\}(1 - \sin\Theta)^3$	for $1 < a/c \le 2$
g_2	=	$1 + \{0.08 + 0.15(a/B)^2\}(1 - \cos\Theta)^3$	for $0.2 \le a/c \le 1$
g_2	=	$1 + \{0.08 + 0.15(c/B)^2\}(1 - \cos\Theta)^3$	for $1 \le a/c \le 2$
f_Θ	=	$\{(a/c)^2\cos^2\Theta + \sin^2\Theta\}^{0.25}$	for $0.2 \le a/c \le 1$
f_Θ	=	$\{(c/a)^2\sin^2\Theta + \cos^2\Theta\}^{0.25}$	for $1 < a/c \le 2$

For Bending loading

Solution

$$M_b = HM_m \tag{A.43}$$

where M_m is given by equation (A.42 and:

H	=	$H_1 + (H_2 - H_1)\sin^q \Theta$	
q	=	$0.2 + (a/c) + 0.6(a/B)$	for $0.2 \le a/c \le 1$
q	=	$0.2 + (c/a) + 0.6(a/B)$	for $1 < a/c \le 2$
H_1	=	$1 - 0.34(a/B) - 0.11(a/c)(a/B)$	for $0.2 \le a/c \le 1$
H_1	=	$1 - \{0.04 + 0.41(c/a)\}(a/B) + \{0.55 - 1.93(c/a)^{0.75} + 1.38(c/a)^{1.5}\}(a/B)^2$	for $1 < a/c \le 2$
H_2	=	$1 + G_1(a/B) + G_2(a/B)^2$	

where

G_1	=	$-1.22 - 0.12(a/c)$	for $0.2 \le a/c \le 1$
G_1	=	$-2.11 + 0.77(c/a)$	for $1 < a/c \le 2$
G_2	=	$0.64 - 1.05(a/c)^{0.75} + 0.47(a/c)^{1.5}$	for $0.2 \le a/c \le 1$
G_2	=	$0.64 - 0.72(c/a)^{0.75} + 0.14(c/a)^{1.5}$	for $1 < a/c \le 2$

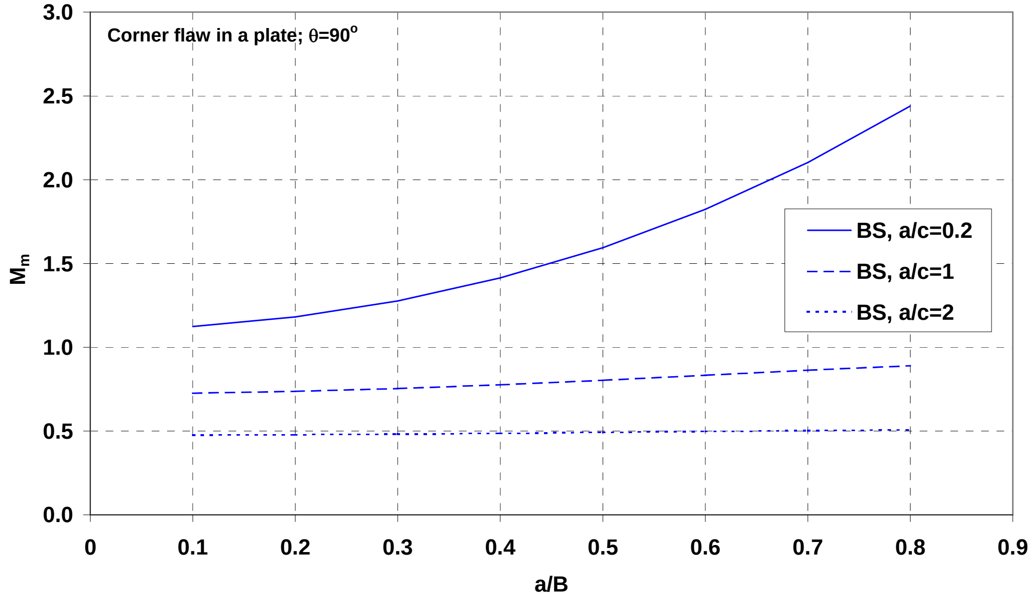
Validity limits:

$0.2 \le a/c \le 2$, $a/B < 1$, $0 \le \Theta \le \Pi/2$ and $c/W < 0.5$.

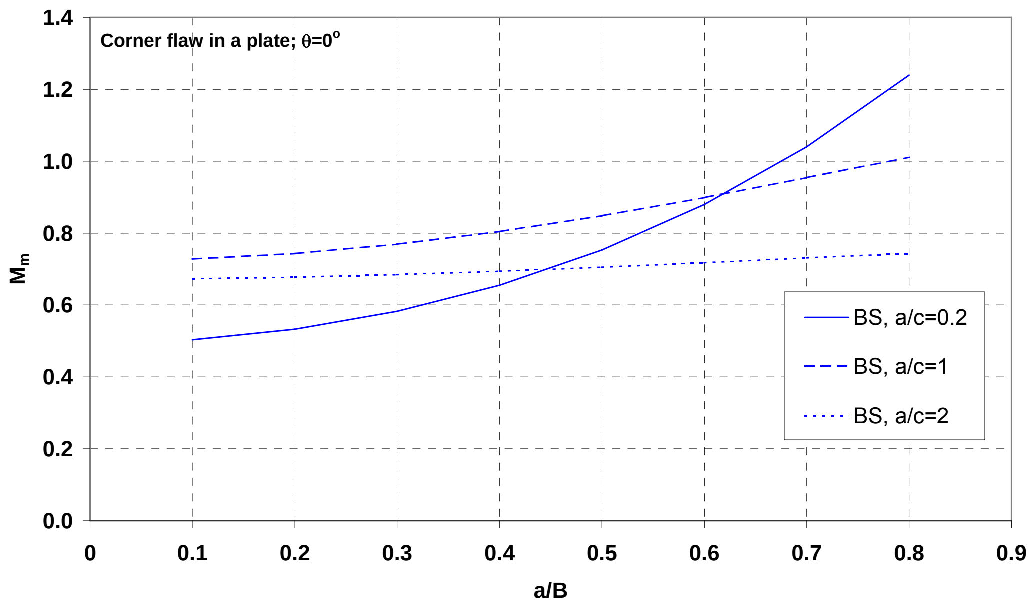
R6, API579

No solution available.

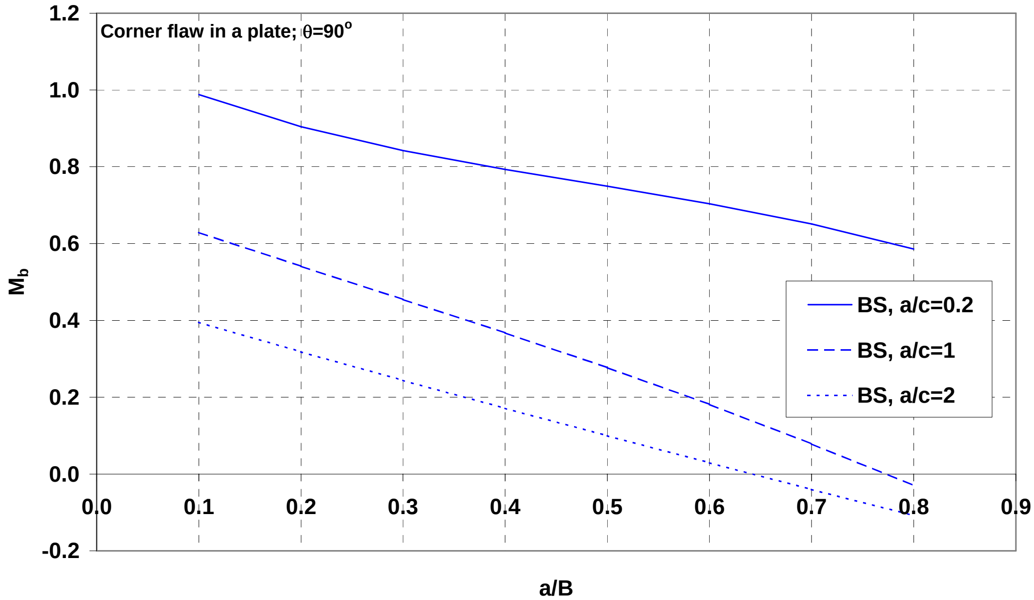
Figure A.8 Normalised K-solution for a corner crack in an infinite plate



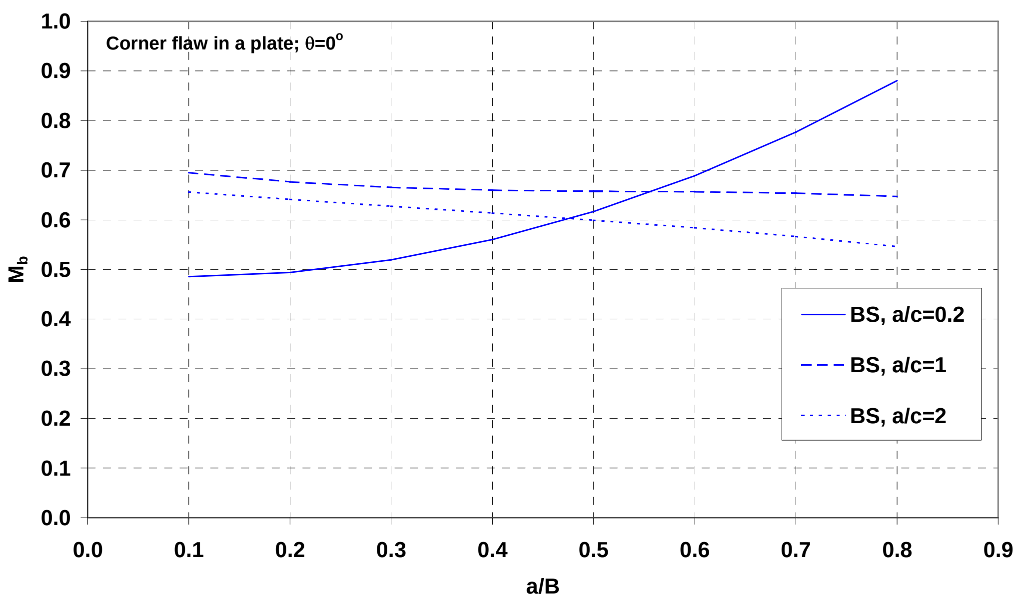
a) Membrane stress, θ=90°



b) Membrane stress, θ=0°

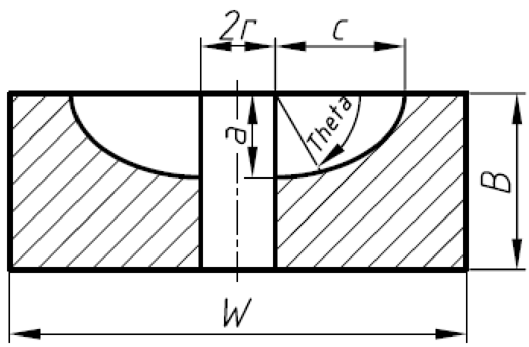


c) Bending stress, θ=90º



d) Bending stress, θ=0º

A.2.7 Corner Crack at a Hole (symmetric)



BS 7910 Solution [A.24]

Equations (A.1 to (A.6 give the stress intensity factor, where M = 1 and:

$$f_w = \left[\sec(\pi r / W) \sec\left\{ \frac{\pi(2r + nc)}{4(W/2 - c) + 2nc} \sqrt{(a/B)} \right\} \right]^{0.5} \quad \text{(A.44)}$$

where $n = 2$ for two symmetric cracks. Equation (A.44 differs from most of the other equations given for f_W in this Annex, in that it accounts both for finite width effects and the stress concentrating effect of the hole. Solutions for M_m and M_b are given in equations (A.45 and (A.46.

For membrane loading

Conditions

$0.2 \leq a/c \leq 2$
$a/B < 1$
$0.5 \leq r/B \leq 2$
$2(r + c)/W \leq 0.5$
$0 \leq \Theta \leq \Pi/2$

Solution

$$M_m = \left\{ M_1 + M_2 (a/B)^2 + M_3 (a/B)^4 \right\} g_1 g_2 g_3 g_4 f_\Theta / \Phi \quad \text{(A.45)}$$

where Φ is defined in equation (A.14 and (A.15

M_1	=	$1.13 - 0.09(a/c)$	for $0.2 \leq a/c \leq 1$
M_1	=	$\{1 + 0.04(c/a)\} \sqrt{(c/a)}$	for $1 < a/c \leq 2$
M_2	=	$-0.54 + 0.89/(0.2 + a/c)$	for $0.2 \leq a/c \leq 1$
M_2	=	$0.2(c/a)^4$	for $1 < a/c \leq 2$
M_3	=	$0.5 - 1/(0.65 + a/c) + 14(1 - a/c)^{24}$	for $0.2 \leq a/c \leq 1$

M_3	=	$-0.11(c/a)^4$	for $1 < a/c \leq 2$
g_1	=	$1 + \{0.1 + 0.35(a/B)^2\}(1 - \sin\Theta)^2$	for $0.2 \leq a/c \leq 1$
g_1	=	$1 + \{0.1 + 0.35(c/a)(a/B)^2\}(1 - \sin\Theta)^2$	for $1 < a/c \leq 2$
g_2	=	$(1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4)/(1 + 0.13\lambda^2)$	

where

λ	=	$1/\{1 + (c/r)\cos(\mu\Theta)\}$	
μ	=	0.85	
g_3	=	$(1 + 0.04a/c)\{1 + 0.1(1 - \cos\Theta)^2\}\{0.85 + 0.15(a/B)^{0.25}\}$	for $0.2 \leq a/c \leq 1$
g_3	=	$(1.13 - 0.09c/a)\{1 + 0.1(1 - \cos\Theta)^2\}\{0.85 + 0.15(a/B)^{0.25}\}$	for $1 < a/c \leq$??
g_4	=	$1 - 0.7(1 - a/B)(a/c - 0.2)(1 - a/c)$	for $0.2 \leq a/c \leq 1$
g_4	=	1	for $1 < a/c \leq 2$
f_Θ	=	$\{(a/c)^2\cos^2\Theta + \sin^2\Theta\}^{0.25}$	for $0.2 \leq a/c \leq 1$
f_Θ	=	$\{(c/a)^2\sin^2\Theta + \cos^2\Theta\}^{0.25}$	for $1 < a/c \leq 2$

For bending loading

Solution

$$M_b = HM_m \quad \text{(A.46)}$$

where

M_m is given in equation (A.45;

μ	=	$0.85 - 0.25(a/B)^{0.25}$	
H	=	$H_1 + (H_2 - H_1)\sin^q\Theta$	
q	=	$0.1 + 1.3a/B + 1.1a/c - 0.7(a/c)(a/B)$	for $0.2 \leq a/c \leq 1$
q	=	$0.2 + c/a + 0.6a/B$	for $1 < a/c \leq 2$
H_1	=	$1 + G_{11}(a/B) + G_{12}(a/B)^2 + G_{13}(a/B)^3$	
H_2	=	$1 + G_{21}(a/B) + G_{22}(a/B)^2 + G_{23}(a/B)^3$	

σ_0 σ_0 where

G_{11}	=	$-0.43 - 0.74(a/c) - 0.84(a/c)^2$	for $0.2 \leq a/c \leq 1$
G_{11}	=	$-2.07 + 0.06(c/a)$	for $1 < a/c \leq 2$
G_{12}	=	$1.25 - 1.19(a/c) + 4.39(a/c)^2$	for $0.2 \leq a/c \leq 1$
G_{12}	=	$4.35 + 0.16(c/a)$	for $1 < a/c \leq 2$
G_{13}	=	$-1.94 + 4.22(a/c) - 5.51(a/c)^2$	for $0.2 \leq a/c \leq 1$
G_{13}	=	$-2.93 - 0.3(c/a)$	for $1 < a/c \leq 2$
G_{21}	=	$-1.5 - 0.04(a/c) - 1.73(a/c)^2$	for $0.2 \leq a/c \leq 1$
G_{21}	=	$-3.64 + 0.37(c/a)$	for $1 < a/c \leq 2$
G_{22}	=	$1.71 - 3.17(a/c) + 6.84(a/c)^2$	for $0.2 \leq a/c \leq 1$
G_{22}	=	$5.87 - 0.49(c/a)$	for $1 < a/c \leq 2$
G_{23}	=	$-1.28 + 2.71(a/c) - 5.22(a/c)^2$	for $0.2 \leq a/c \leq 1$
G_{23}	=	$-4.32 + 0.53(c/a)$	for $1 < a/c \leq 2$

Validity limits

$0.2 \leq a/c \leq 2$

$a/B < 1$

$0.5 \leq r/B \leq 2$

$2(r+c)/W \leq 0.5$

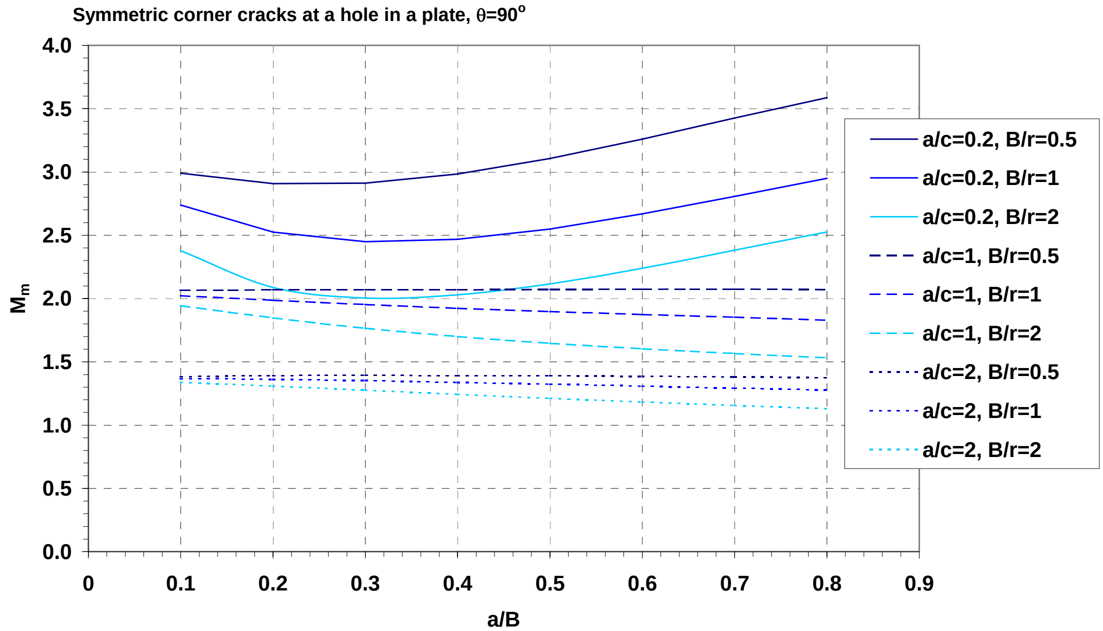
$0 \leq \theta \leq \pi/2$

R6

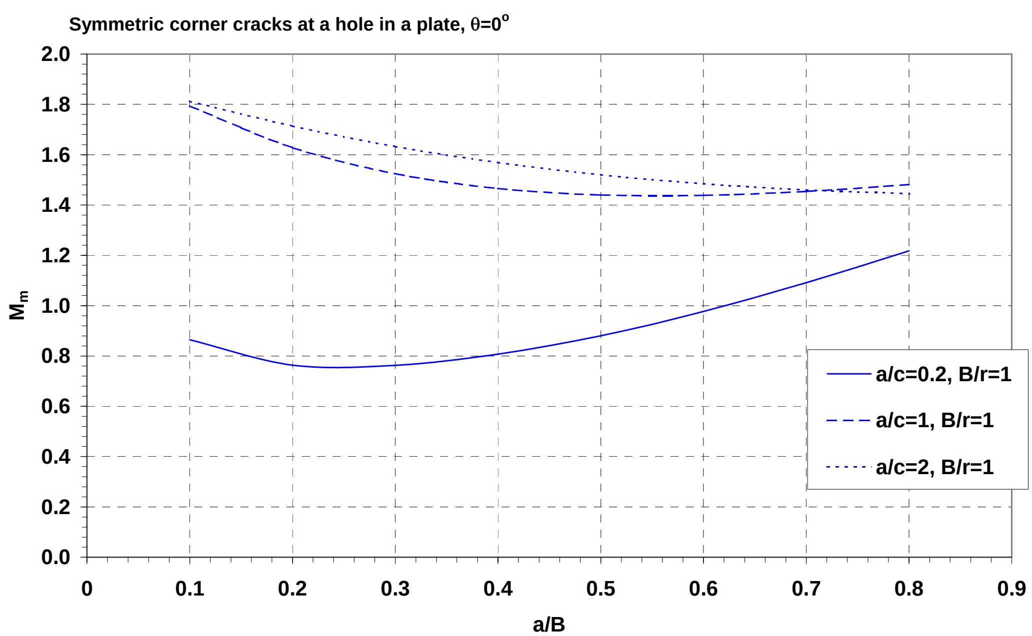
No solution available.

Figure A.9 shows the normalised K-solution for a pair of symmetric flaws in an infinite plate, for various ratios of B/r (where r is the radius of the hole) and a/c, the flaw aspect ratio.

Figure A.9 Normalised K-solution for a pair of symmetric corner cracks at a hole

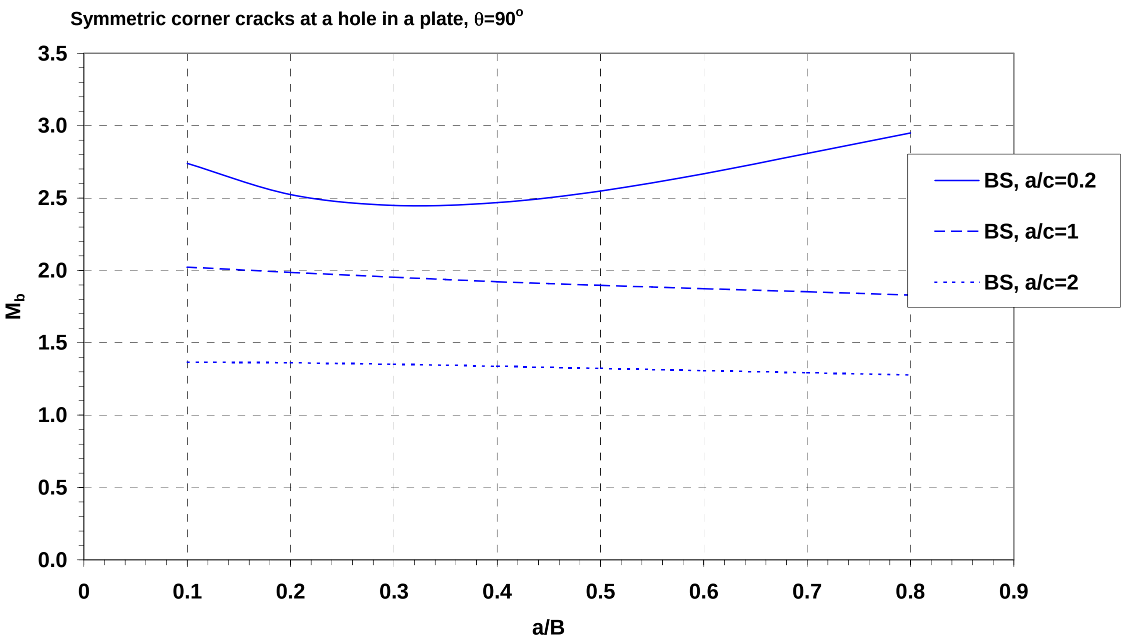


a) Membrane stress, θ=90°

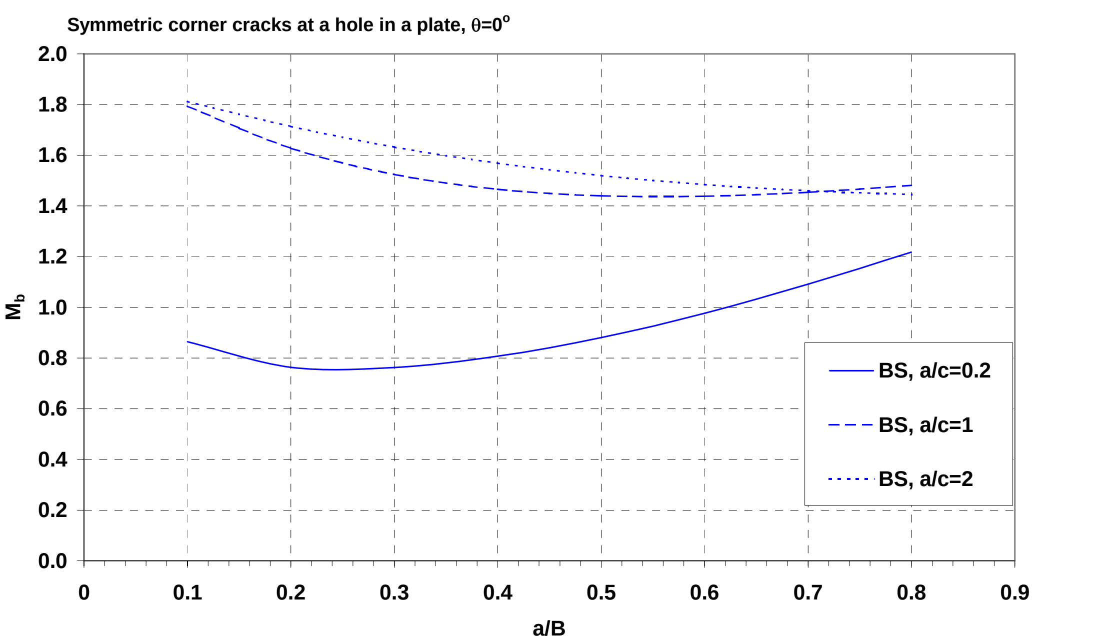


b) Membrane stress, θ=0°

Normalised K-solution for a pair of symmetric corner cracks at a hole (cont'd)

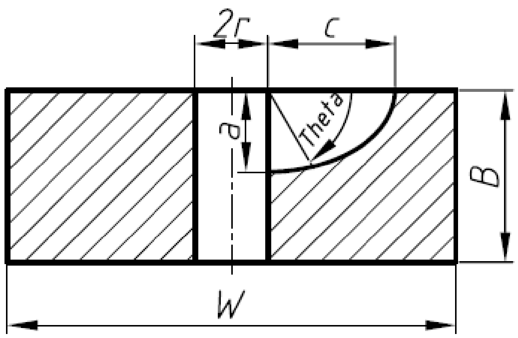


c) Bending stress, θ=90°, B/r=1



d) Bending stress, θ=0°, B/r=1

A.2.8 Corner Crack at a Hole (single)



BS 7910 Solution [A.24]

The stress intensity factor for a single corner crack at a hole ($K_{single\ crack}$) may be estimated from $K_{symmetric\ crack}$ (Section A.2.7) using the following expression:

$$f_w = \left[\sec\left(\pi r / W\right) \sec\left\{ \frac{\pi\left(2r + nc\right)}{4\left(W/2 - c\right) + 2nc} \sqrt{\left(a/B\right)} \right\} \right]^{0.5} \quad \text{(A.47)}$$

where n=1 for a single flaw. Equation (A.47 differs from most of the other equations given for f_W in this Annex, in that it accounts both for finite width effects and the stress concentrating effect of the hole.

$K_{symmetric\ crack}$ for an infinite plate with a hole is found simply by modifying the equation for a symmetric crack as follows:

$$K_{single\ crack} = K_{symmetric\ crack} \left(\frac{\frac{4}{\pi} + \frac{ac}{2Br}}{\frac{4}{\pi} + \frac{ac}{Br}} \right)^{0.5} \quad \text{(A.48)}$$

where $K_{summetric\ crack}$ is found from equations (A.1 to (A.6 with M_m and M_b from equations (A.45 and (A.46.

Validity limits

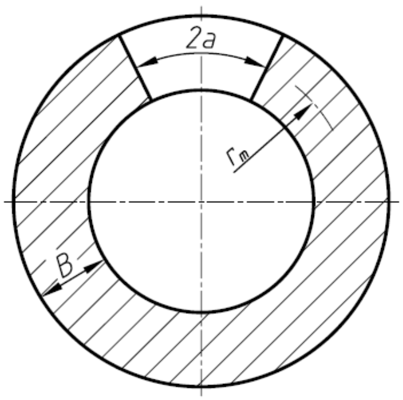
as for the symmetric crack

R6

No solution available.

A.3 Spheres

A.3.1 Through-thickness Equatorial Crack



BS 7910 Solution [A.25]

The stress intensity factor solution is calculated from equations (A.1 to (A.6 where:

$M = f_w = 1$;

M_m and M_b are given in Table A.7.

Table A.7 BS 7910 solutions for M_m and M_b for a through-thickness crack in a spherical shell

$B/r_i = 0.05$					$B/r_i = 0.1$				
2a/B	M_m(o)	M_b(o)	M_m(i)	M_b(i)	2a/B	M_m(o)	M_b(o)	M_m(i)	M_b(i)
0.0	1.000	1.000	1.000	–1.000	0.0	1.000	1.000	1.000	–1.000
2.0	1.144	1.020	0.941	–0.995	2.0	1.240	1.031	0.919	–0.993
4.0	1.401	1.050	0.897	–0.992	4.0	1.637	1.074	0.894	–0.993
6.0	1.700	1.080	0.895	–0.993	6.0	2.083	1.111	0.944	–0.997
8.0	2.020	1.106	0.932	–0.996	8.0	2.549	1.143	1.059	–1.003
10.0	2.351	1.130	1.003	–1.001	10.0	3.016	1.170	1.231	–1.011
15.0	3.186	1.180	1.309	–1.014	15.0	4.124	1.226	1.915	–1.031
20.0	3.981	1.219	1.799	–1.028	20.0	5.084	1.272	2.968	–1.050
NOTE (o) is for the intersection of the crack with the outside surface, and (i) the inner.									

Range of application: $0 \leq 2a/B \leq 20$
$0.05 \leq B/r_i \leq 0.1$

Validity limits

$0 \leq 2a/B \leq 20$

$0.05 \leq B/r_i \leq 0.1$

R6 Solution [A.26][A.27]

The stress distribution consists of a uniform stress P_m and a through-wall bending stress P_b. The stress intensity factors at Points A (internal surface) and B (external surface) in the sketch above are as follows (the bending stress is assumed positive at the surface of interest):

$$K_A = \left(P_m G_3 + P_b H_3\right)\sqrt{\pi a} \tag{A.49}$$

$$K_B = \left(P_m G_4 + P_b H_4\right)\sqrt{\pi a} \tag{A.50}$$

where:

$$p = \frac{a}{\sqrt{r_m B}}$$

where:

$$r_m = \left(r_i + B/2\right)$$

and:

$$G_3 = 1 - 0.26066\rho + 0.88766\rho^2 + 0.015826\rho^3 - 0.025266\rho^4 + \frac{\left(2.99573 - \ln(r_m/B)\right)}{1.38629}(0.26785\rho - 0.39378\rho^2 + 0.383574\rho^3 - 0.095384\rho^4) \tag{A.51}$$

$$G_4 = 1 + 0.41551\rho + 0.82404\rho^2 - 0.45458\rho^3 + 0.076714\rho^4 + \frac{\left(2.99573 - \ln(r_m/B)\right)}{1.38629}(-0.05409\rho - 0.24698\rho^2 + 0.35622\rho^3 - 0.099022\rho^4) \tag{A.52}$$

$$H_3 = 0.967 - 2.5204\rho + 6.8405\rho^2 - 10.214\rho^3 + 8.0057\rho^4 - 3.1394\rho^5 + 0.48611\rho^6 + \frac{\left(4.60517 - \ln(r_m/B)\right)}{2.99573}(0.183 + 0.4921\rho - 3.7129\rho^2 + 7.1292\rho^3 - 6.2412\rho^4 + 2.59325\rho^5 - 0.414499\rho^6) \tag{A.53}$$

$$H_4 = 0.92 - 1.3932\rho + 2.7674\rho^2 - 3.0555\rho^3 + 1.7261\rho^4 - 0.45873\rho^5 + 0.043402\rho^6 \tag{A.54}$$

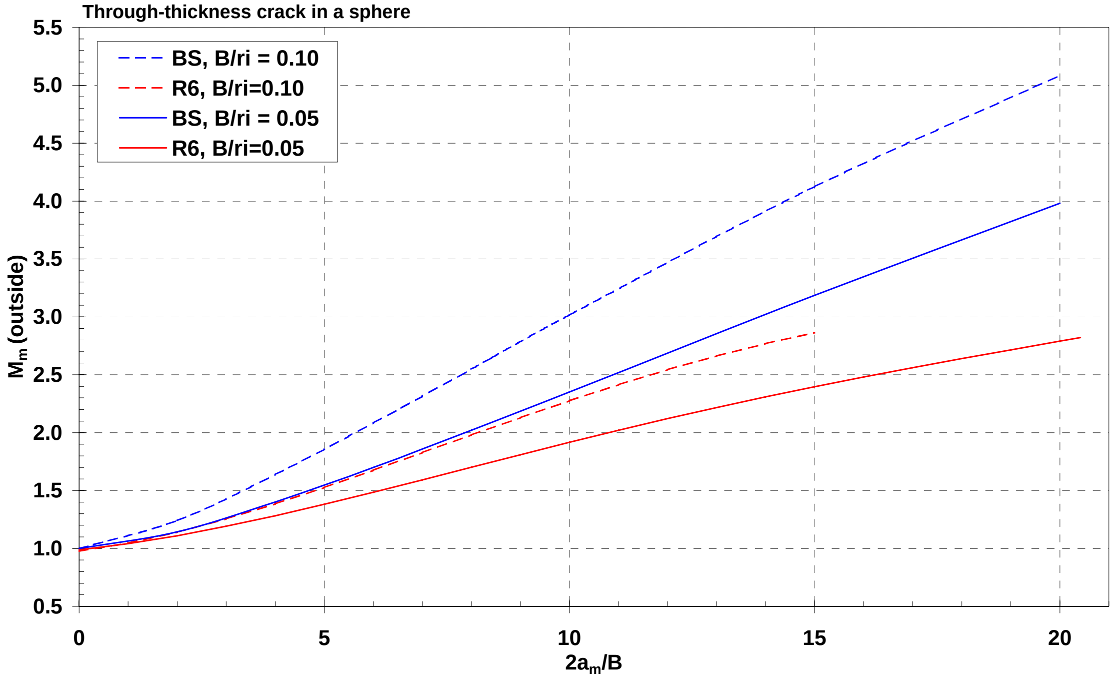
Validity limits

$0 \leq \rho \leq 2.2$, thin shells ($B/r_m \leq 0.1$. Note that the lower limit of B/r_m is not known, so the solution must be used with caution below $B/r_m=0.01$.

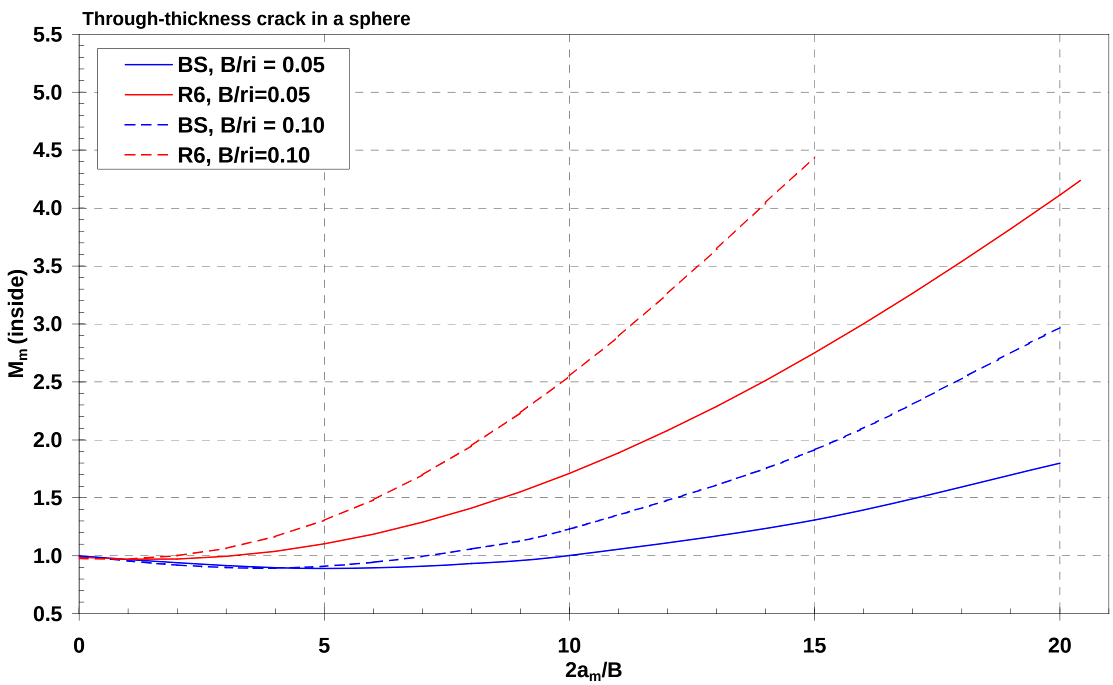
Plots

Figure A.10 shows the BS 7910 and R6 solutions for two ratios of B/r_i: 0.05 and 0.1. The two sets of results differ widely, especially under bending stress. Note that the R6 solutions are reasonably consistent with the trends for through-thickness circumferential and axial flaws in cylinders (see Sections A.4.1.1 and A.4.2.1).

Figure A.10 Normalised K-solution for a through-thickness crack in a sphere

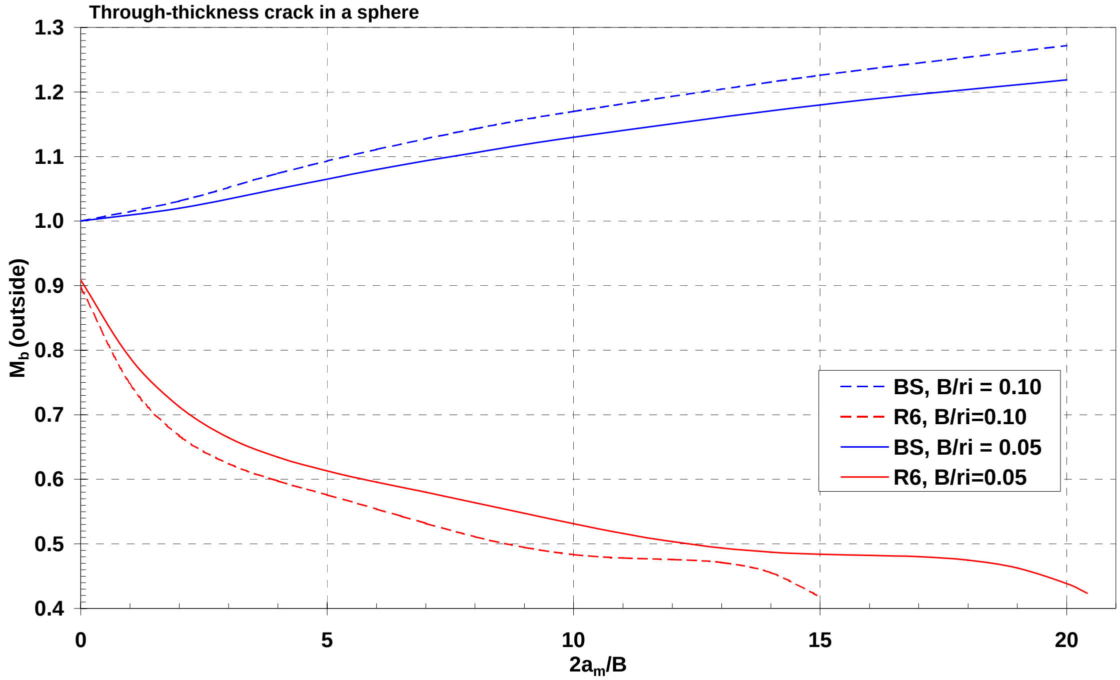


a) Membrane stress, outside

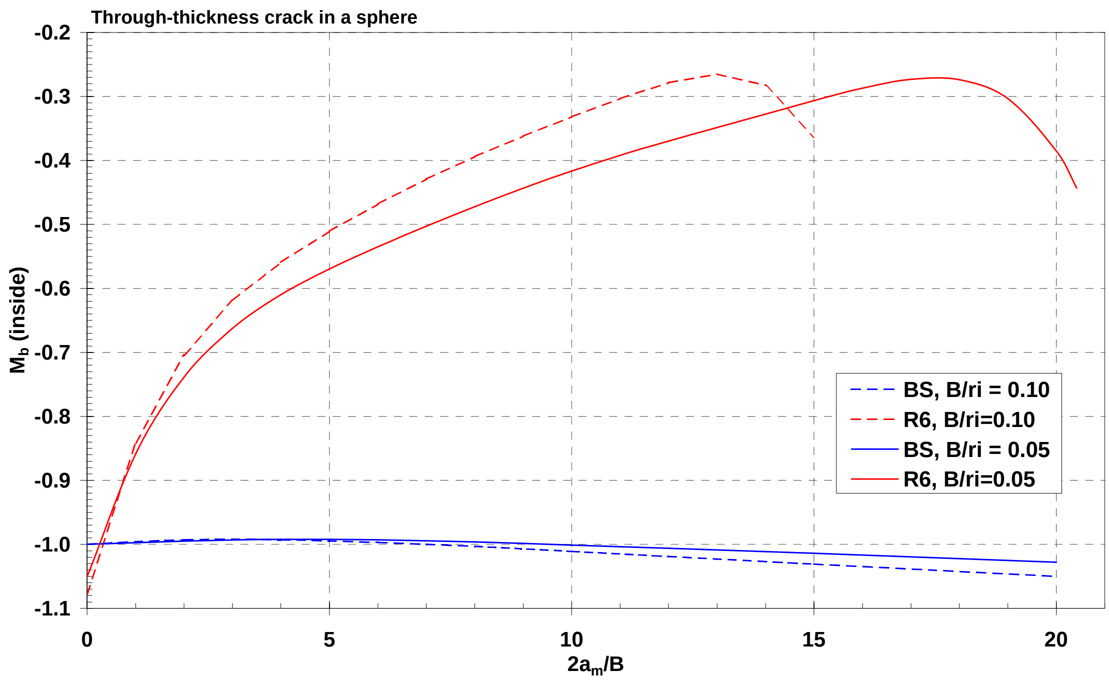


b) Membrane stress, inside

Normalised K-solution for a through-thickness crack in a sphere (cont'd)



c) Bending stress, outside



d) Bending stress, inside

A.3.2 Surface Crack

BS 7910 Solution

Flat plate solutions for M_m ((A.13) and M_b ((A.16) are recommended, with.

$$M = \frac{1-\left\{a/\left(BM_T\right)\right\}}{1-\left(a/B\right)} \quad \text{(A.55)}$$

where:

$$M_T = \left\{1+3.2\left(c^2/2r_mB\right)\right\}^{0.5} \quad \text{(A.56)}$$

Validity* limits*:

$0 \le a/2c \le 1.0$

$0 \le \theta \le \pi$

$a/B < 1.0$ for $0.1 \le a/2c \le 1.0$

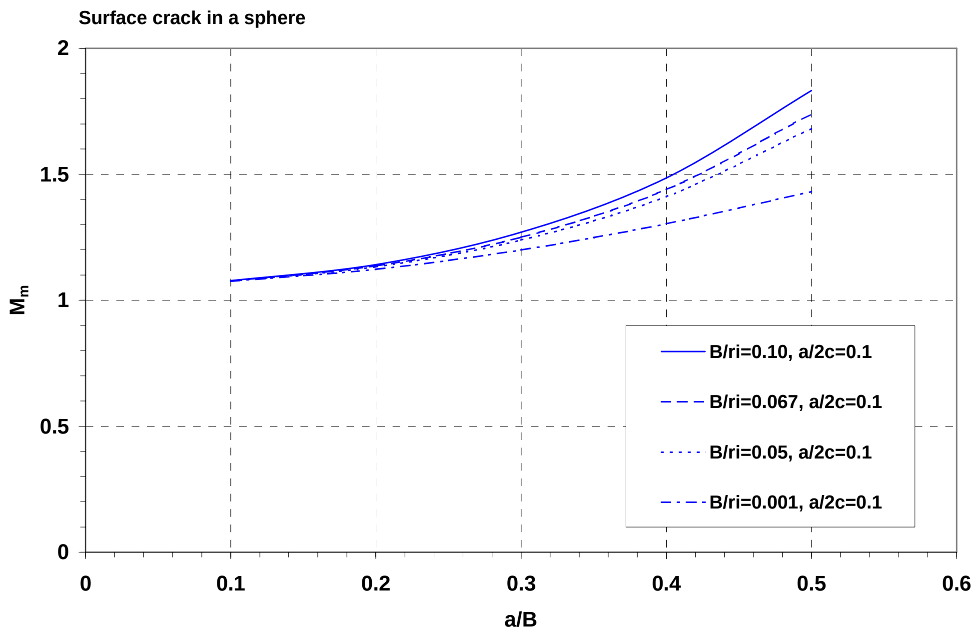
R6 solution

For internal and external part-circumferential equatorial surface flaws, R6 refers to API 579 solutions. However, it does not recommend their use in assessments.

Plots

Figure A.11 shows the solution as a function of crack depth for various B/r_i ratios and for a constant crack shape, a/2c=0.1. A tighter radius (higher B/r_i) is associated with a higher value of M_m; for lower B/r_i ratios, the solution matches that for a surface flaw in a plate.

Figure A.11 Normalised K-solution for a surface crack in a sphere



a) Membrane stress, θ=90º

A.3.3 Embedded Crack

BS 7910 Solution

M=1

Flat plate solutions for M_m ((A.13) and M_b ((A.16) are recommended

Validity limits:

For M_m

$0 \leq a/2c \leq 1.0$

$2c/W < 0.5$

$-\pi \leq \theta \leq \pi$

$a/B' < 0.625$ for $0 \leq a/2c \leq 0.1$ where $B' = 2a + 2p$

For M_b

$0 \leq a/2c \leq 0.5$

$\Theta = \Pi / 2$

(ie solution only refers to the ends of the minor axis of the elliptical crack).

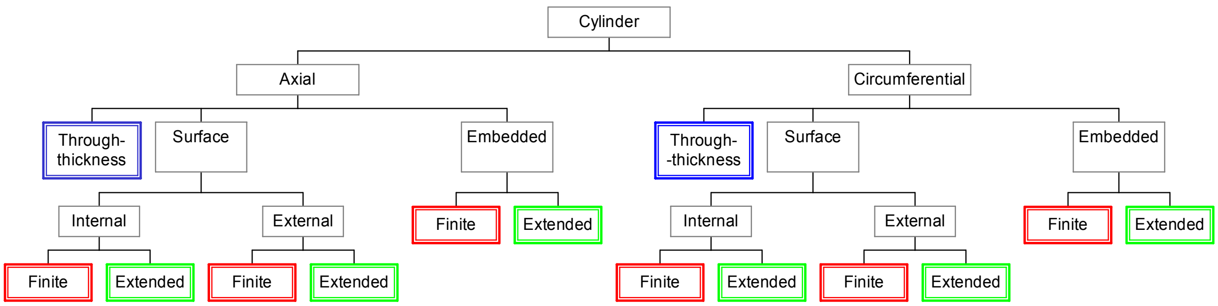
R6

No solution available

A.4 Pipes or Cylinders

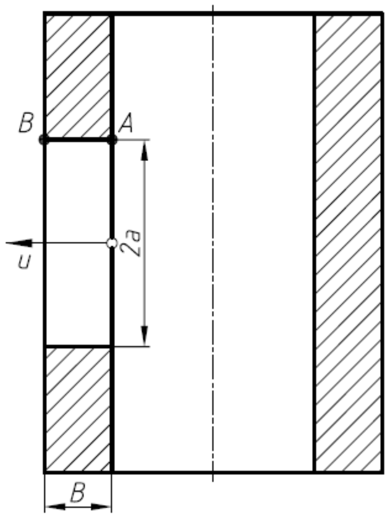
This section summarises K-solutions for cylinders and pipes with both axial and circumferential cracks; see Figure A.12. Guidance is also given on situations not covered by the current solutions, where flat plate solutions (with a bulging correction factor, where necessary) can be used.

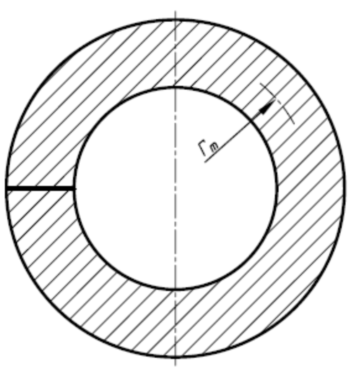
Figure A.12 Summary of cylindrical geometries considered in this section



A.4.1 Pipes or Cylinders with Axial Cracks

A.4.1.1 Through-thickness cracks





BS 7910 solution [A.28]

The stress intensity factor solution is calculated from equation (A.1 to (A.6

where

$K_I = K_I^{pressure} + K_I^{bending}$

M = 1 [Note, bulging is taken into account by the parameter λ: (see equation (A.59) and:

$$M_m \; M_1 + M_2 \text{ at the outer surface and } M_1 - M_2 \text{ at the inner surface} \quad \text{(A.57)}$$

$$M_b = M_3 + M_4 \text{ at the outer surface and } M_3 - M_4 \text{ at the inner surface} \quad \text{(A.58)}$$

Where:

$K_I^{pressure}$ and $K_I^{bending}$ are calculated from equation (A.1 to (A.6 and represent, respectively, contributions to K_I of pressure-induced membrane stresses and through-wall bending stresses.

M_1 to M_4 are given in Table A.8a-d for pressure and bending loading, in terms of λ:

$$\lambda = \left\{12\left(1-\nu^2\right)\right\}^{0.25} \frac{a}{\sqrt{r_m}} \tag{A.59}$$

NOTE The stress intensity magnification factors at the outside (o) and inside (i) surfaces are given by $M_m^* + M_b^*$, and $M_m^* - M_b^*$ respectively. These solutions are valid for long cylinders, or pressure vessels with closed ends.

Table A.8 BS 7910 Coefficients for axial through-thickness cracks in cylinders

a) M_1 for pressure loading

Parameter, λ	B/r_m=0.2	B/r_m=0.1	B/r_m=0.05	B/r_m=0.02	B/r_m=0.01
0.000	1.000	1.000	1.000	1.000	1.000
0.862		1.158			
0.910					1.264
1.016	1.433		1.249		
1.285				1.383	
1.818					1.609
1.928				1.663	
2.012		1.636			
2.032	1.912		1.691		
3.636					2.543
3.856				2.642	
4.024		2.604			
4.065	3.133		2.709		
5.784				3.613	
6.036		3.527			
6.097	4.116		3.65*		
6.362					3.927
7.712				4.534	
7.926	4.980				
8.048		4.377			
8.130			4.605		
8.186					4.799
9.959	5.873				
9.998					5.628
10.162			5.463		
10.283				5.688	
11.816					6.416
11.991	6.687				
12.072		5.874			
12.194			6.257		
12.211				6.503	

* BS 7910 and the original reference by France et al give a value of 3.369 for λ=6.097, B/r_m=0.05. This value lies outside the smooth trend of the other points, and an error is suspected. Substitution of the value M_1 =3.65 (obtained by interpolation), produces a smooth curve as shown in Figure A.13.

b) M_2 for Pressure loading

Parameter, λ	B/r_m=0.2	B/r_m=0.1	B/r_m=0.05	B/r_m=0.02	B/r_m=0.01
0.000	0.000	0.000	0.000	0.000	0.000
0.862		0.093			
0.910					0.143
1.016	0.098		0.125		
1.285				0.165	
1.818					0.229
1.928				0.205	
2.012		0.156			
2.032	0.143		0.182		
3.636					0.218
3.856				0.161	
4.024		0.041			
4.065	–0.030		0.089		
5.784				–0.077	
6.036		–0.264			
6.097	–0.419		-0.2*		
6.362					–0.126
7.712				–0.436	
7.926	–0.851				
8.048		–0.684			
8.130			–0.622		
8.186					–0.475
9.959	–1.358				
9.998					–0.884
10.162			–1.122		
10.283				–1.034	
11.816					–1.339
11.991	–1.829				
12.072		–1.718			
12.194			–1.700		
12.211				–1.543	

* BS 7910 and the original reference by France et al give a value of –0.399 for λ=6.097, B/r_m=0.05. This value lies outside the smooth trend of the other points, and an error is suspected. Substitution of the value M_2 =-0.2 (obtained by interpolation), produces a smooth curve as shown in Figure A.13.

***c) M_3* for Bending loading**

Parameter, λ	B/r_m=0.2	B/r_m=0.1	B/r_m=0.05	B/r_m=0.02	B/r_m=0.01
0.000	0.000	0.000	0.000	0.000	0.000
0.862		0.040			
0.910					0.025
1.016	0.053		0.040		
1.285				0.042	
1.818					0.055
1.928				0.060	
2.012		0.075			
2.032	0.083		0.068		
3.636					0.095
3.856				0.097	
4.024		0.109			
4.065	0.121		0.103		
5.784				0.119	
6.036		0.128			
6.097	0.139		0.123		
6.362					0.127
7.712				0.134	
7.926	0.150				
8.048		0.138			
8.130			0.135		
8.186					0.139
9.959	0.161				
9.998					0.147
10.162			0.143		
10.283				0.145	
11.816					0.151
11.991	0.171				
12.072		0.150			
12.194			0.146		
12.211				0.150	

d) M_4 for Bending loading

Parameter, λ	B/r_m=0.2	B/r_m=0.1	B/r_m=0.05	B/r_m=0.02	B/r_m=0.01
0.000	1.000	1.000	1.000	1.000	1.000
0.862		0.694			
0.910					0.637
1.016	0.701		0.659		
1.285				0.629	
1.818					0.598
1.928				0.600	
2.012		0.608			
2.032	0.604		0.602		
3.636					0.527
3.856				0.529	
4.024		0.517			
4.065	0.493		0.524		
5.784				0.474	
6.036		0.453			
6.097	0.417		0.467		
6.362					0.448
7.712				0.430	
7.926	0.364				
8.048		0.403			
8.130			0.421		
8.186					0.407
9.959	0.314				
9.998					0.374
10.162			0.382		
10.283				0.381	
11.816					0.348
11.991	0.276				
12.072		0.328			
12.194			0.355		
12.211				0.353	

Validity limits

Range of application: $0 \le \lambda \le 12.211$
$0.01 \le B/r_m \le 0.2$

R6 solution [A.26]

K is defined for a stress distribution P which varies linearly through the cylinder wall with the co-ordinate u (see sketch above), and which does not vary along the length of the cylinder:

$$P(u) = P_m + P_b(2u/B - 1) \quad \text{for } 0 \le u \le B \tag{A.60}$$

where P_m is the average uniform membrane (hoop) stress and P_b is the maximum through wall bending stress. The stress intensity factors at the Points A (inside wall) and B (outside wall) are given by

$$K_I^A = P_m . \sqrt{\pi a}\,(G_1(p) - g_1(p)) + P_b\sqrt{\pi a}\,(-H_1(p) + h_1(p)) \tag{A.61}$$

$$K_I^B = P_m . \sqrt{\pi a}\,(G_1(\rho) + g_1(\rho)) + P_b\sqrt{\pi a}(H_1(\rho) + h_1(\rho)) \tag{A.62}$$

where:

$$\rho = a / \sqrt{r_m / B} \tag{A.63}$$

with r_m being the mean radius of the cylinder. The functions G_1, g_1, H_1, h_1 are given by:

$$\begin{aligned} G_1(\rho) &= \sqrt{1 + 0.7044\rho + 0.8378\rho^2} \\ g_1(\rho) &= -0.035211 + 0.39394\rho - 0.20036\rho^2 + 0.028085\rho^3 \\ &\quad - 0.0018763\rho^4 + \frac{(3.912 - \ln(r_m/B))}{1.6094} . \begin{pmatrix} 0.01556 - 0.05202\rho + \\ 0.0381\rho^2 - 0.012782\rho^3 \\ +0.001246\rho^4 \end{pmatrix} \end{aligned} \tag{A.64}$$

$$H_1(\rho) = 0.76871 - 0.27718\rho + 0.14343\rho^2 - 0.037505\rho^3 + 0.0035194\rho^4 + \frac{\left(3.912 - \ln\left(r_m/B\right)\right)}{1.6094} \cdot \begin{pmatrix} 0.09852 - 0.16404\rho + \\ 0.10378\rho^2 - 0.027703\rho^3 \\ +0.002597\rho^4 \end{pmatrix} \quad (A.65)$$

$$h_1(\rho) = -0.0030702 + 0.074457\rho - 0.018716\rho^2 + 0.0025344\rho^3 - 0.00014028\rho^4 + \frac{\left(3.912 - \ln\left(r_m/B\right)\right)}{1.6094} \cdot \begin{pmatrix} 0.0005847 + 0.010301\rho \\ -0.007184\rho^2 \\ +0.0019107\rho^3 \\ -0.00017655\rho^4 \end{pmatrix}$$

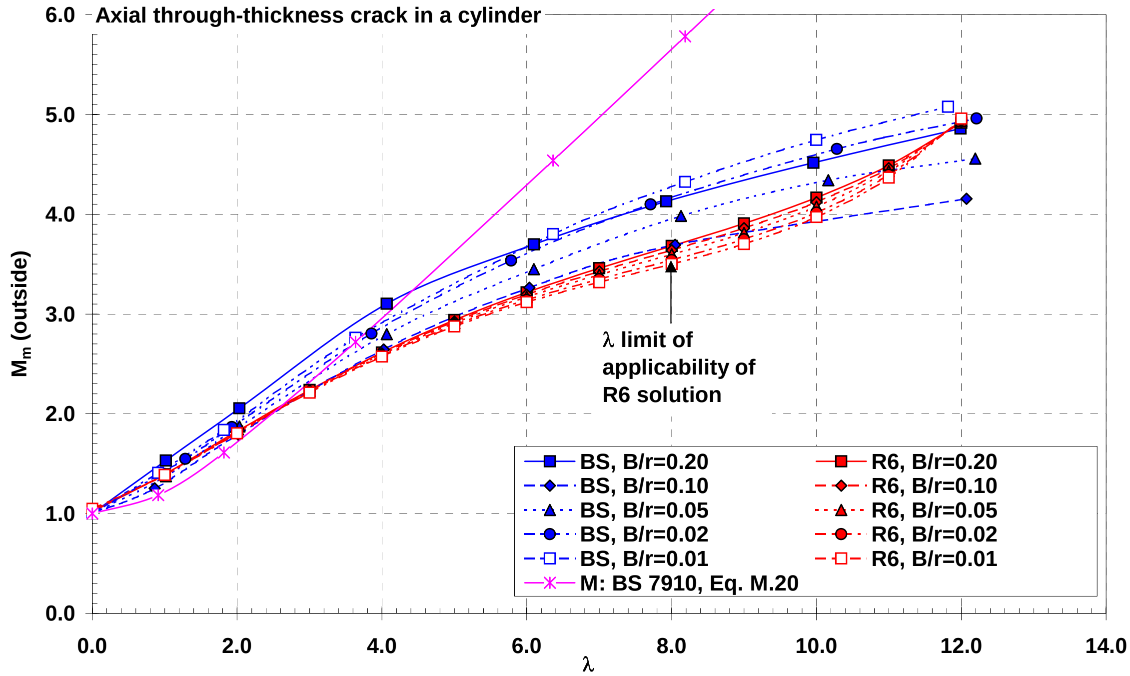
Plots

Figure A.13 shows the BS 7910 and R6 solutions for various B/r_m ratios. If the geometry of the cylinder of interest falls outside the range shown, BS 7910 suggests the use of a flat plate solution, with a bulging factor M given by:

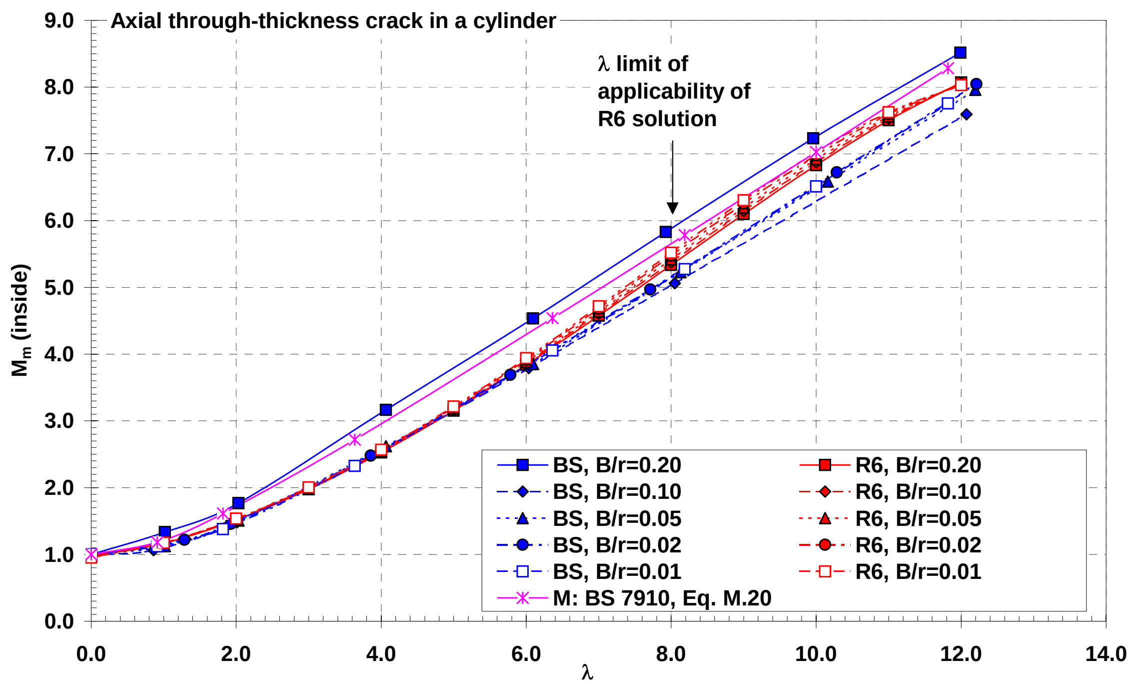
$$M = \sqrt{1 + 3.2\frac{a^2}{2r_m B}} \quad (A.66)$$

This flat plate solution is shown for comparison with the BS and R6 solutions. For high values of λ, the solutions for the outer surface diverge, with the flat plate solutions overestimating K relative to the geometry-specific solutions.

Figure A.13 Normalised K-solution for an axial through-thickness crack in a cylinder

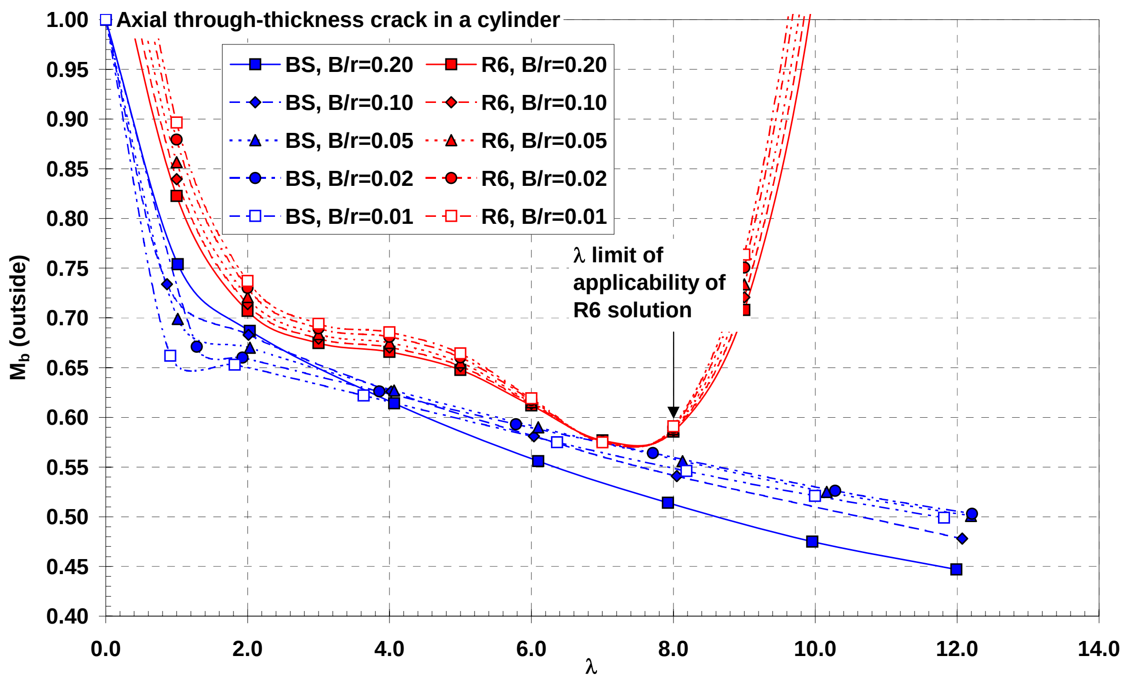


a) Outside surface, pressure loading

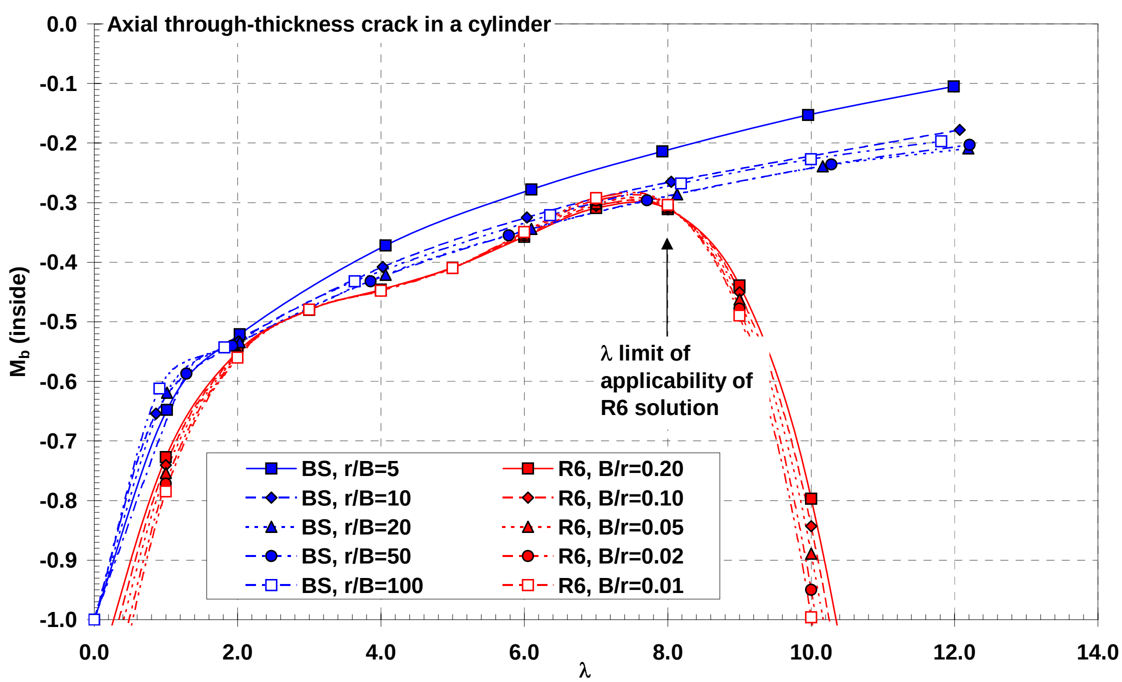


b) Inside surface, pressure loading

Normalised K-solution for an axial through-thickness crack in a cylinder (cont'd)



c) Outside surface, bending loading

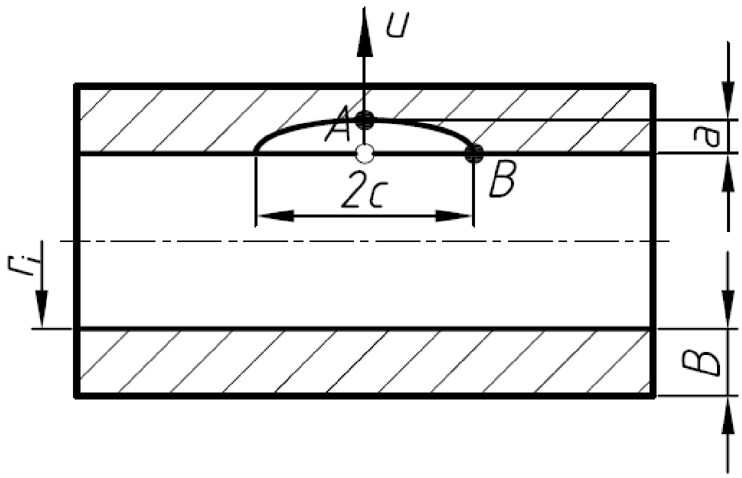


d) Inside surface, bending loading

A.4.1.2 Surface cracks

A.4.1.2.1 Internal axial surface crack

a) Finite crack



BS 7910 Solution [A.15][A.29][A.30]

The stress intensity factor solution is calculated from equations (A.1 to (A.6 where:

$M = f_w = 1;$

M_m and M_b for the deepest point in the crack (Point A, ie θ=90º) and for the points where the crack intersects the free surface (Point B, ie θ=0º) are given in Table A.9

Table A.9 BS 7910 solutions for M_m and M_b for an axial internal surface crack in cylinder

a/2*c* = 0.5, *B*/r_i = 0.1					*a*/2*c* = 0.5, *B*/r_i = 0.25				
a/*B*	M_m(A)	M_b(A)	M_m(B)	M_b(B)	*a*/*B*	M_m(A)	M_b(A)	M_m(B)	M_b(B)
0.0	0.663	0.663	0.729	0.729	0.0	0.663	0.663	0.729	0.729
0.2	0.647	0.464	0.726	0.676	0.2	0.643	0.461	0.719	0.669
0.4	0.661	0.291	0.760	0.649	0.4	0.656	0.288	0.745	0.638
0.6	0.677	0.110	0.804	0.623	0.6	0.677	0.107	0.785	0.610
0.8	0.694	-0.080	0.859	0.599	0.8	0.704	-0.079	0.838	0.585
a/2*c* = 0.2, *B*/r_i = 0.1					*a*/2*c* = 0.2, *B*/r_i = 0.25				
0.0	0.951	0.951	0.662	0.662	0.0	0.951	0.951	0.662	0.662
0.2	0.932	0.698	0.676	0.632	0.2	0.919	0.688	0.669	0.627
0.4	1.016	0.519	0.768	0.651	0.4	0.998	0.506	0.759	0.644
0.6	1.109	0.316	0.896	0.674	0.6	1.110	0.311	0.889	0.666
0.8	1.211	0.090	1.060	0.700	0.8	1.255	0.103	1.060	0.694
a/2*c* = 0.1, *B*/r_i = 01					*a*/2*c* = 0.1, *B*/r_i = 0.25				
0.0	1.059	1.059	0.521	0.521	0.0	1.059	1.059	0.521	0.521
0.2	1.062	0.806	0.578	0.548	0.2	1.045	0.791	0.577	0.547
0.4	1.260	0.677	0.695	0.597	0.4	1.240	0.663	0.698	0.599
0.6	1.500	0.515	0.876	0.660	0.6	1.514	0.515	0.887	0.665
0.8	1.783	0.320	1.123	0.737	0.8	1.865	0.348	1.144	0.745
a/2*c* = 0.05, *B*/r_i = 0.1					*a*/2*c* = 0.05, *B*/r_i = 0.25				
0.0	1.103	1.103	0.384	0.384	0.0	1.103	1.103	0.384	0.384
0.2	1.172	0.897	0.451	0.429	0.2	1.153	0.881	0.451	0.428
0.4	1.494	0.834	0.582	0.503	0.4	1.470	0.816	0.585	0.504
0.6	1.985	0.765	0.820	0.623	0.6	2.003	0.765	0.830	0.627
0.8	2.737	0.689	1.219	0.810	0.8	2.864	0.749	1.242	0.819
a/2*c* = 0.025, *B*/r_i = 0.1					*a*/2*c* = 0.025, *B*/r_i = 0.25				
0.0	1.120	1.120	0.275	0.275	0.0	1.120	1.120	0.275	0.275
0.2	1.231	0.946	0.335	0.318	0.2	1.211	0.929	0.334	0.318
0.4	1.701	0.971	0.469	0.406	0.4	1.674	0.950	0.471	0.407
0.6	2.619	1.080	0.765	0.584	0.6	2.285	1.079	0.774	0.587
0.8	4.364	1.301	1.374	0.919	0.8	3.163	1.081	1.400	0.928

Validity limits:

Range of application:
$0 \leq a/B \leq 0.8$
$0.025 \leq a/2c \leq 0.5$
$0.1 \leq B/r_i \leq 0.25$
$2c/W \leq 0.15$

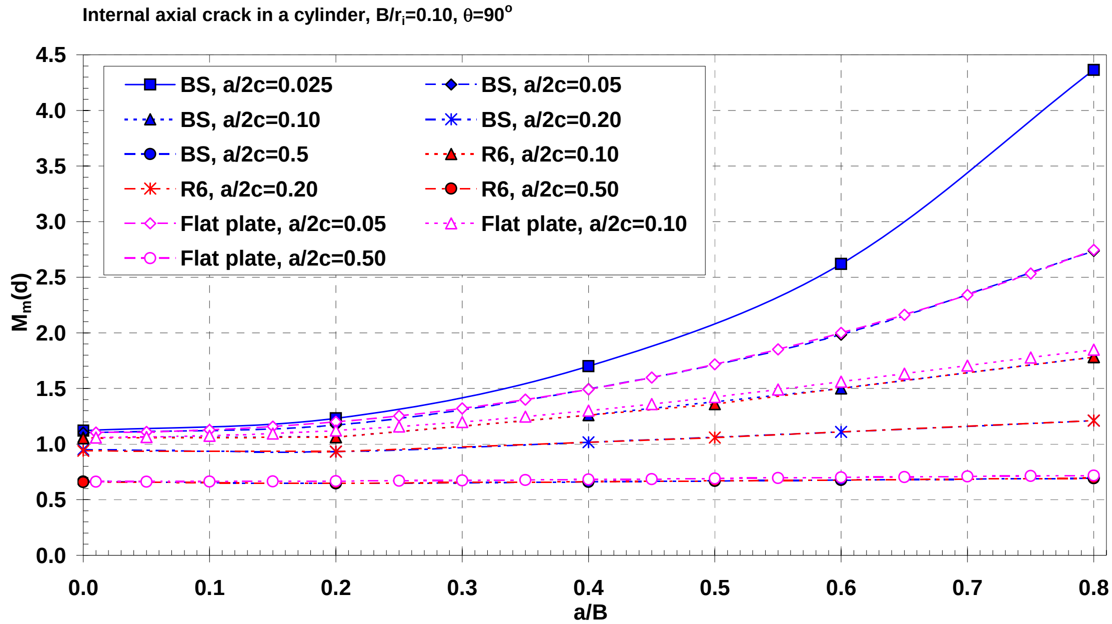
R6 solution [A.31]

Tabulated geometry functions are given in R6 for restricted values of a/2c and B/r_i. These are not repeated here, since for uniform membrane and bending stresses they coincide with the values given by BS 7910, and there are no solutions for arbitrary through-wall stress distributions.

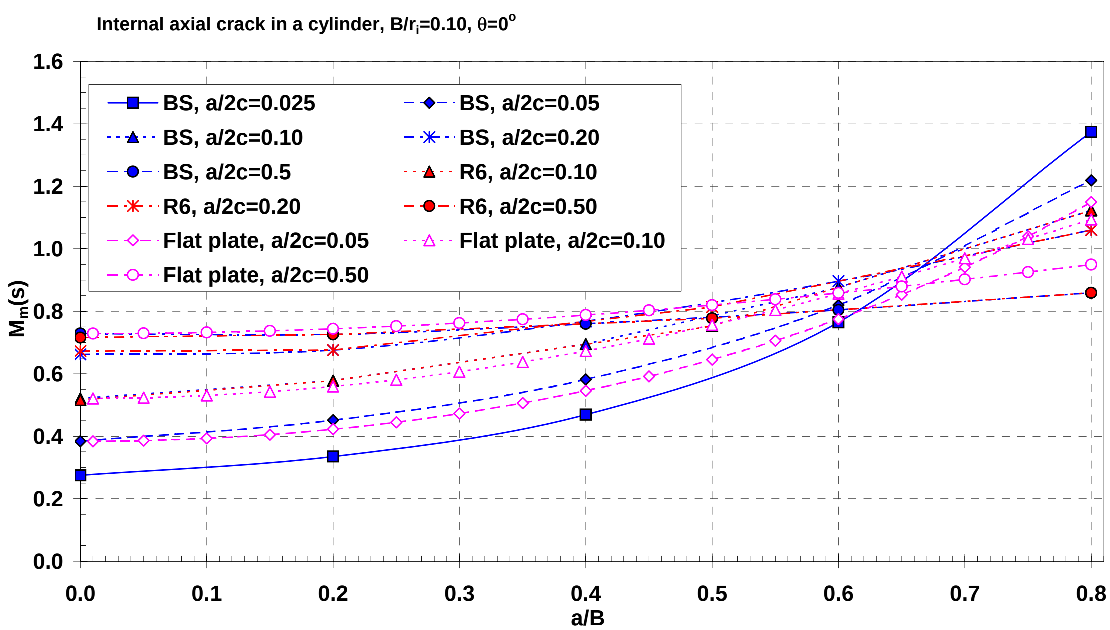
Plots

Figure A.14 shows the BS 7910 and R6 solutions under membrane and bending stress, at θ=90º (Point A) and θ=0º (Point B), for various B/r_i and a/2c ratios. The BS 7910 flat plate solutions (membrane stress only) are shown for comparison, and give a good approximation to the geometry-specific solutions, especially for B/r_i=0.1.

Figure A.14 Normalised K-solution for an internal axial crack in a cylinder

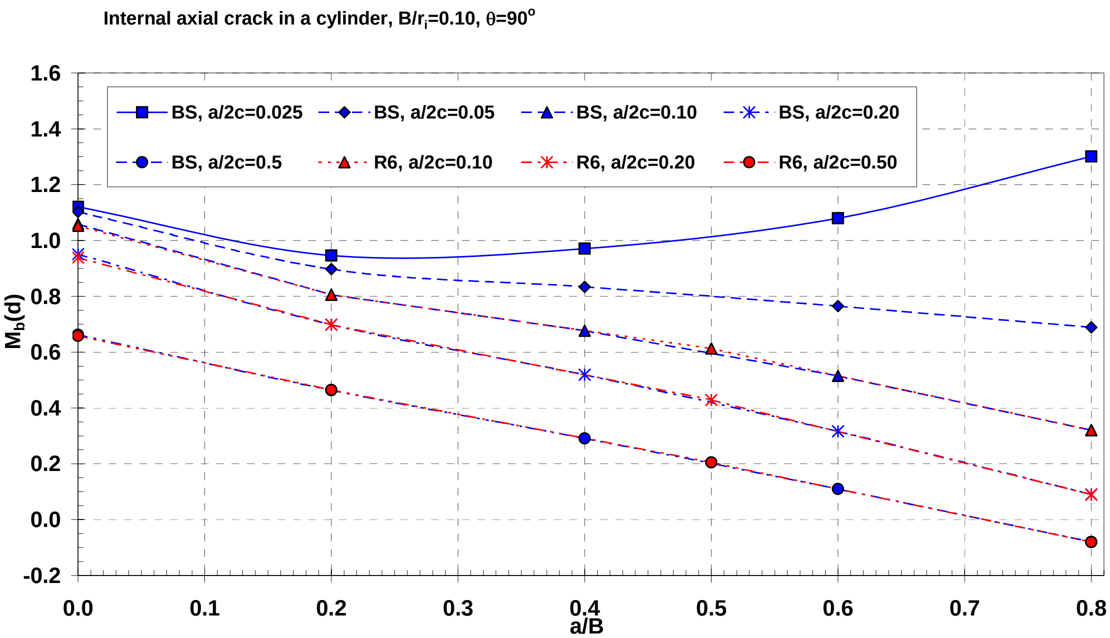


a) Membrane stress, θ=90º, B/r_i=0.1

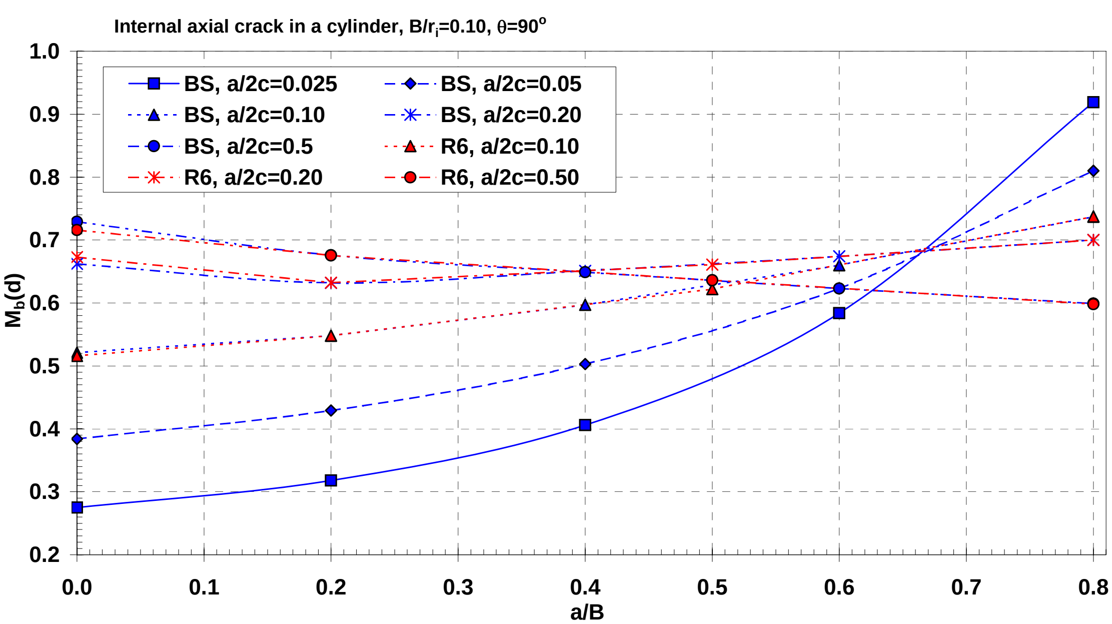


b) Membrane stress, θ=0º, B/r_i=0.1

Normalised K-solution for an internal axial crack in a cylinder (cont'd)

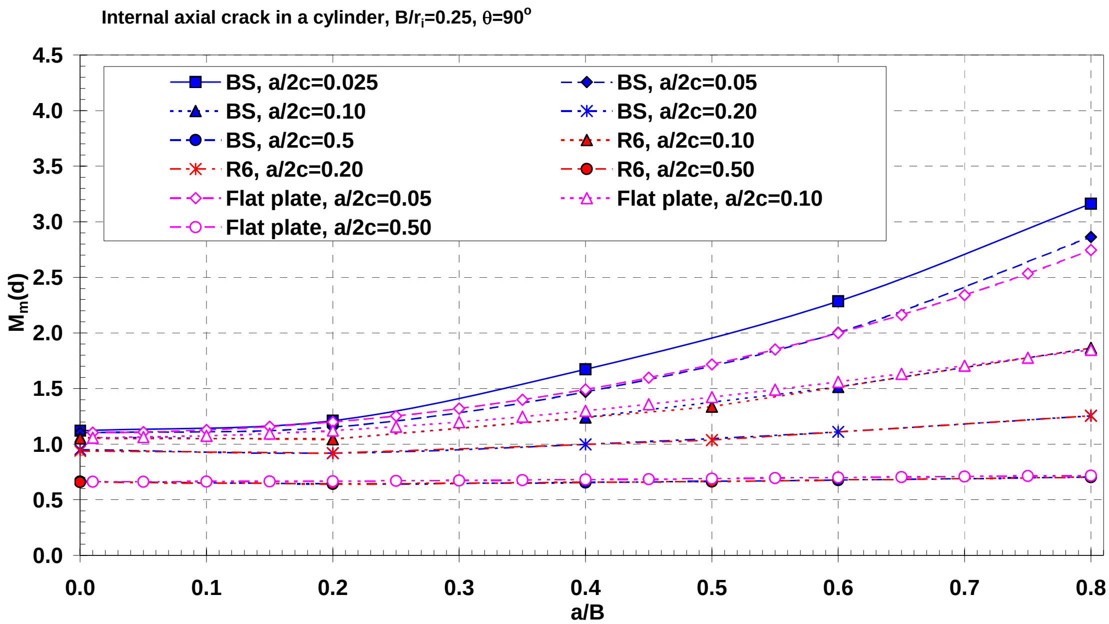


c) Bending stress, θ=90°, B/r_i=0.1

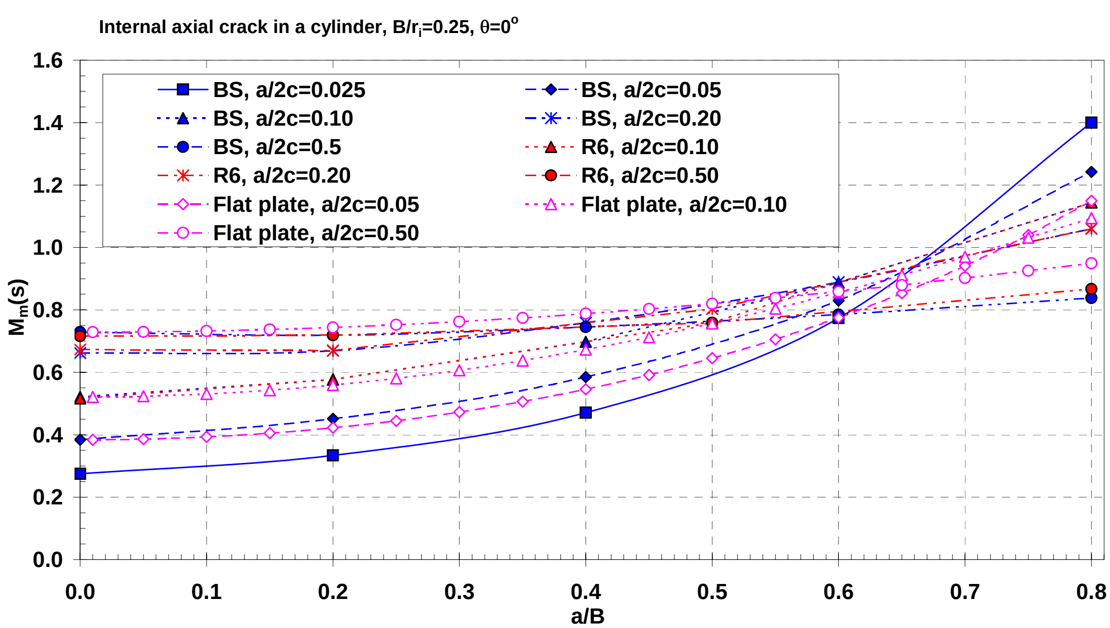


d) Bending stress, θ=0°, B/r_i=0.1

Normalised K-solution for an internal axial crack in a cylinder (cont'd)

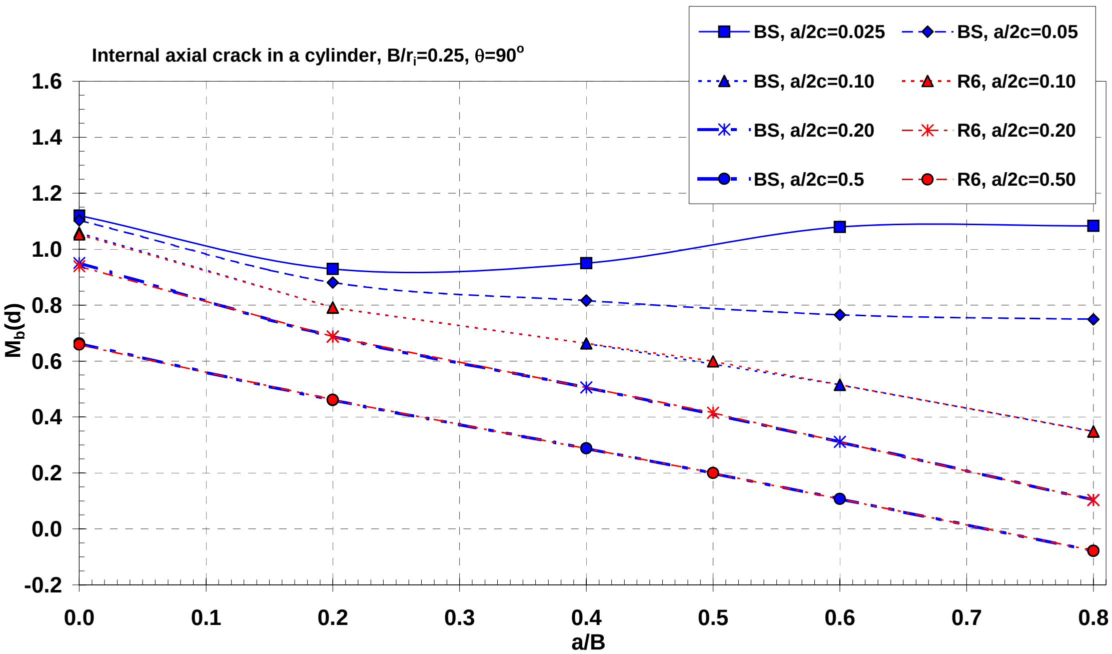


e) Membrane stress, θ=90º, B/r_i=0.25

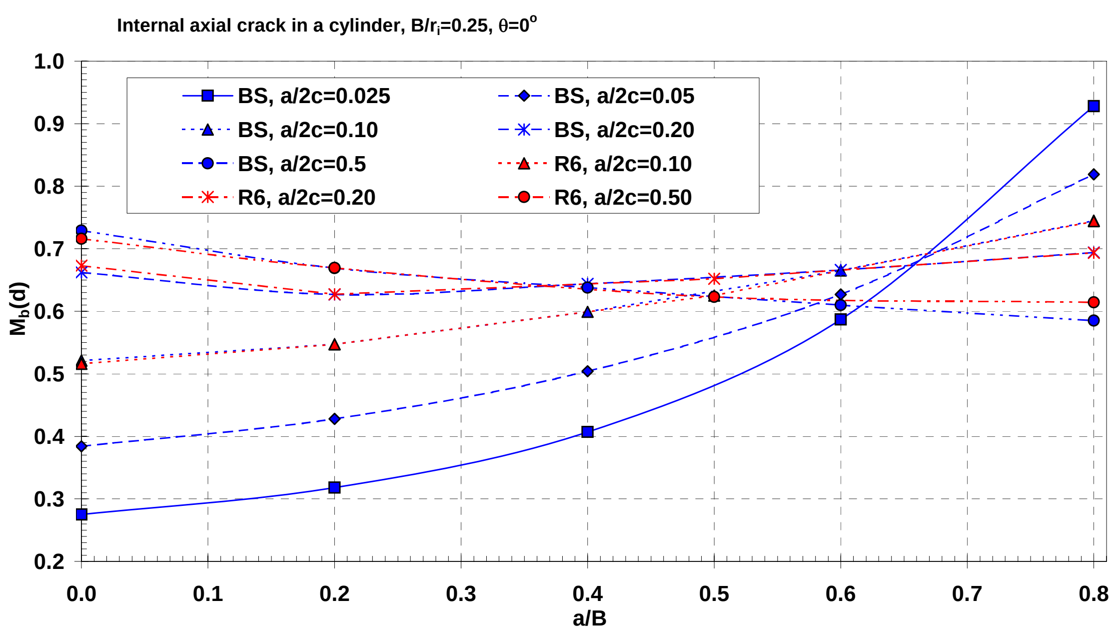


f) Membrane stress, θ=0º, B/r_i=0.25

Normalised K-solution for an internal axial crack in a cylinder (cont'd)

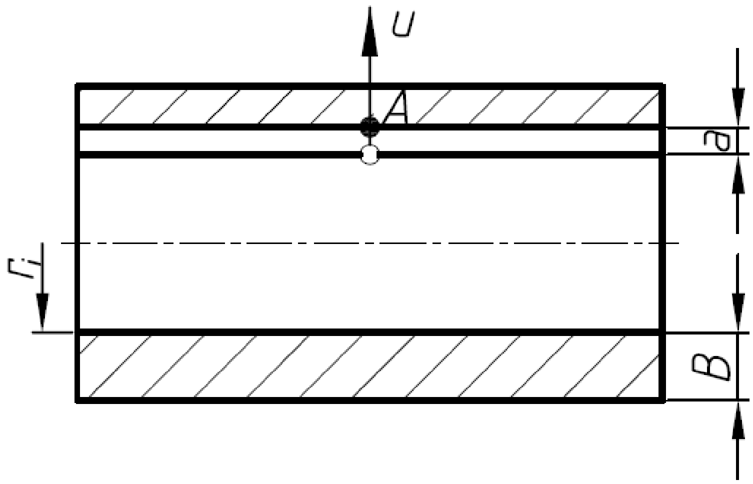


g) Bending stress, θ=90°, B/r_i=0.25



h) Bending stress, θ=0°, B/r_i=0.25

b) Extended Crack



BS 7910 Solution [A.29][A.30][A.31]
The stress intensity factor solution is calculated from equations (A.1 to (A.6:

Where:

$M = f_w = 1$;

M_m and M_b are given in Table A.10 for the deepest point in the crack.

Table A.10 BS 7910 solutions for M_m and M_b for an extended internal axial surface crack in cylinder

B/r_i = 0.1			B/r_i = 0.25		
a/B	M_m	M_b	a/B	M_m	M_b
0.0	1.122	1.122	0.0	1.122	1.122
0.2	1.380	1.018	0.2	1.304	1.002
0.4	1.930	1.143	0.4	1.784	1.033
0.6	2.960	1.484	0.6	2.566	1.094
0.8	4.820	1.990	0.8	3.461	0.949

Validity limits:

$0.0 \le a/B \le 0.8$

$0.1 \le B/r_i \le 0.25$

R6 Solution [A.19]

The stress intensity factor K_I is given by:

$$K_I = \frac{1}{\sqrt{2\pi a}} \int_0^a P(u) \sum_{i=1}^{i=3} f_i\left(a/B, B/r_i\right)\left(1-\frac{u}{a}\right)^{i-\frac{3}{2}} du \qquad \text{(A.67)}$$

The stress distribution P = P(u) is to be taken normal to the prospective crack plane in an uncracked cylinder, where u=0 denotes the inside of the cylinder.

The geometry functions f_i (i = 1 to 3) are given in Table A.11 for the deepest point of the crack (Point A). Note that the solution includes very thick-walled cylinders, eg B/r_i=2.

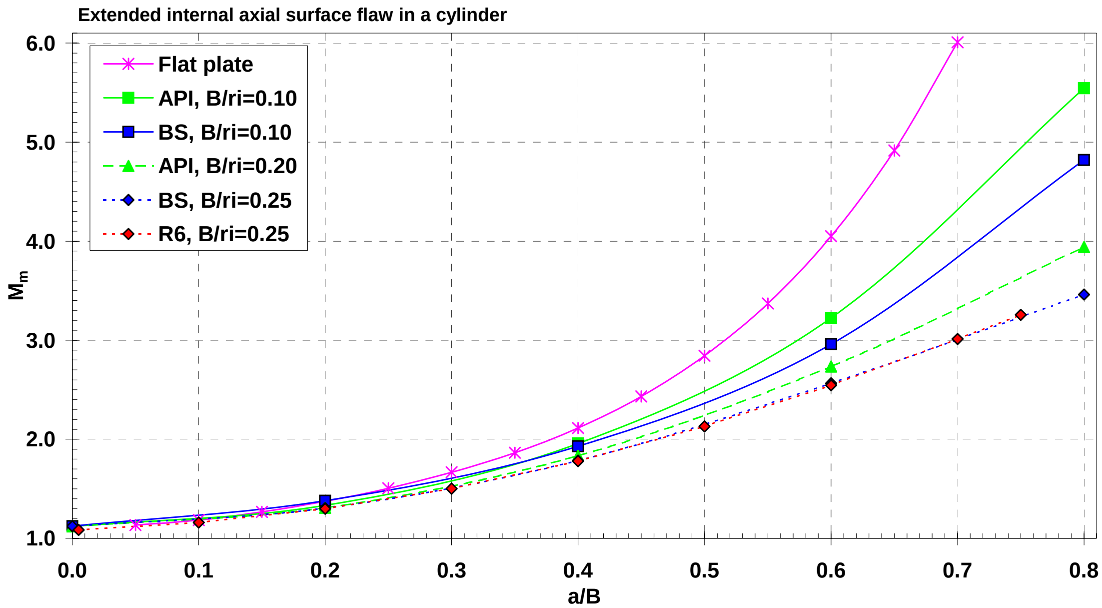
Table A.11 R6 Geometry functions for an extended internal axial surface crack in a cylinder.

***a*/B**	***B/r*** B/r_i **= 2**			B/r_i **= 1**		
0	2.000	1.328	0.220	2.000	1.336	0.220
0.1	2.000	0.890	0.155	2.000	1.271	0.184
0.2	2.000	0.895	0.193	2.000	1.566	0.237
0.3	2.000	1.032	0.252	2.000	1.997	0.360
0.4	2.000	1.329	0.210	2.000	2.501	0.542
0.5	2.000	1.796	0.093	2.000	3.072	0.762
0.6	2.000	2.457	-0.074	2.000	3.807	0.892
0.7	2.000	3.597	-0.618	2.000	4.877	0.825
0.75	2.000	4.571	-1.272	2.000	5.552	0.786
a/B	B/r_i **= 0.5**			B/r_i **= 0.25**		
0	2.000	1.340	0.219	2.000	1.340	0.219
0.1	2.000	1.519	0.212	2.000	1.659	0.217
0.2	2.000	2.119	0.322	2.000	2.475	0.358
0.3	2.000	2.934	0.551	2.000	3.615	0.709
0.4	2.000	3.820	1.066	2.000	4.982	1.499
0.5	2.000	4.692	1.853	2.000	6.455	2.936
0.6	2.000	5.697	2.600	2.000	7.977	5.018
0.7	2.000	6.995	3.224	2.000	9.513	7.637
0.75	2.000	7.656	3.733	2.000	10.24	9.134

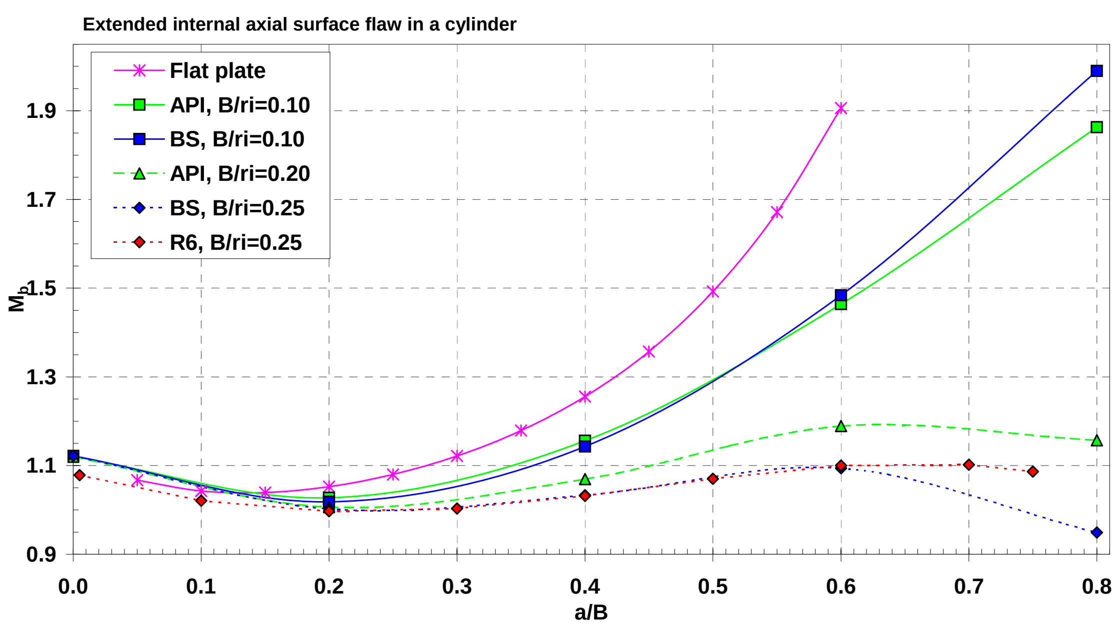
Plots

Figure A.14 shows the BS 7910 solution for two B/r_i ratios, 0.1 and 0.25. The BS 7910 solution for an extended surface flaw in a flat plate is shown for comparison. Also shown are the R6 solutions for B/r_i=0.25 and the API solutions for B/r_i=0.25 and B/r_i=0.1.

Figure A.15 Normalised K-solution for an extended internal axial crack in a cylinder



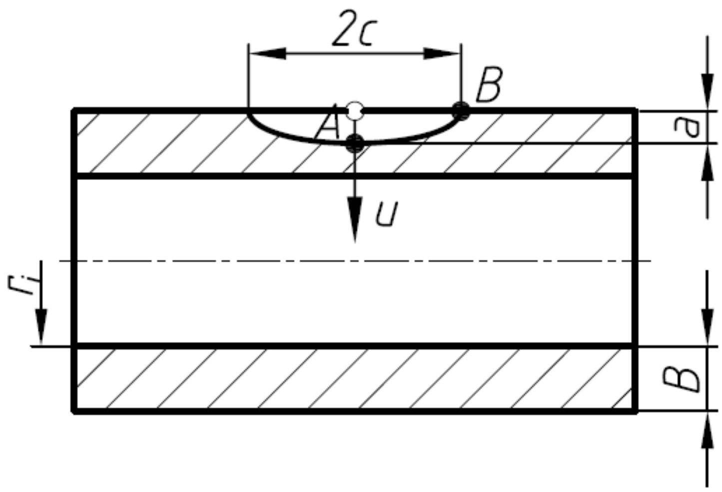
a) Membrane stress



b) Bending stress

A.4.1.2.2 External axial surface crack

a) Finite crack



BS 7910 Solution [A.29][A.30]

The stress intensity factor solution is calculated from equations (A.1 to (A.6:

Where:

$M = f_w = 1$;

M_m and M_b for the deepest point in the crack (Point A, θ=90º) and for the points where the crack intersects the free surface (Point B, θ=0º) are given in Table A.12. The coefficients are in general a little higher than those for an internal axial surface crack (Table A.9).

Table A.12 BS 7910 solutions for M_m and M_b for an axial external surface crack in a cylinder

$a/2c = 0.5$, $B/r_i = 0.1$					$a/2c = 0.5$, $B/r_i = 0.25$				
a/B	**M_m(A)**	**M_b(A)**	**M_m(B)**	**M_b(B)**	***a/B***	**M_m(A)**	**M_b(A)**	**M_m(B)**	**M_b(B)**
0.0	0.663	0.663	0.729	0.729	0.0	0.663	0.663	0.729	0.729
0.2	0.653	0.470	0.736	0.685	0.2	0.656	0.473	0.741	0.689
0.4	0.675	0.301	0.783	0.666	0.4	0.683	0.307	0.793	0.673
0.6	0.695	0.122	0.846	0.649	0.6	0.710	0.131	0.864	0.659
0.8	0.712	-0.068	0.926	0.634	0.8	0.736	-0.055	0.954	0.647
$a/2c = 0.2$, $B/r_i = 0.1$					**$a/2c = 0.2$, $B/r_i = 0.25$**				
0.0	0.951	0.951	0.662	0.662	0.0	0.951	0.951	0.662	0.662
0.2	0.953	0.716	0.685	0.641	0.2	0.964	0.726	0.689	0.644
0.4	1.077	0.561	0.799	0.673	0.4	1.110	0.582	0.806	0.678
0.6	1.213	0.377	0.970	0.715	0.6	1.289	0.417	0.982	0.721
0.8	1.361	0.167	1.198	0.769	0.8	1.502	0.230	1.217	0.775
$a/2c = 0.1$, $B/r_i = 0.1$					**$a/2c = 0.1$, $B/r_i = 0.25$**				
0.0	1.059	1.059	0.521	0.521	0.0	1.059	1.059	0.521	0.521
0.2	1.092	0.831	0.583	0.552	0.2	1.106	0.844	0.583	0.552
0.4	1.370	0.750	0.706	0.606	0.4	1.410	0.776	0.693	0.598
0.6	1.735	0.644	0.912	0.681	0.6	1.838	0.693	0.867	0.659
0.8	2.188	0.514	1.202	0.780	0.8	2.390	0.595	1.105	0.736
$a/2c = 0.05$, $B/r_i = 0.1$					**$a/2c = 0.05$, $B/r_i = 0.25$**				
0.0	1.103	1.103	0.384	0.384	0.0	1.103	1.103	0.384	0.384
0.2	1.206	0.926	0.455	0.432	0.2	1.222	0.939	0.455	0.432
0.4	1.624	0.923	0.592	0.510	0.4	1.672	0.955	0.581	0.504
0.6	2.295	0.957	0.853	0.643	0.6	2.432	1.029	0.811	0.622
0.8	3.360	1.108	1.305	0.857	0.8	3.670	1.128	1.199	0.809
$a/2c = 0.025$, $B/r_i = 0.1$					**$a/2c = 0.025$, $B/r_i = 0.25$**				
0.0	1.120	1.120	0.275	0.275	0.0	1.120	1.120	0.275	0.275
0.2	1.266	0.976	0.338	0.321	0.2	1.282	0.991	0.338	0.321
0.4	1.849	1.075	0.477	0.412	0.4	1.753	1.011	0.468	0.407
0.6	2.628	1.349	0.796	0.602	0.6	2.581	1.107	0.757	0.583
0.8	4.090	1.549	1.471	0.972	0.8	3.839	1.153	1.352	0.918

Validity limits:

$0 \leq a/B \leq 0.8$

$0.025 \leq a/2c \leq 0.5$

$0.1 \leq B/r_i \leq 0.25$

$2c/W \leq 0.15$

R6 Solution [A.18][A.31]

Solutions are given in the form of tables for B/r_i ratios of between 0.1 and 0.25 and a/2c ratios of 0.1 and 0.5, for positions θ=90 (Point A) and θ=0º (Point B).

The stress intensity factor K_I is given by:

$$K_I + \sqrt{\pi a}\sum_{i=0}^{3} \sigma_i f_i\left(\frac{a}{B},\frac{a}{2c},\frac{B}{r_i}\right) \quad (A.68)$$

P_i (i = 0 to 3) are stress components which define the stress distribution P according to

$$\sigma = \sigma(u) = \sum_{i=0}^{3} \sigma_i \left(\frac{u}{a}\right)^i \qquad for\, 0 \le u \le a \tag{A.69}$$

where P is to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is the distance from the external surface of the cylinder.

The geometry functions f_i (i = 0 to 3) are given in Table A.13 and Table A.14 for the deepest point of the crack (A), and at the intersection of the crack with the free surface (B), respectively.

Table A.13 R6 Geometry functions at Point *A* for a finite external axial surface crack in a cylinder

***a/2c=0.5*, B/r$_i$=0.25**				
a/B				
0	0.659	0.471	0.387	0.337
0.2	0.656	0.459	0.377	0.327
0.5	0.697	0.473	0.384	0.331
0.8	0.736	0.495	0.398	0.342
***a/2c=0.5*, B/r$_i$=0.1**				
a/B				
0	0.659	0.471	0.387	0.337
0.2	0.653	0.457	0.376	0.327
0.5	0.687	0.470	0.382	0.330
0.8	0.712	0.487	0.394	0.340
***a/2c=0.2*, B/r$_i$=0.25**				
a/B				
0	0.939	0.580	0.434	0.353
0.2	0.964	0.596	0.461	0.387
0.5	1.183	0.672	0.500	0.410
0.8	1.502	0.795	0.568	0.455
***a/2c=0.2*, B/r$_i$=0.1**				
a/B				
0	0.939	0.580	0.434	0.353
0.2	0.953	0.591	0.459	0.386
0.5	1.139	0.656	0.491	0.405
0.8	1.361	0.746	0.543	0.439
***a/2c=0.1*, B/r$_i$=0.25**				
a/B				
0	1.053	0.606	0.443	0.357
0.2	1.107	0.658	0.499	0.413
0.5	1.562	0.820	0.584	0.465
0.8	2.390	1.122	0.745	0.568
***a/2c=0.1*, B/r$_i$=0.1**				
a/B				
0	1.053	0.606	0.443	0.357
0.2	1.092	0.652	0.496	0.411
0.5	1.508	0.799	0.571	0.457
0.8	2.188	1.047	0.704	0.541

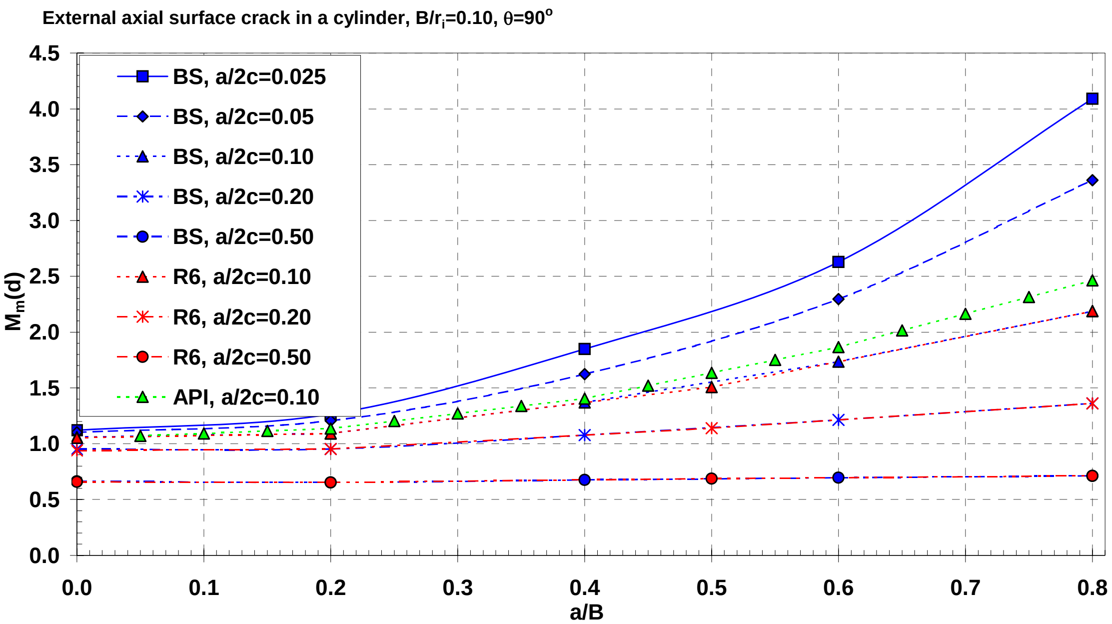
Table A.14 Geometry functions at Point B for a finite external axial surface crack in a cylinder.

a/2c=0.5, **B/r_i=0.25**				
a/B				
0	0.716	0.118	0.041	0.022
0.2	0.741	0.130	0.049	0.026
0.5	0.819	0.155	0.061	0.033
0.8	0.954	0.192	0.078	0.041
a/2c=0.5, **B/r_i=0.1**				
a/B				
0	0.716	0.118	0.041	0.022
0.2	0.736	0.129	0.048	0.025
0.5	0.807	0.150	0.059	0.031
0.8	0.926	0.182	0.072	0.038
a/2c=0.2, **B/r_i=0.25**				
a/B				
0	0.673	0.104	0.032	0.015
0.2	0.690	0.113	0.039	0.019
0.5	0.864	0.170	0.068	0.036
0.8	1.217	0.277	0.117	0.064
a/2c=0.2, **B/r_i=0.1**				
a/B				
0	0.673	0.104	0.032	0.015
0.2	0.685	0.111	0.039	0.019
0.5	0.856	0.167	0.066	0.035
0.8	1.198	0.269	0.112	0.061
***a/2c* =0.1**, **B/r_i=0.25**				
a/B				
0	0.516	0.069	0.017	0.009
0.2	0.583	0.076	0.022	0.010
0.5	0.748	0.128	0.047	0.024
0.8	1.105	0.230	0.092	0.049
a/2c=0.1, **B/r_i=0.1**				
a/t				
0	0.516	0.069	0.017	0.009
0.2	0.583	0.076	0.022	0.010
0.5	0.768	0.135	0.051	0.027
0.8	1.202	0.264	0.109	0.059

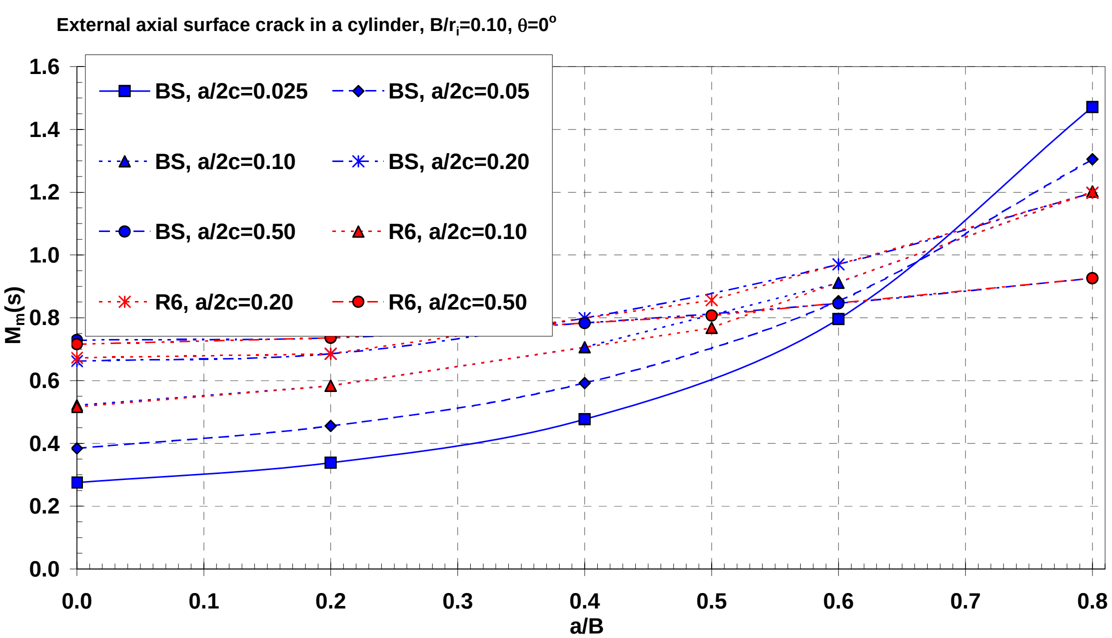
Plots

Figure A.16 shows solutions (BS 7910, R6 and selected API solutions) for various a/2c ratios, with B/r_i=0.10 and 0.25. For geometries outside the B/r_i validity limits described in this section, a possible approach is to use the BS 7910 flat plate solution multiplied by a bulging factor as given by equation (A.55 and (A.56. This tends to overestimate the stress intensity relative to the solutions shown in this section.

Figure A.16 Normalised K-solution for an external axial crack in a cylinder

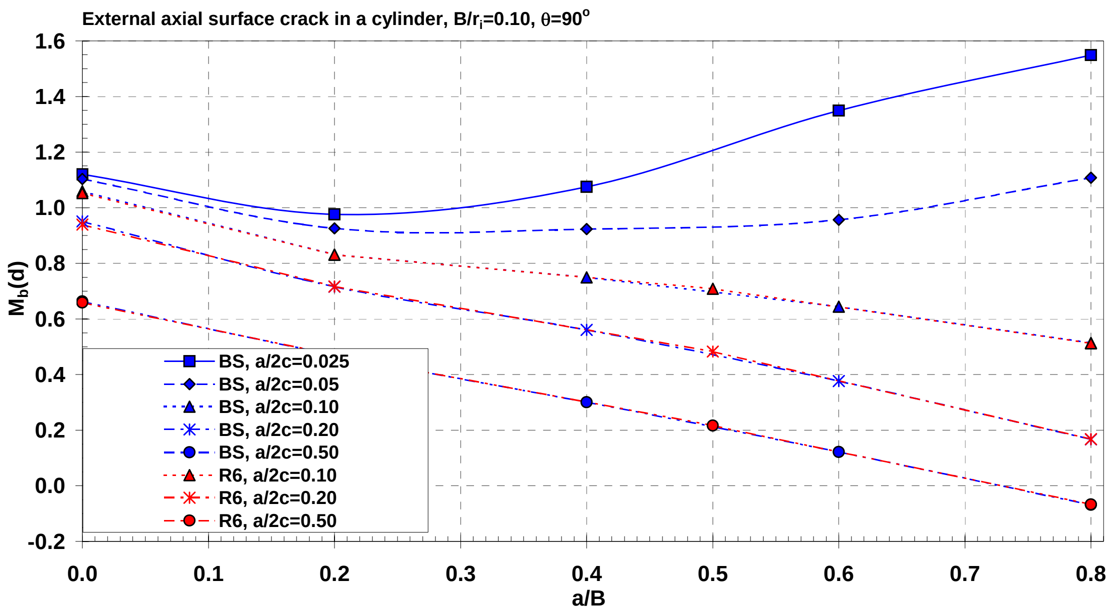


a) Membrane stress, B/r_i=0.1, θ=90°

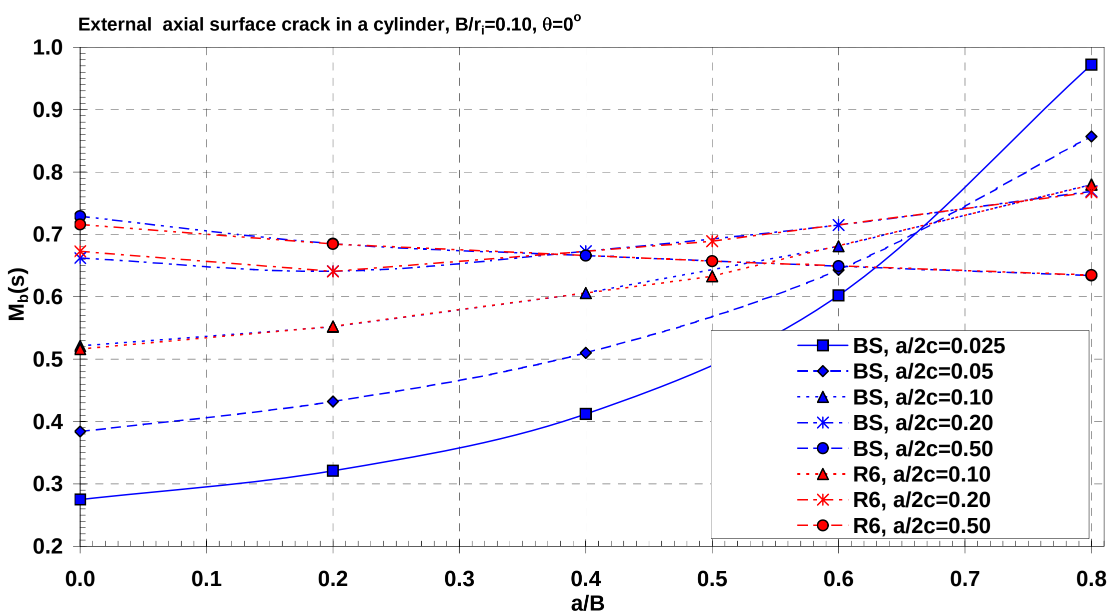


b) Membrane stress, B/r_i=0.1, θ=0°

Normalised K-solution for an external axial crack in a cylinder (cont'd)

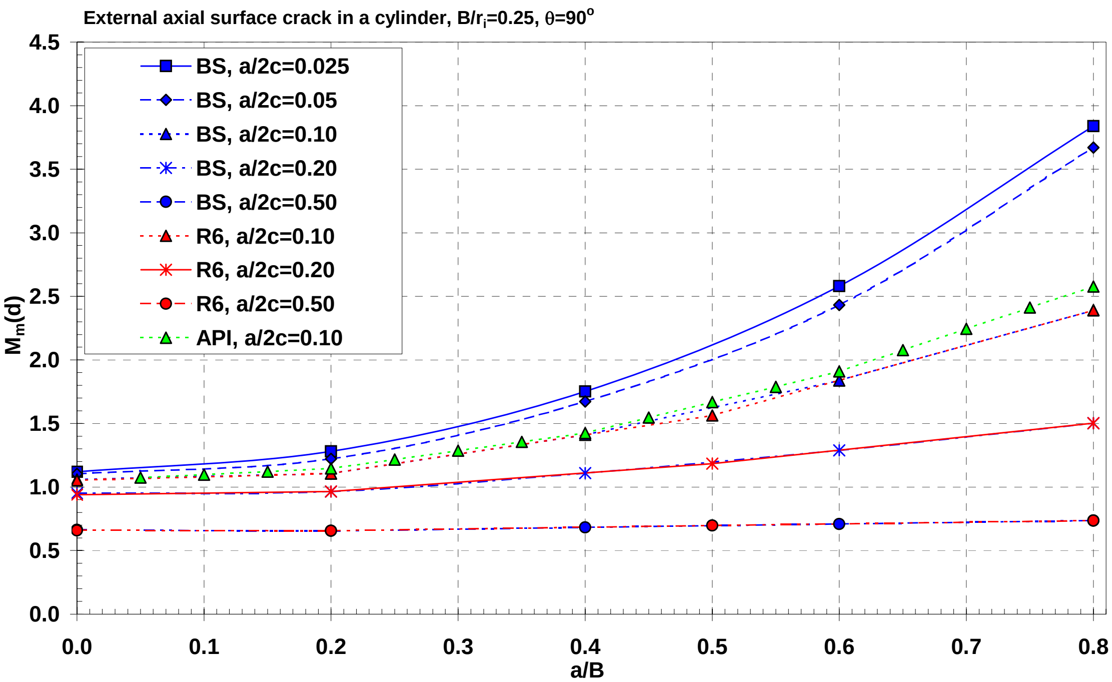


c) Bending stress, B/r_i=0.1, θ=90°

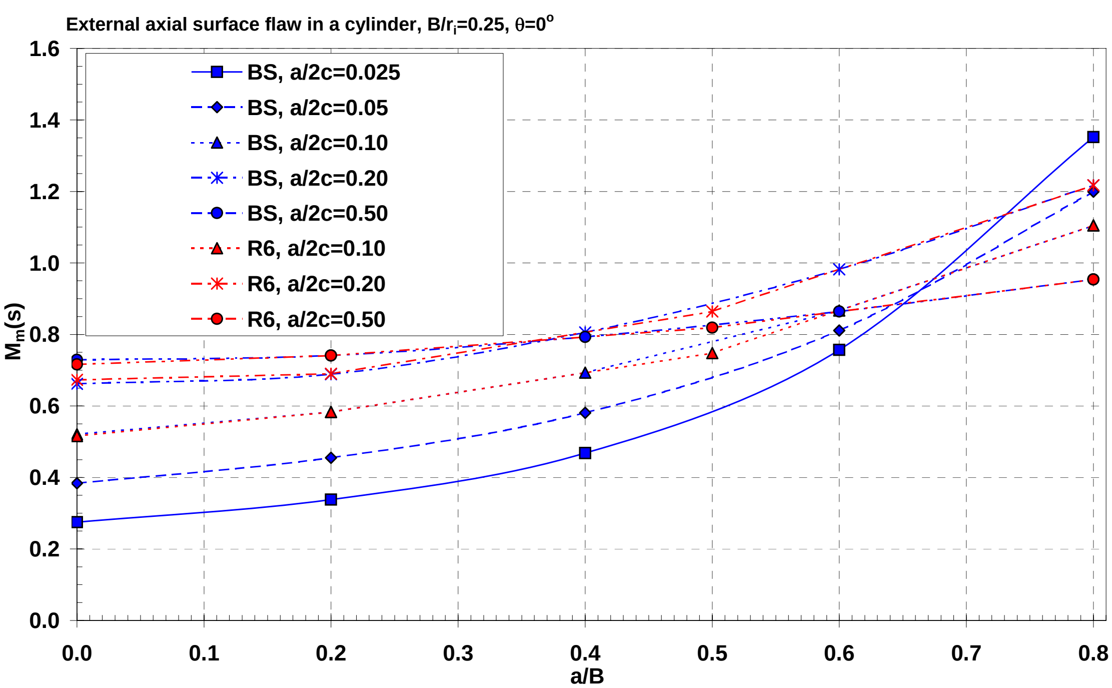


d) Bending stress, B/r_i=0.1, θ=0°

Normalised K-solution for an external axial crack in a cylinder (cont'd)

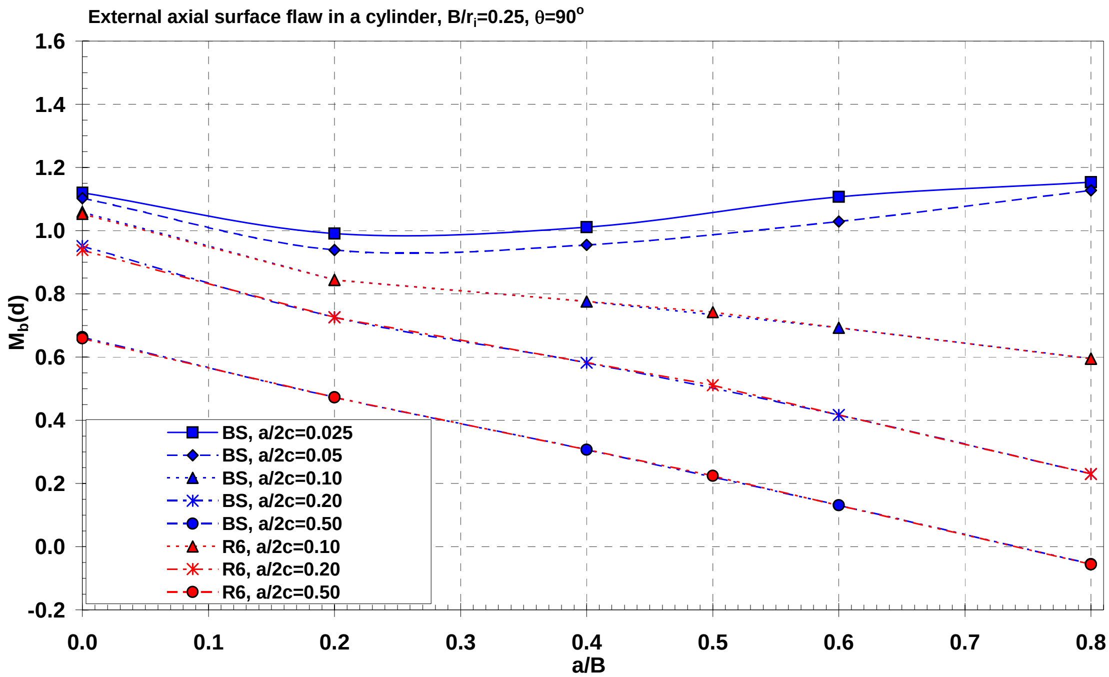


e) Membrane stress, B/r_i=0.25, θ=90°

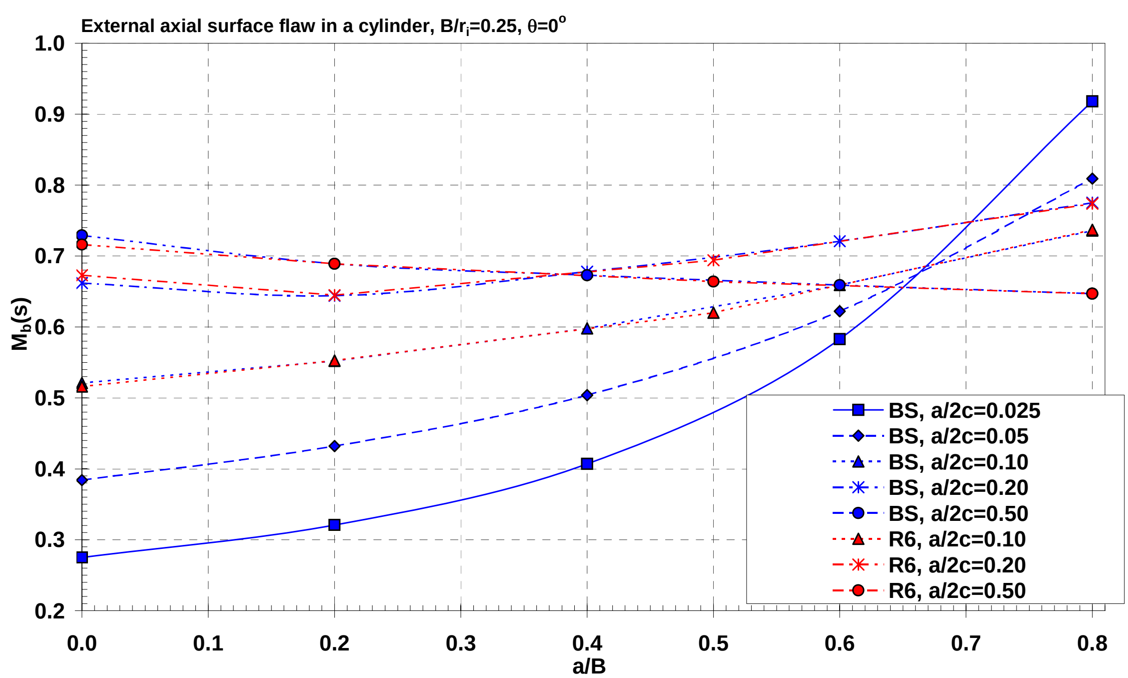


f) Membrane stress, B/r_i=0.25, θ=0°

Normalised K-solution for an external axial crack in a cylinder (contd')

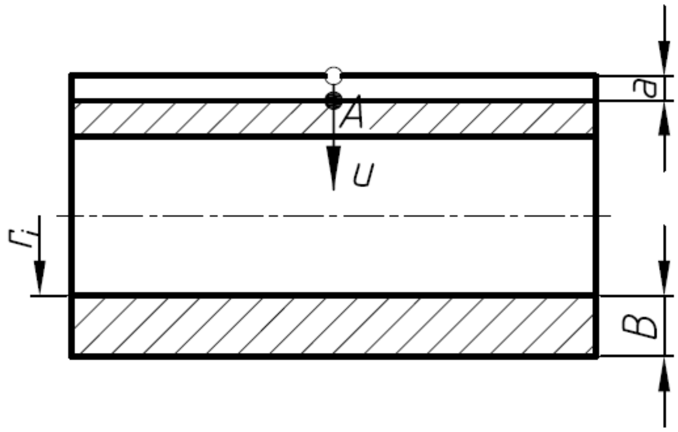


g) Bending stress, $B/r_i=0.25$, $\theta=90^o$



h) Bending stress, $B/r_i=0.25$, $\theta=0^o$

b) Extended crack



BS 7910 Solution [A.32][A.33]

The stress intensity factor solution is calculated from equations (A.1 to (A.6:

Where:

$M = f_w = 1$;

M_m and M_b are given in Table A.15 (note that this is identical to Table A.10, ie BS 7910 assumes the same solutions for long external and long internal axial flaws), and specifically states M=1, ie no bulging correction factor should be applied.

Table A.15 BS 7910 solutions for M_m and M_b for an extended axial crack in a cylinder

***B/r*$_i$ = 0.1**			***B/r*$_i$ = 0.25**		
a/B	**M_m**	**M_b**	***a/B***	**M_m**	**M_b**
0.0	1.122	1.122	0.0	1.122	1.122
0.2	1.380	1.018	0.2	1.304	1.002
0.4	1.930	1.143	0.4	1.784	1.033
0.6	2.960	1.484	0.6	2.566	1.094
0.8	4.820	1.990	0.8	3.461	0.949

Validity limits:

$0.0 \leq a/B \leq 0.8$

$0.1 \leq B/r_i \leq 0.25$

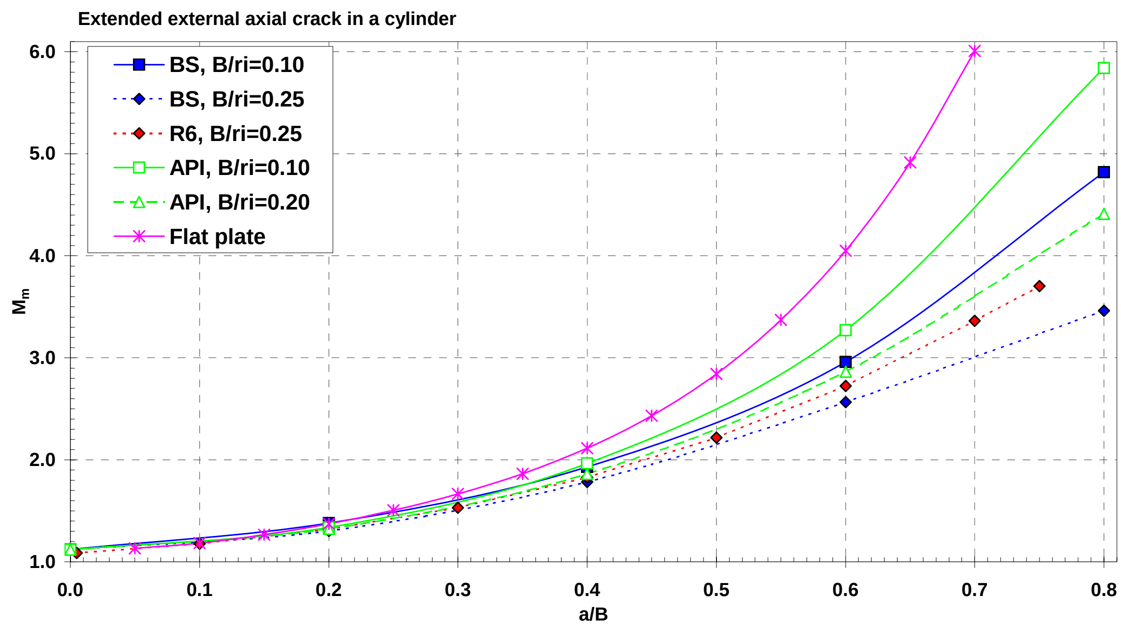
R6 Solution [A.34]

a closed form solution is not given; the solution is provided in integral form, covering B/r_i ratios between 0.25 and 2. Alternative solutions are cited for thinner-walled cylinders.

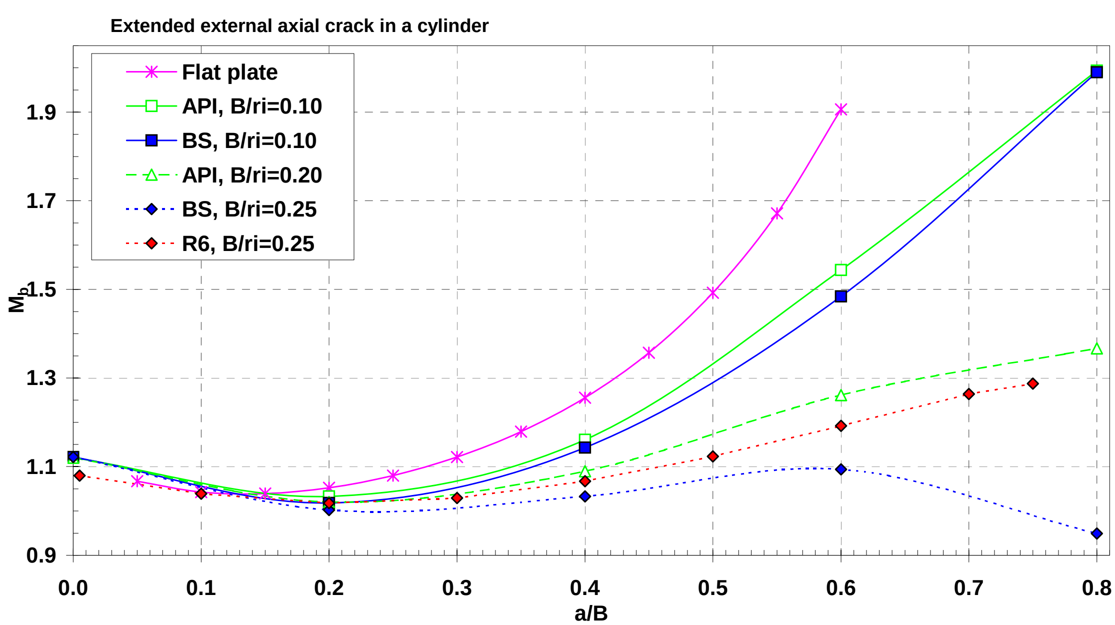
Plots

Figure A.17 compares the solutions recommended by BS, R6 and API. A BS 7910 flat plate solution for a fully extended surface flaw (cf Figure A.3) is shown for comparison. Note that both the API and R6 solutions show a higher value of M_m for external than for internal flaws (cf Figure A.15). The R6 solutions are recommended.

Figure A.17 Normalised K-solution for an extended external axial crack in a cylinder



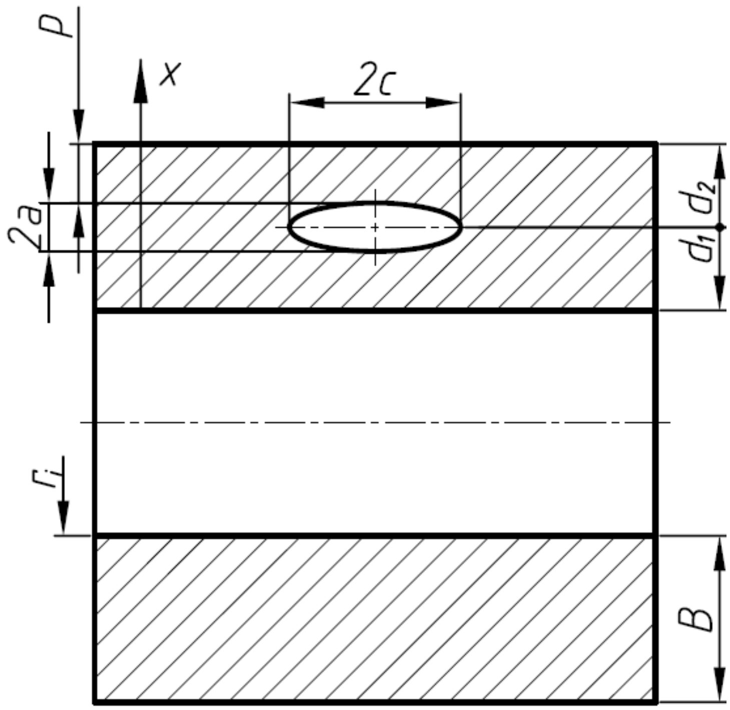
a) Membrane stress



b) Bending stress

A.4.1.3 Embedded crack

a) Finite crack



BS 7910 Solution

For internal pressure only, BS 7910 suggests using a flat plate solution with bulging correction factor, M=1.

Validity limits:

For membrane loading:

$0 \leq a/2c \leq 1.0$

$2c/W < 0.5$

$-\pi \leq \theta \leq \pi$

$a/B' < 0.625(a/c+0.6)$ for $0 \leq a/2c \leq 0.1$, where $B' = 2a + 2p$

For bending loading:

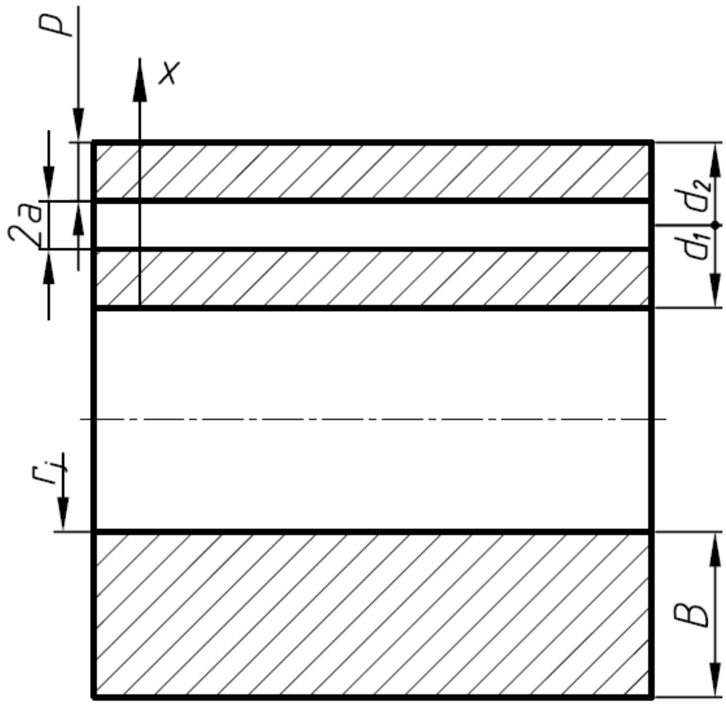
$0 \leq a/2c \leq 0.5$

$\theta = \pi/2$

R6

No solution available.

b) Extended Crack



BS 7910

No solution available.

R6

No solution available.

API 579 solution [A.21]

The flat plate solution for a through-wall 4th order polynomial stress distribution is used for cylinders and spheres when $B/r_i \leq 0.2$. The finite width correction factor should be set to 1.

Validity limits: (BS 7910 terminology)

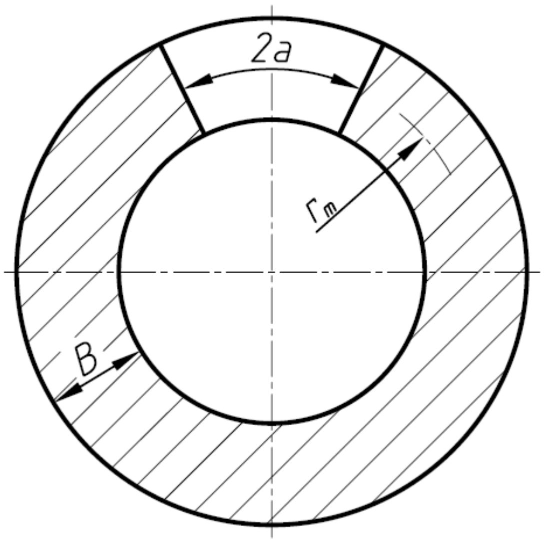
$p/B \geq 0.2$ when $p+a \leq B/2$

$(B-(p+2a))/B \geq 0.2$ when $B-(p+2a) \leq B/2$

$0.25 \leq (p+a)/B \leq 0.75$

A.4.2 Pipes or Cylinders with Circumferential Cracks

A.4.2.1 Through-thickness crack



BS 7910 Solution: [A.28]

The stress intensity factor solution is calculated from equations (A.1 to (A.6

Where:

$K_I = K_I^{pressure} + K_I^{bending}$;
$M = 1$;
$M_m = M_1 + M_2$ at the outside surface, $M_1 - M_2$ at the inside surface
$M_b = M_3 + M_4$ at the outside surface, $M_3 – M_4$ at the inside surface

where

$K_I^{pressure}$ and $K_I^{bending}$ are calculated from equations (A.1 to (A.6 and represent, respectively, contributions to K_I of pressure-induced membrane stresses and through-wall bending stresses.

The coefficients $M_1 – M_4$ are given in Table A.16 for pressure and bending loading, in terms of the parameter λ referred to in equation (A.59. For membrane loading, P_m should be multiplied by a factor of β, where:

$$\beta = \left\{ \frac{2r}{a} \tan\left(\frac{a}{2r_m} \right) \right\}^{0.5} \qquad \text{(A.70)}$$

Table A.16 BS 7910 Solution for circumferential through-thickness cracks in cylinders

a) M_1 for Pressure loading

Parameter, λ	B/r_m=0.2	B/r_m=0.1	B/r_m=0.05	B/r_m=0.02	B/r_m=0.01
0.000	1.000	1.000	1.000	1.000	1.000
0.177	1.248				
0.251		1.032			
0.355	1.290		1.050		
0.502		1.066			
0.561				1.061	
0.709			1.085		
0.793					1.088
1.064	1.406				
1.122				1.096	
1.505		1.192			
1.586					1.139
1.596	1.522				
2.128			1.276		
2.257		1.324			
2.306	1.723				
3.193	2.044		1.469		
3.261		1.545			
3.365				1.378	
3.902	2.367				
4.515		1.864			
4.612			1.752		
4.759					1.425
4.789	2.883				
5.498	3.414				
5.518		2.164			
6.385	4.301		2.140		
6.772		2.641			
7.139					1.732
7.776		3.117			
7.804			2.495		
9.032		3.917			
9.578			3.040		
10.096				2.588	
10.997			3.580		
12.770			4.502		
15.143				3.623	

b) M_2 for Pressure loading

Parameter, λ	B/r_m=0.2	B/r_m=0.1	B/r_m=0.05	B/r_m=0.02	B/r_m=0.01
0.000	0.000	0.000	0.000	0.000	0.000
0.177	0.069				
0.251		0.035			
0.355	0.077		0.034		
0.502		0.057			
0.561				0.045	
0.709			0.069		
0.793					0.063
1.064	0.140				
1.122				0.087	
1.505		0.121			
1.586					0.102
1.596	0.153				
2.128			0.116		
2.257		0.108			
2.306	0.112				
3.193	–0.014		0.041		
3.261		0.019			
3.365				0.043	
3.902	–0.158				
4.515		–0.153			
4.612			–0.119		
4.759					–0.082
4.789	–0.385				
5.498	–0.622				
5.518		–0.328			
6.385	–1.015		–0.318		
6.772		–0.528			
7.139					–0.277
7.776		–0.747			
7.804			–0.485		
9.032		–1.071			
9.578			–0.762		
10.096				–0.585	
10.997			–0.944		
12.770			–1.281		
15.143				–1.126	

c) M_3 for Bending loading

Parameter, λ	B/r_m=0.2	B/r_m=0.1	B/r_m=0.05	B/r_m=0.02	B/r_m=0.01
0.000	0.000	0.000	0.000	0.000	0.000
0.177	0.023				
0.251		0.021			
0.355	0.037		0.015		
0.502		0.028			
0.561				0.013	
0.709			0.025		
0.793					0.012
1.064	0.064				
1.122				0.026	
1.505		0.054			
1.586					0.025
1.596	0.079				
2.128			0.048		
2.257		0.063			
2.306	0.092				
3.193	0.106		0.052		
3.261		0.069			
3.365				0.043	
3.902	0.117				
4.515		0.074			
4.612			0.052		
4.759					0.032
4.789	0.135				
5.498	0.156				
5.518		0.079			
6.385	0.191		0.054		
6.772		0.088			
7.139					0.033
7.776		0.100			
7.804			0.059		
9.032		0.119			
9.578			0.065		
10.096				0.046	
10.997			0.068		
12.770			0.078		
15.143				0.029	

d) M_4 for Bending loading

Parameter, λ	B/r_m=0.2	B/r_m=0.1	B/r_m=0.05	B/r_m=0.02	B/r_m=0.01
0.000	1.000	1.000	1.000	1.000	1.000
0.177	0.918				
0.251		0.828			
0.355	0.816		0.750		
0.502		0.733			
0.561				0.673	
0.709			0.666		
0.793					0.633
1.064	0.624				
1.122				0.587	
1.505		0.544			
1.586					0.544
1.596	0.533				
2.128			0.465		
2.257		0.450			
2.306	0.441				
3.193	0.361		0.373		
3.261		0.364			
3.365				0.364	
3.902	0.315				
4.515		0.299			
4.612			0.301		
4.759					0.293
4.789	0.270				
5.498	0.239				
5.518		0.264			
6.385	0.203		0.249		
6.772		0.230			
7.139					0.228
7.776		0.205			
7.804			0.218		
9.032		0.179			
9.578			0.187		
10.096				0.184	
10.997			0.182		
12.770			0.161		
15.143				0.205	

Validity limits:

Range of application: $0 \leq \lambda \leq 15.143$
$0.01 \leq B/r_m \leq 0.2$

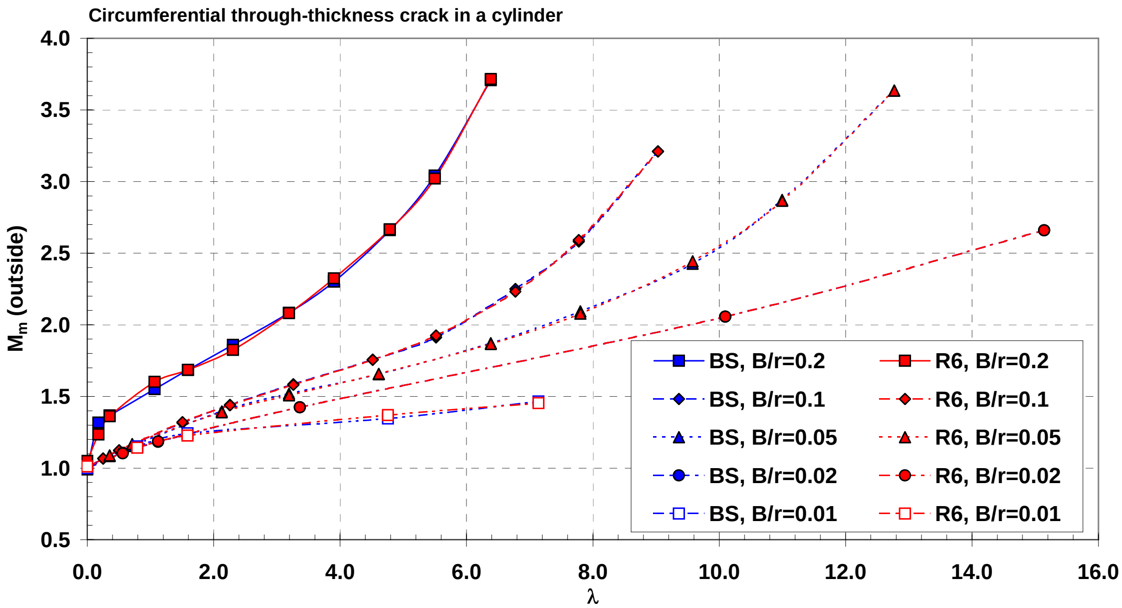
R6 solution [A.7][A.35]

The R6 solution is based on similar work to that described for the BS 7910 solution, although it is presented in a slightly different way. Consequently it is not repeated here.

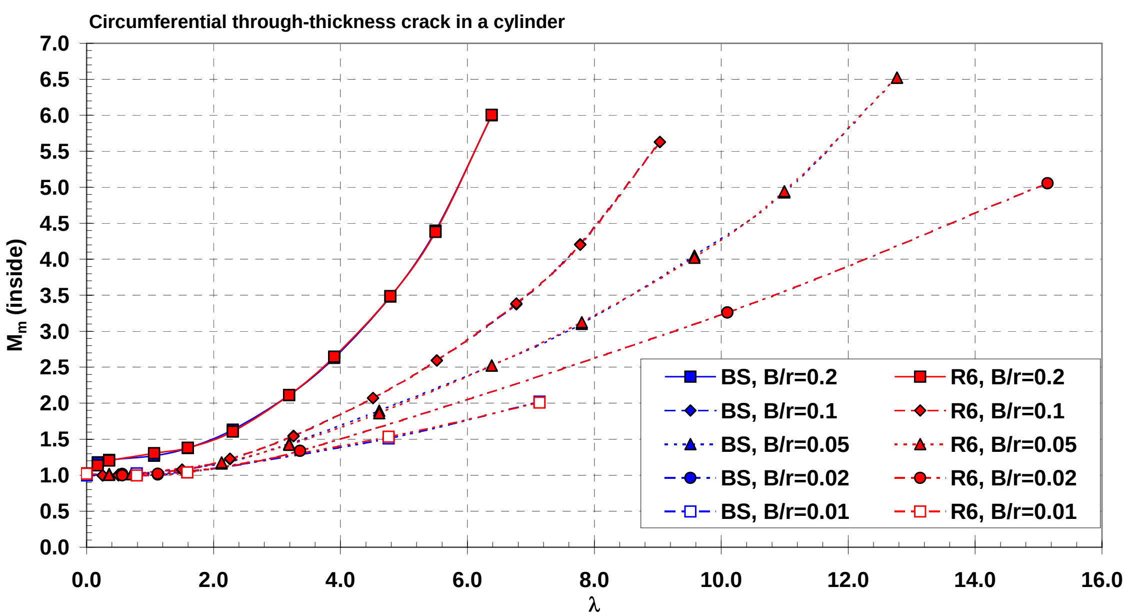
Plots

Figure A.18 shows the solution as a function of λ (defined in (A.59) for various B/r_m ratios.

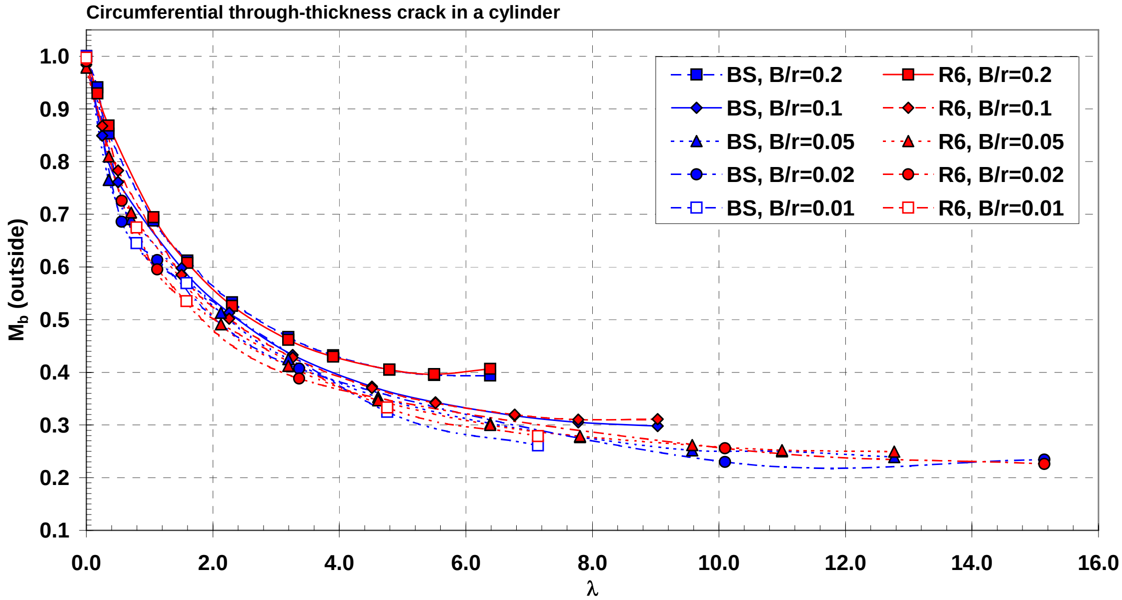
Figure A.18 Normalised K-solution for a circumferential through-thickness crack in a cylinder



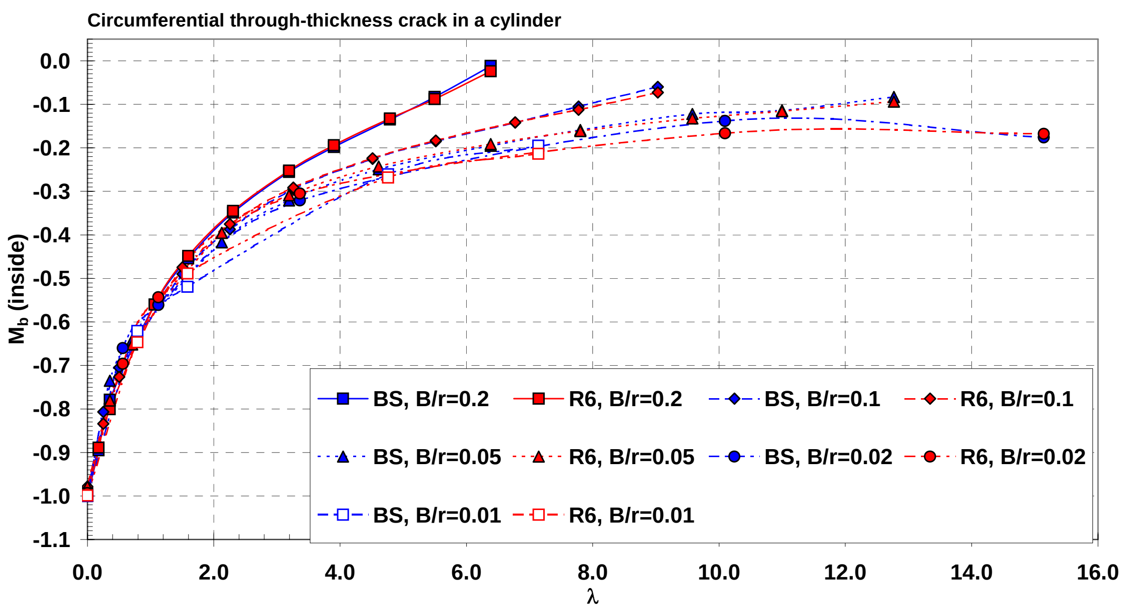
a) M_m on the outside of the cylinder



b) M_m on the inside of the cylinder



c) M_b on the outside of the cylinder

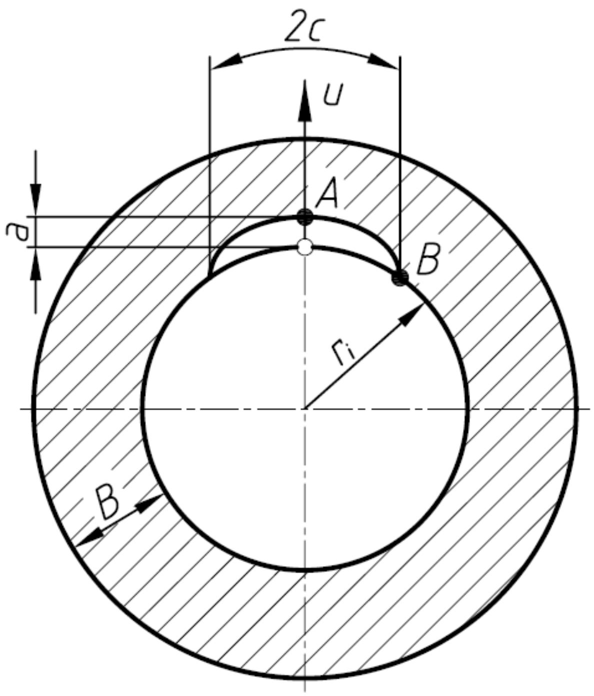


d) M_b on the inside of the cylinder

A.4.2.2 Surface cracks

A.4.2.2.1 Internal circumferential surface crack

a) Finite crack



BS 7910 Solution: [A.15][A.29][A.36][A.37]

The stress intensity factor solution is calculated from equations (A.1 to (A.6

Where:

$M = f_w = 1$

M_m and M_b for the deepest point in the crack (A, θ=90º) and for the points where the crack intersects the free surface (B, θ=0º) are given in Table A.17.

Table A.17 BS 7910 M_m and M_b solutions for a circumferential internal surface crack in a cylinder

a/2*c* = 0.5, *B*/r_i = 0.1					*a*/2*c* = 0.5, *B*/r_i = 0.2				
a/*B*	M_m(A)	M_b(A)	M_m(B)	M_b(B)	*a*/*B*	M_m(A)	M_b(A)	M_m(B)	M_b(B)
0.0	0.663	0.663	0.729	0.729	0.0	0.663	0.663	0.729	0.729
0.2	0.667	0.574	0.681	0.623	0.2	0.667	0.582	0.681	0.623
0.4	0.670	0.327	0.706	0.528	0.4	0.670	0.334	0.706	0.528
0.6	0.686	0.140	0.733	0.431	0.6	0.686	0.117	0.733	0.431
0.8	0.702	-0.105	0.764	0.332	0.8	0.702	-0.099	0.764	0.332
a/2*c* = 0.25, *B*/r_i = 0.1					*a*/2*c* = 0.25, *B*/r_i = 0.2				
0.0	0.896	0.896	0.697	0.697	0.0	0.896	0.896	0.697	0.697
0.2	0.999	0.731	0.731	0.628	0.2	1.004	0.735	0.731	0.628
0.4	1.031	0.504	0.801	0.563	0.4	1.030	0.503	0.801	0.563
0.6	1.121	0.306	0.889	0.502	0.6	1.124	0.305	0.889	0.502
0.8	1.148	0.014	0.993	0.445	0.8	1.192	0.027	0.993	0.445
a/2*c* = 0.1, *B*/r_i = 0.1					*a*/2*c* = 0.1, *B*/r_i = 0.2				
0.0	1.059	1.059	0.521	0.521	0.0	1.059	1.059	0.521	0.521
0.2	1.168	0.870	0.617	0.623	0.2	1.144	0.851	0.617	0.623
0.4	1.375	0.736	0.835	0.591	0.4	1.318	0.698	0.835	0.591
0.6	1.599	0.561	1.048	0.556	0.6	1.517	0.515	1.048	0.556
0.8	1.803	0.269	1.255	0.519	0.8	1.782	0.253	1.255	0.519
a/2*c* = 0.05, *B*/r_i = 0.1					*a*/2*c* = 0.05, *B*/r_i = 0.2				
0.0	1.103	1.103	0.384	0.384	0.0	1.103	1.103	0.384	0.384
0.2	1.219	0.921	0.482	0.487	0.2	1.214	0.903	0.482	0.487
0.4	1.529	0.829	0.700	0.498	0.4	1.382	0.776	0.700	0.498
0.6	1.939	0.677	0.981	0.525	0.6	1.661	0.624	0.981	0.525
0.8	2.411	0.479	1.363	0.570	0.8	2.031	0.386	1.363	0.570

A global bending moment on the cylinder can be included [A.46] by adding the following stress to P_m:

$$M_{global} = (r_i + a)/\Pi\left\{(r_i + B)^4 - r_i^4\right\} \quad \text{(A.71)}$$

Validity limits:

Range of application:

$0 \leq a/B \leq 0.8$

$0.05 \leq a/2c \leq 0.5$

$0.1 \leq B/r_i \leq 0.2$

R6 solution [A.18][A.38]

The stress intensity factor K_I is given by

$$K_I = \sqrt{\pi a}\left(\sum_{i=0}^{3} P_i f_i\left(\frac{a}{B}, \frac{a}{2c}, \frac{B}{r_i}\right) + P_{bg} f_{bg}\left(\frac{a}{B}, \frac{a}{2c}, \frac{B}{r_i}\right)\right) \quad \text{(A.72)}$$

where P_i (i = 0 to 3) are stress components which define the axisymmetric stress distribution P according to

$$P = P(u) = \sum_{i=0}^{3} P_i \left(\frac{u}{a}\right)^i \qquad \text{for } 0 \le u \le a \tag{A.73}$$

and P_{bg} is the global bending stress, ie the maximum outer fibre bending stress. The stresses P and P_{bg} are to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is the distance from the inner surface of the cylinder as defined in the figure above. The solution for the global bending stress assumes that the crack is symmetrically positioned about the global bending axis so that the maximum stress occurs at u=B above Point A. The geometry functions f_i (i = 0 to 3) and f_{bg} are given in Table A.18 and for the deepest point of the crack (A), and at the intersection of the crack with the free surface (B), respectively.

Table A.18 R6 Geometry functions at Point A for a part circumferential internal surface crack in a cylinder.

***a/2c=0.5*, B/r_i=0.2**					
a/B					
0	0.659	0.471	0.387	0.337	0.549
0.2	0.665	0.460	0.371	0.316	0.570
0.4	0.682	0.471	0.381	0.327	0.600
0.6	0.700	0.481	0.390	0.335	0.632
0.8	0.729	0.506	0.410	0.352	0.675
a/2c=0.5, B/r_i=0.1					
a/B					
0	0.659	0.471	0.387	0.337	0.599
0.2	0.664	0.459	0.370	0.315	0.613
0.4	0.680	0.469	0.379	0.325	0.636
0.6	0.696	0.478	0.387	0.333	0.659
0.8	0.714	0.497	0.403	0.347	0.685
***a/2c=0.25*, B/r_i=0.2**					
a/B					
0	0.886	0.565	0.430	0.352	0.738
0.2	0.890	0.556	0.424	0.347	0.761
0.4	0.934	0.576	0.440	0.362	0.817
0.6	0.991	0.602	0.457	0.377	0.885
0.8	1.066	0.653	0.496	0.409	0.973
***a/2c=0.25*, B/r_i=0.1**					
a/B					
0	0.886	0.565	0.430	0.352	0.806
0.2	0.895	0.557	0.424	0.347	0.825
0.4	0.947	0.580	0.441	0.363	0.883
0.6	1.008	0.605	0.458	0.377	0.950
0.8	1.062	0.647	0.492	0.406	1.012
***a/2c=0.125*, B/r_i=0.2**					
a/B					
0	1.025	0.600	0.441	0.356	0.854
0.2	1.041	0.625	0.469	0.381	0.890
0.4	1.142	0.666	0.496	0.403	0.995
0.6	1.274	0.718	0.527	0.427	1.126
0.8	1.463	0.813	0.589	0.471	1.310

R6 Geometry functions at Point A for a part circumferential internal surface crack in a cylinder (continued).

a/2c=0.125, B/r_i=0.1					
a/B					
0	1.025	0.600	0.441	0.356	0.931
0.2	1.053	0.629	0.471	0.382	0.970
0.4	1.180	0.678	0.502	0.407	1.097
0.6	1.335	0.737	0.536	0.431	1.253
0.8	1.482	0.814	0.587	0.469	1.402
a/2c=0.063, B/r_i=0.2					
a/B					
0	1.079	0.635	0.473	0.388	0.899
0.2	1.130	0.665	0.493	0.398	0.964
0.4	1.294	0.732	0.537	0.433	1.120
0.6	1.521	0.820	0.587	0.468	1.321
0.8	1.899	0.987	0.690	0.541	1.633
a/2c=0.063, B/r_i=0.1					
a/B					
0	1.079	0.635	0.473	0.388	0.981
0.2	1.150	0.672	0.498	0.401	1.059
0.4	1.366	0.756	0.549	0.441	1.267
0.6	1.643	0.859	0.606	0.479	1.531
0.8	1.972	1.002	0.694	0.541	1.842
a/2c=0.031, B/r_i=0.2					
a/B					
0	1.101	0.658	0.499	0.413	0.918
0.2	1.180	0.690	0.512	0.414	1.004
0.4	1.377	0.775	0.564	0.453	1.188
0.6	1.707	0.902	0.638	0.505	1.430
0.8	2.226	1.137	0.783	0.609	1.794
a/2c=0.031, B/r_i=0.2					
a/B					
0	1.101	0.658	0.499	0.413	1.001
0.2	1.209	0.701	0.518	0.418	1.112
0.4	1.490	0.810	0.582	0.464	1.377
0.6	1.887	0.958	0.665	0.520	1.737
0.8	2.444	1.187	0.799	0.613	2.219

Table A.19 R6 Geometry functions at Point B for a part circumferential internal surface crack in a cylinder

***a/2c=0.5*, B/r$_i$ = 0.2**					
a/B					
0	0.718	0.117	0.041	0.020	0.598
0.2	0.746	0.125	0.046	0.023	0.625
0.4	0.774	0.133	0.051	0.026	0.652
0.6	0.822	0.147	0.058	0.031	0.696
0.8	0.876	0.161	0.064	0.034	0.746
***a/2c=0.5*, B/r$_i$ = 0.1**					
a/B					
0	0.716	0.116	0.041	0.020	0.652
0.2	0.747	0.125	0.046	0.023	0.682
0.4	0.778	0.134	0.051	0.026	0.712
0.6	0.831	0.148	0.058	0.031	0.763
0.8	0.890	0.163	0.064	0.033	0.820
a/2c=*0.25*, B/r$_i$ = 0.2					
a/B					
0	0.664	0.091	0.029	0.013	0.555
0.2	0.716	0.108	0.039	0.019	0.599
0.4	0.768	0.125	0.049	0.025	0.643
0.6	0.852	0.152	0.062	0.033	0.712
0.8	0.944	0.179	0.075	0.040	0.788
***a/2c=0.25*, B/r$_i$ = 0.1**					
a/B					
0	0.657	0.089	0.030	0.014	0.598
0.2	0.719	0.109	0.040	0.020	0.656
0.4	0.781	0.129	0.050	0.026	0.714
0.6	0.883	0.160	0.066	0.035	0.809
0.8	0.995	0.191	0.079	0.042	0.913
a/2c=0.125, B/r$_i$ = 0.2					
a/B					
0	0.541	0.054	0.014	0.004	0.461
0.2	0.598	0.072	0.023	0.010	0.496
0.4	0.655	0.090	0.032	0.016	0.531
0.6	0.737	0.116	0.045	0.023	0.576
0.8	0.846	0.151	0.062	0.033	0.634
***a/2c=0.5*, B/r$_i$ = 0.2**					
a/B					
0	0.718	0.117	0.041	0.020	0.598
0.2	0.746	0.125	0.046	0.023	0.625
0.4	0.774	0.133	0.051	0.026	0.652
0.6	0.822	0.147	0.058	0.031	0.696
0.8	0.876	0.161	0.064	0.034	0.746

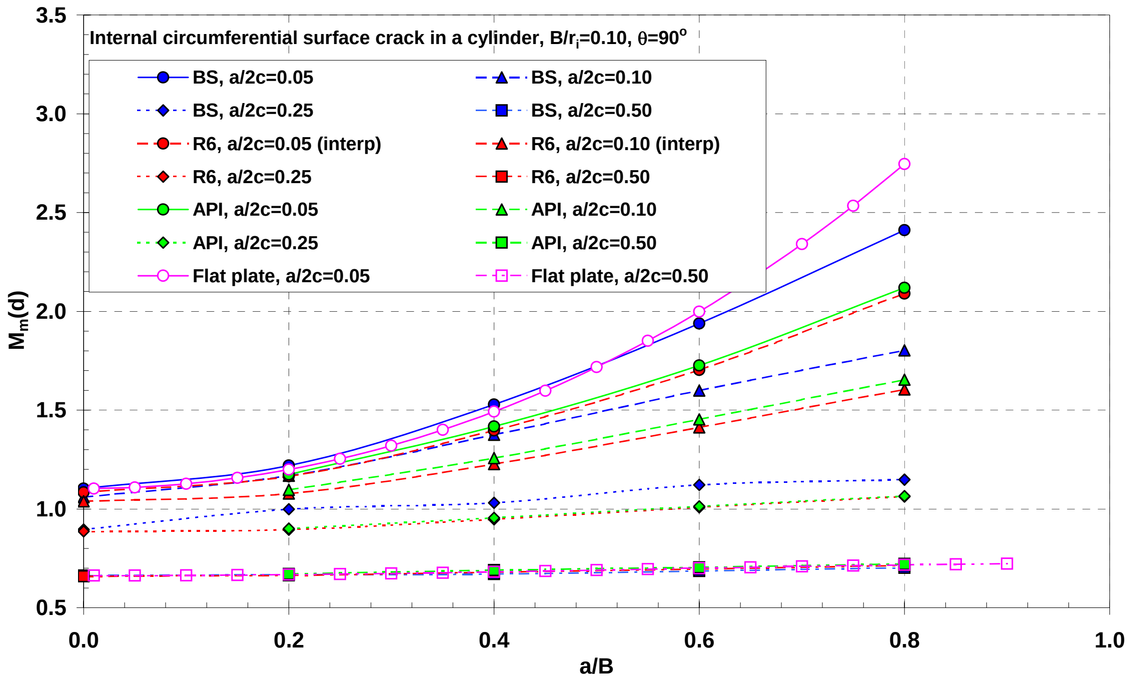
R6 Geometry functions at Point B for a part circumferential internal surface crack in a cylinder (continued).

***a/2c=0.5*, B/r$_i$ = 0.1**					
a/B					
0	0.716	0.116	0.041	0.020	0.652
0.2	0.747	0.125	0.046	0.023	0.682
0.4	0.778	0.134	0.051	0.026	0.712
0.6	0.831	0.148	0.058	0.031	0.763
0.8	0.890	0.163	0.064	0.033	0.820
***a/2c=0.25*, B/r$_i$ = 0.2**					
a/B					
0	0.664	0.091	0.029	0.013	0.555
0.2	0.716	0.108	0.039	0.019	0.599
0.4	0.768	0.125	0.049	0.025	0.643
0.6	0.852	0.152	0.062	0.033	0.712
0.8	0.944	0.179	0.075	0.040	0.788
***a/2c=0.25*, B/r$_i$ = 0.1**					
a/B					
0	0.657	0.089	0.030	0.014	0.598
0.2	0.719	0.109	0.040	0.020	0.656
0.4	0.781	0.129	0.050	0.026	0.714
0.6	0.883	0.160	0.066	0.035	0.809
0.8	0.995	0.191	0.079	0.042	0.913
***a/2c=0.125*, B/r$_i$ = 0.2**					
a/B					
0	0.541	0.054	0.014	0.004	0.461
0.2	0.598	0.072	0.023	0.010	0.496
0.4	0.655	0.090	0.032	0.016	0.531
0.6	0.737	0.116	0.045	0.023	0.576
0.8	0.846	0.151	0.062	0.033	0.634

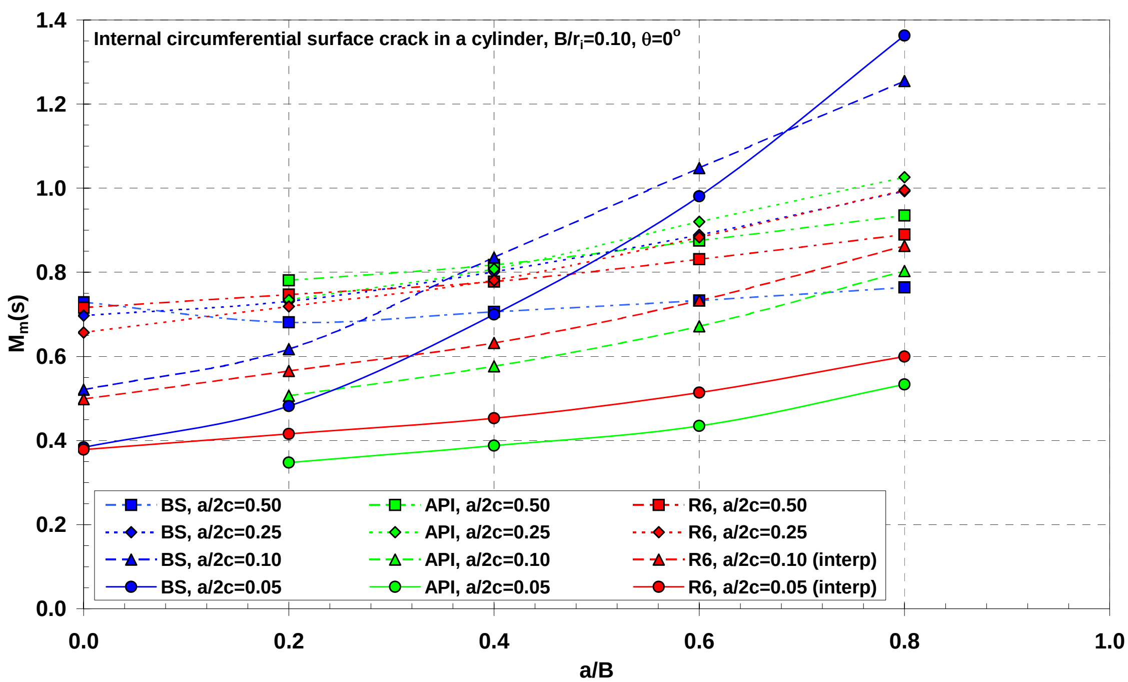
Plots

Figure A.19 (x-ref) shows the solutions for various a/2c and B/r$_i$. BSI, R6 and API solutions are included (R6 results were obtained by extrapolation of the tabulated solutions); BSI solutions for a surface crack in a flat plate (infinite radius, B/r$_i$$\to$0) are included for comparison. For the solution at θ=90º (M_m(A) and M_m(B)) R6 and API solutions are very close for a given geometry, with BS 7910 solutions slightly higher. For the surface (Point B, θ=0º), the BS solutions show rather different trends from the others. As has been pointed out by Smith ([A.13], the determination of stress intensity by elastic FEA as the crack front approaches the surface presents particular difficulties, and this may be a factor in the discrepancy.

Figure A.19 Normalised K-solution for an internal circumferential crack in a cylinder

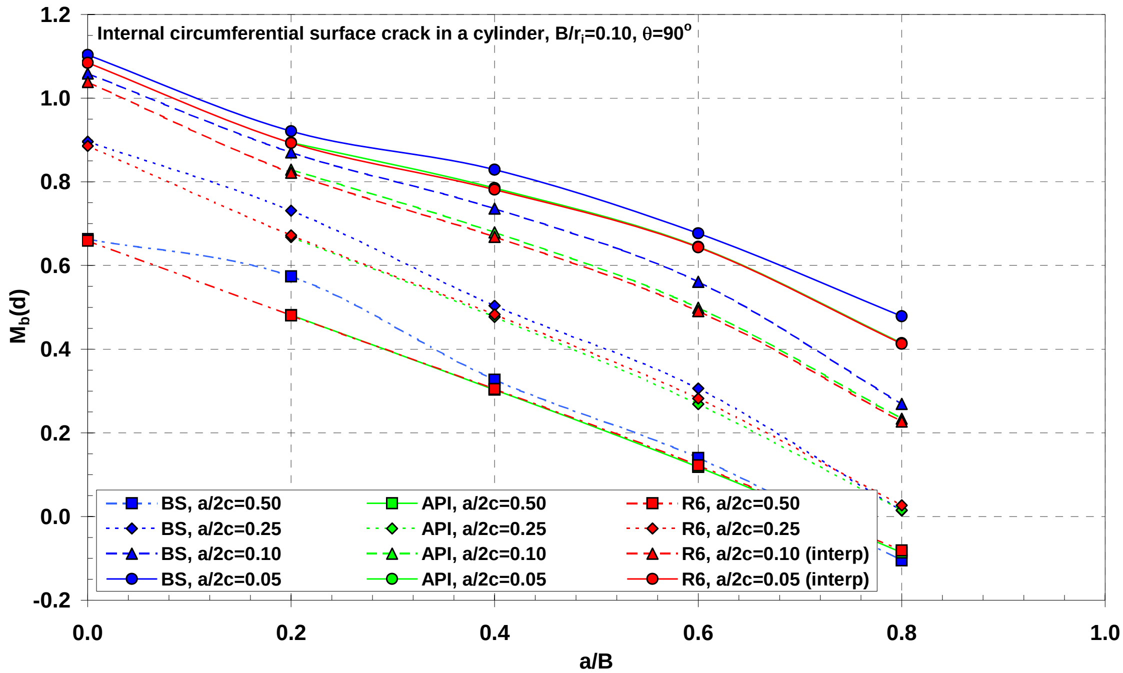


a) Membrane stress, θ=90º, B/r_i=0.1

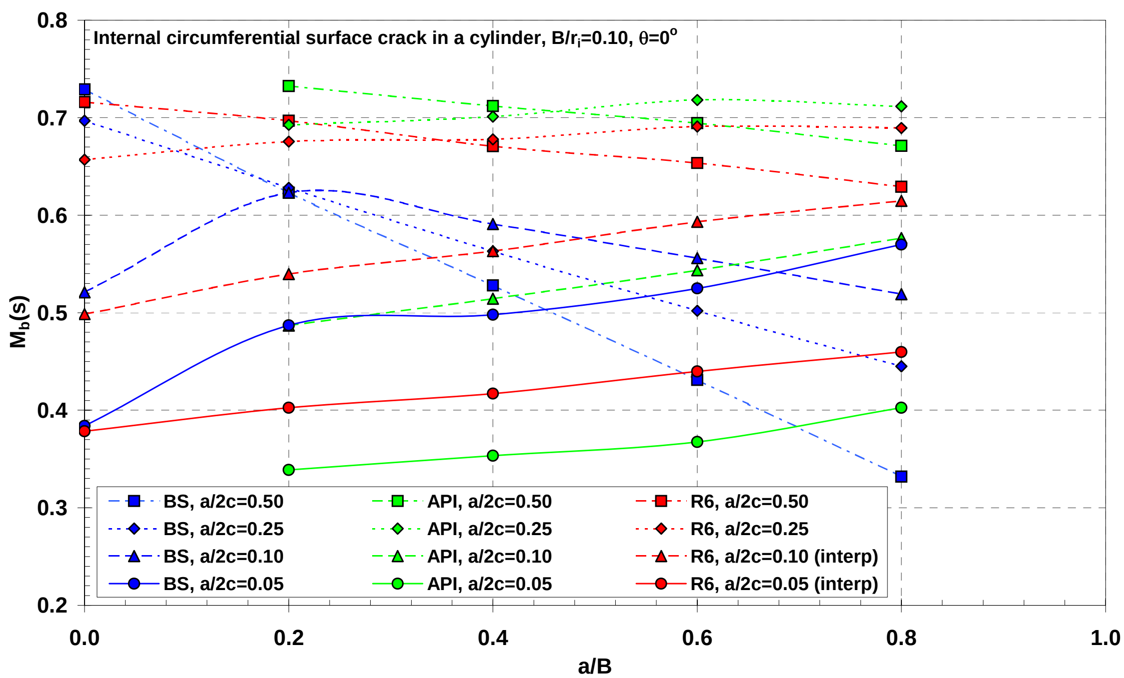


b) Membrane stress, θ=0º, B/r_i=0.1

Normalised K-solution for an internal circumferential crack in a cylinder (cont'd)

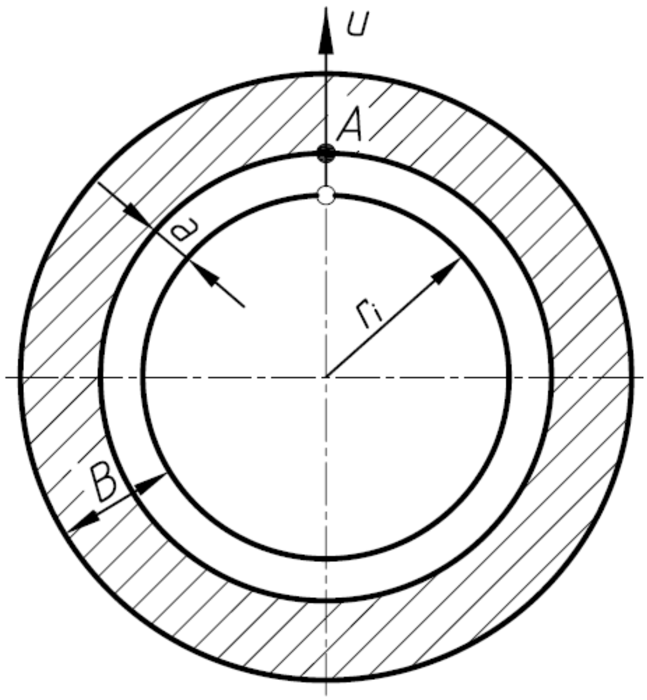


c) Bending stress, θ=90°, B/r_i=0.1



d) Bending stress, θ=0°, B/r_i=0.1

b) Extended crack



BS 7910 Solution [A.36][A.32][A.39]

The stress intensity factor solution is calculated from equations (A.1 to (A.6

Where:

$M = f_w = 1$;

M_m and M_b are given in Table A.20:

Table A.20 BS 7910 M_m and M_b solutions for an extended circumferential internal surface crack in a cylindrical shell

$B/r_i = 0.1$			$B/r_i = 0.2$		
a/B	M_m	M_b	a/B	M_m	M_b
0.0	1.122	1.122	0.0	1.122	1.122
0.2	1.261	0.954	0.2	1.215	0.933
0.4	1.582	0.909	0.4	1.446	0.810
0.6	2.091	0.810	0.6	1.804	0.650
0.8	2.599	0.600	0.8	2.280	0.411

Validity limits:

Range of application: $0 \leq a/B \leq 0.8$
$0.1 \leq B/r_i \leq 0.2$

R6 Solution [A.19]

The stress intensity factor K_I is given by:

$$K_I = \frac{1}{\sqrt{2\pi a}} \int_0^a P(u) \sum_{i=1}^{i=3} f_i \left(a/B, B/r_i\right) \left(1 - \frac{u}{a}\right)^{i-\frac{3}{2}} du \qquad \text{(A.74)}$$

The stress distribution P = P(u) is to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is the distance from the inner surface of the cylinder as shown above.

The geometry functions f_i (i = 1 to 3) are given in Table A.21 for the deepest point of the crack

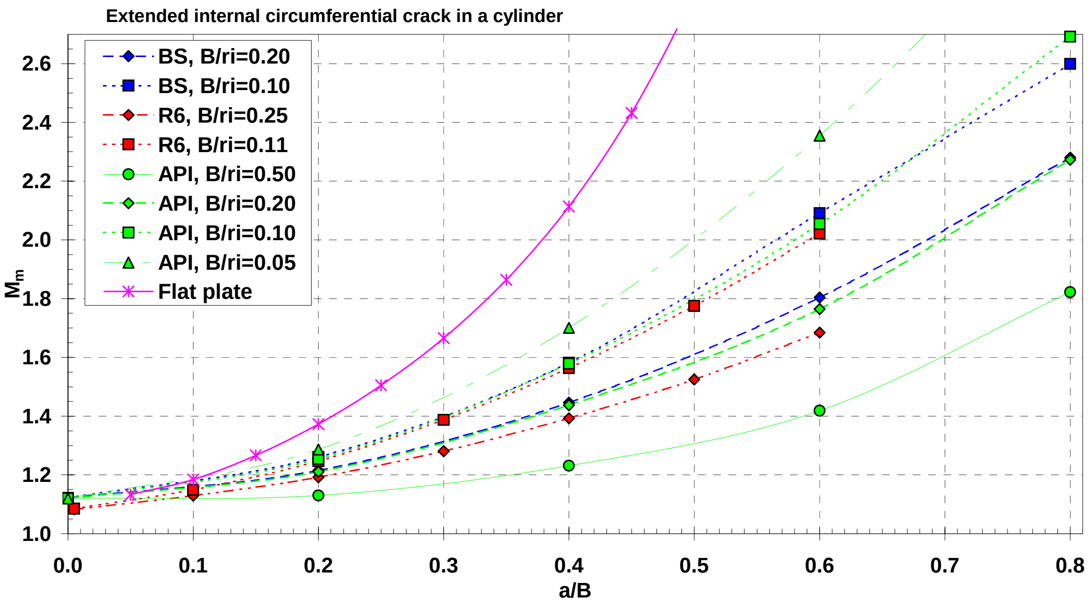
Table A.21 R6 geometry functions for an extended internal circumferential crack in a cylinder.

a/B	B/r_i = 0.43		
0	2.000	1.327	0.218
0.1	2.000	1.337	0.200
0.2	2.000	1.543	0.201
0.3	2.000	1.880	0.228
0.4	2.000	2.321	0.293
0.5	2.000	2.879	0.373
0.6	2.000	3.720	0.282
a/B	B/r_i = 0.25		
0	2.000	1.336	0.218
0.1	2.000	1.460	0.206
0.2	2.000	1.839	0.241
0.3	2.000	2.359	0.353
0.4	2.000	2.976	0.556
0.5	2.000	3.688	0.837
0.6	2.000	4.598	1.086
a/B	B/r_i = 0.11		
0	2.000	1.346	0.219
0.1	2.000	1.591	0.211
0.2	2.000	2.183	0.279
0.3	2.000	2.966	0.518
0.4	2.000	3.876	0.956
0.5	2.000	4.888	1.614
0.6	2.000	5.970	2.543

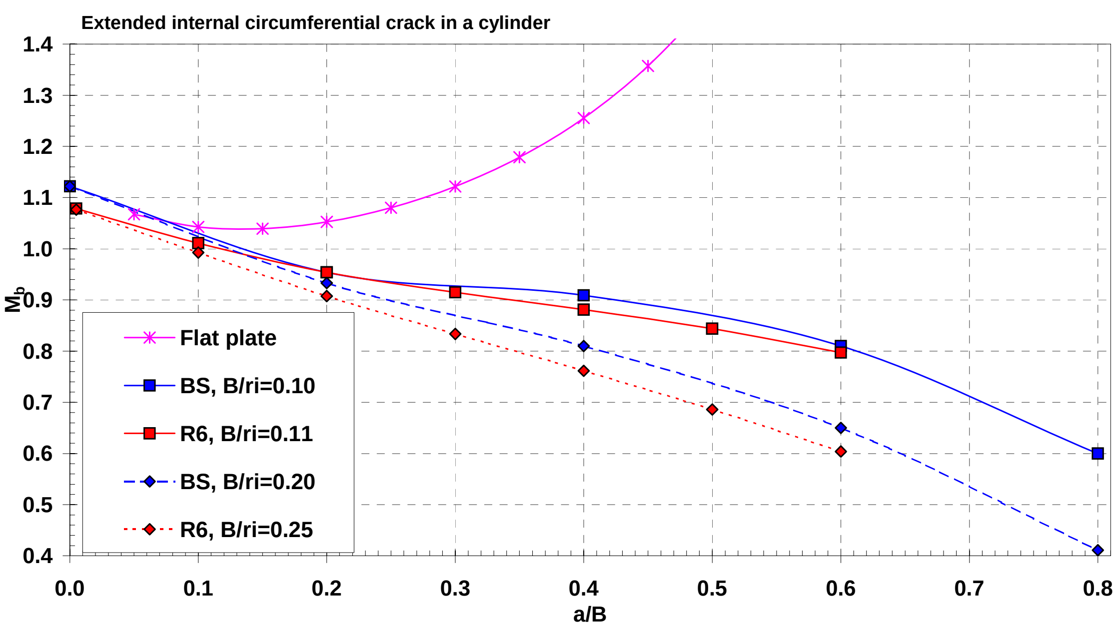
Plots

Figure A.20 shows the BSI, R6 and API solutions; the solutions are very similar between procedures for $B/r_i \approx 0.1$ $B/r_i \approx 0.2$, but API offers a wider range of geometries (up to $0.001 \leq B/r_i \leq 0.5$). The BS 7910 solution for an extended surface flaw in a flat plate is shown for comparison and is seen to overestimate M_m relative to the geometry-specific solutions.

Figure A.20 Normalised K-solution for an extended internal circumferential crack in a cylinder



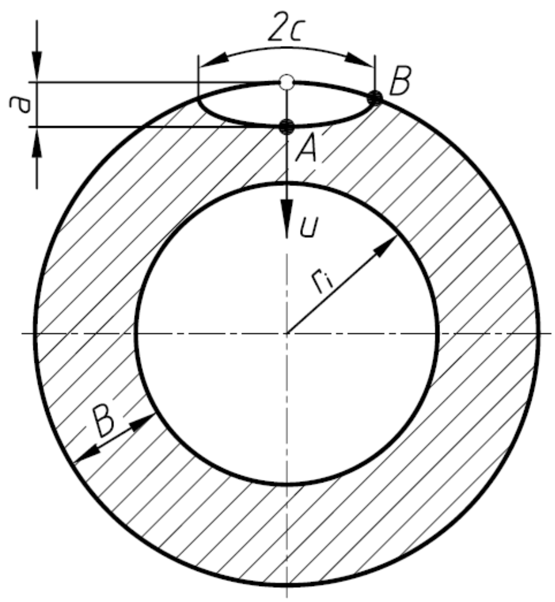
a) Membrane stress



Bending stress

A.4.2.2.2 External circumferential surface crack

a) Finite crack



BS 7910 Solution

It is recommended to use the flat plate solution given in Section A.2.2.
M = 1

Validity limits:

$0 \le a/2c \le 1.0$
$0 \le \Theta \le \Pi$

and

$a/B < 1.25\,(a/c + 0.6)$	for $0 \le a/2c \le 0.1$
$a/B < 1.0$	for $0.1 \le a/2c \le 1.0$

R6 Solution [A.14][A.38]

The stress intensity factor K_I is given by

$$K_I = \sqrt{\pi a}\left(\sum_{i=0}^{3} P_i f_i\left(\frac{a}{B},\frac{a}{2c},\frac{B}{r_i}\right) + P_{bg} f_{bg}\left(\frac{a}{B},\frac{a}{2c},\frac{B}{r_i}\right)\right) \qquad (A.75)$$

where P_i (i = 0 to 3) are stress components which define the axisymmetric stress distribution P according to:

$$P = P(u) = \sum_{i=0}^{3} P_i \left(\frac{u}{a}\right)^i \qquad \text{for } 0 \le u \le a \tag{A.76}$$

and P_{bg} is the global bending stress, ie the maximum outer fibre bending stress. The stresses P and P_{bg} are to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is the distance from the external surface of the cylinder as shown in the figure above. The solution for global bending stress assumes that the crack is symmetrically positioned about the global bending axis so that the maximum stress occurs at u=0. The geometry functions f_i (i = 0 to 3) and f_{bg} are given in and Table A.23 respectively for the deepest point of the crack, and at the intersection of the crack with the free surface.

Table A.22 R6 geometry functions for an external circumferential crack in a cylinder; deepest part of crack (Point A)

***a/2c=0.5*, B/r_i=0.2**					
a/B					
0	0.659	0.471	0.387	0.337	0.659
0.2	0.661	0.455	0.367	0.313	0.645
0.4	0.673	0.462	0.374	0.321	0.642
0.6	0.686	0.467	0.378	0.325	0.638
0.8	0.690	0.477	0.387	0.333	0.626
***a/2c=0.5*, B/r_i=0.1**					
a/B					
0	0.659	0.471	0.387	0.337	0.659
0.2	0.662	0.456	0.368	0.313	0.653
0.4	0.676	0.464	0.376	0.322	0.659
0.6	0.690	0.470	0.381	0.328	0.664
0.8	0.695	0.482	0.392	0.337	0.660
***a/2c=0.25*, B/r_i=0.2**					
a/B					
0	0.886	0.565	0.430	0.352	0.886
0.2	0.905	0.560	0.425	0.347	0.885
0.4	0.972	0.586	0.443	0.363	0.932
0.6	1.060	0.618	0.462	0.378	0.995
0.8	1.133	0.659	0.493	0.403	1.041
***a/2c=0.25*, B/r_i=0.1**					
a/B					
0	0.886	0.565	0.430	0.352	0.886
0.2	0.903	0.559	0.425	0.347	0.891
0.4	0.969	0.586	0.443	0.363	0.947
0.6	1.051	0.616	0.462	0.378	1.016
0.8	1.108	0.654	0.491	0.403	1.059
***a/2c=0.125*, B/r_i=0.2**					
a/B					
0	1.025	0.600	0.441	0.356	1.025
0.2	1.078	0.638	0.476	0.386	1.055
0.4	1.253	0.702	0.513	0.413	1.202
0.6	1.502	0.790	0.561	0.446	1.413
0.8	1.773	0.900	0.625	0.490	1.631

R6 geometry functions for an external circumferential crack in a cylinder; deepest part of crack (cont'd)

***a/2c=0.125*, B/r$_i$=0.1**					
a/B					
0	1.025	0.600	0.441	0.356	1.025
0.2	1.073	0.637	0.475	0.386	1.060
0.4	1.246	0.700	0.512	0.413	1.219
0.6	1.489	0.786	0.559	0.445	1.443
0.8	1.711	0.880	0.616	0.484	1.640

***a/2c=0.063*, B/r$_i$=0.2**					
a/B					
0	1.079	0.635	0.473	0.388	1.079
0.2	1.186	0.685	0.504	0.406	1.162
0.4	1.482	0.797	0.570	0.454	1.419
0.6	1.907	0.951	0.654	0.508	1.779
0.8	2.461	1.166	0.776	0.591	2.220

***a/2c=0.063*, B/r$_i$=0.1**					
a/B					
0	1.079	0.635	0.473	0.388	1.079
0.2	1.182	0.684	0.504	0.405	1.168
0.4	1.491	0.800	0.571	0.454	1.458
0.6	1.949	0.962	0.658	0.511	1.883
0.8	2.479	1.165	0.772	0.587	2.363

***a/2c=0.031*, B/r$_i$=0.2**					
a/B					
0	1.101	0.658	0.499	0.413	1.101
0.2	1.252	0.716	0.525	0.422	1.225
0.4	1.599	0.854	0.607	0.482	1.525
0.6	2.067	1.036	0.713	0.555	1.926
0.8	2.740	1.313	0.875	0.666	2.491

***a/2c=0.031*, B/r$_i$=0.1**					
a/B					
0	1.101	0.658	0.499	0.413	1.101
0.2	1.252	0.716	0.525	0.421	1.237
0.4	1.651	0.869	0.614	0.485	1.611
0.6	2.243	1.089	0.736	0.566	2.157
0.8	3.011	1.387	0.904	0.678	2.845

Table A.23 R6 geometry functions for an external circumferential crack in a cylinder; surface point (Point B)

a/2c=0.5, **B/r$_i$=0.2**					
a/B					
0	0.715	0.117	0.040	0.020	0.717
0.2	0.748	0.125	0.045	0.023	0.744
0.4	0.781	0.133	0.050	0.026	0.771
0.6	0.837	0.147	0.057	0.030	0.821
0.8	0.905	0.163	0.063	0.033	0.880

a/2c=0.5, **B/r$_i$=0.1**					
a/B					
0	0.713	0.117	0.041	0.020	0.713
0.2	0.748	0.125	0.046	0.023	0.745
0.4	0.783	0.133	0.051	0.026	0.777
0.6	0.841	0.149	0.058	0.030	0.832
0.8	0.912	0.166	0.064	0.033	0.898

a/2c=0.25, **B/r$_i$=0.2**					
a/B		f_1^B			
0	0.654	0.088	0.028	0.013	0.657
0.2	0.724	0.110	0.040	0.020	0.719
0.4	0.794	0.132	0.052	0.027	0.781
0.6	0.915	0.168	0.069	0.037	0.888
0.8	1.059	0.208	0.087	0.046	1.012

a/2c=0.25, **B/r$_i$=0.1**					
a/B					
0	0.649	0.087	0.028	0.013	0.649
0.2	0.723	0.110	0.040	0.020	0.720
0.4	0.797	0.133	0.052	0.027	0.791
0.6	0.925	0.172	0.071	0.038	0.912
0.8	1.081	0.215	0.089	0.048	1.058

a/2c=0.125, **B/r$_i$=0.2**					
a/B					
0	0.527	0.047	0.010	0.003	0.537
0.2	0.610	0.074	0.024	0.011	0.603
0.4	0.693	0.101	0.038	0.019	0.669
0.6	0.818	0.139	0.055	0.029	0.762
0.8	0.972	0.185	0.077	0.041	0.868

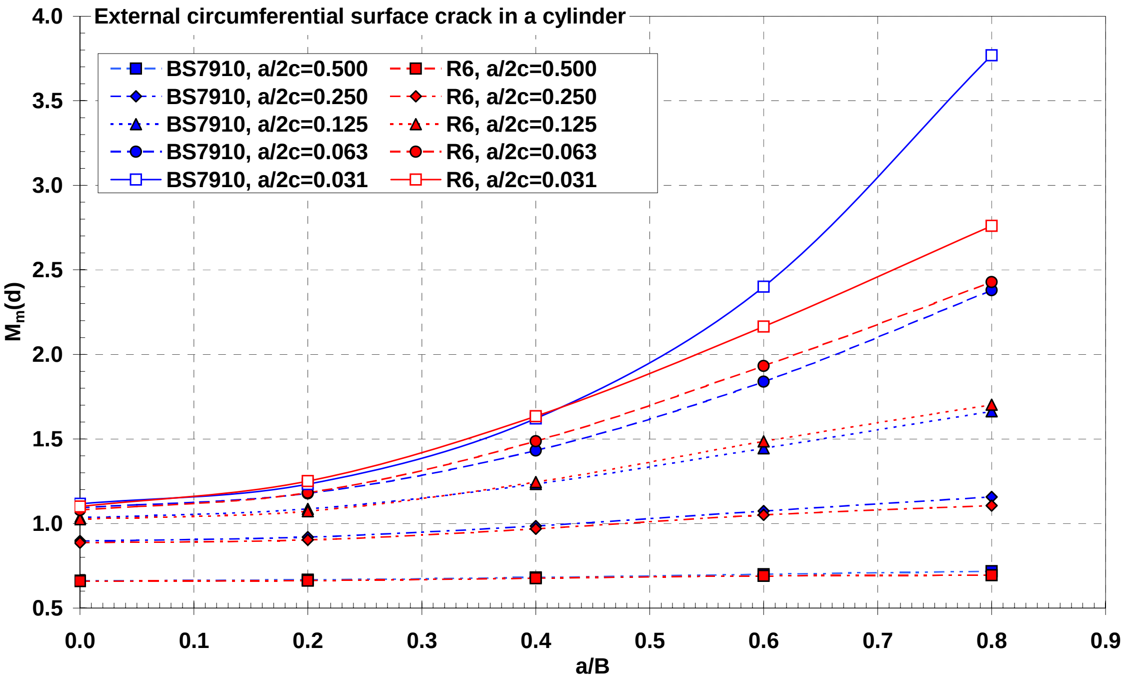
R6 geometry functions for an external circumferential crack in a cylinder; surface point (cont'd)

***a/2c=0.125*, B/r$_i$=0.1**					
a/B					
0	0.518	0.043	0.009	0.002	0.521
0.2	0.610	0.074	0.024	0.011	0.607
0.4	0.702	0.105	0.039	0.020	0.693
0.6	0.856	0.152	0.062	0.033	0.834
0.8	1.060	0.211	0.088	0.047	1.019
***a/2c=0.063*, B/r$_i$=0.2**					
a/B					
0	0.425	0.029	0.004	0.001	0.454
0.2	0.459	0.040	0.010	0.004	0.443
0.4	0.493	0.050	0.016	0.007	0.432
0.6	0.529	0.058	0.018	0.008	0.390
0.8	0.542	0.057	0.016	0.006	0.294
***a/2c=0.063*, B/r$_i$=0.1**					
a/B					
0	0.409	0.023	0.003	0.000	0.417
0.2	0.461	0.040	0.011	0.004	0.455
0.4	0.513	0.057	0.019	0.009	0.493
0.6	0.589	0.078	0.028	0.014	0.542
0.8	0.671	0.099	0.037	0.018	0.582
***a/2c=0.031*, B/r$_i$=0.2**					
a/B					
0	0.307	0.017	0.005	0.000	0.379
0.2	0.306	0.016	0.003	0.000	0.265
0.4	0.305	0.014	0.001	0.000	0.151
0.6	0.299	0.008	0.000	0.000	0.024
0.8	0.292	0.003	0.000	0.000	
***a/2c=0.031*, B/r$_i$=0.1**					
a/B					
0	0.299	0.021	0.002	0.000	0.323
0.2	0.309	0.020	0.003	0.000	0.296
0.4	0.319	0.019	0.004	0.000	0.269
0.6	0.322	0.016	0.002	0.000	0.208
0.8	0.305	0.005	0.000	0.000	0.103

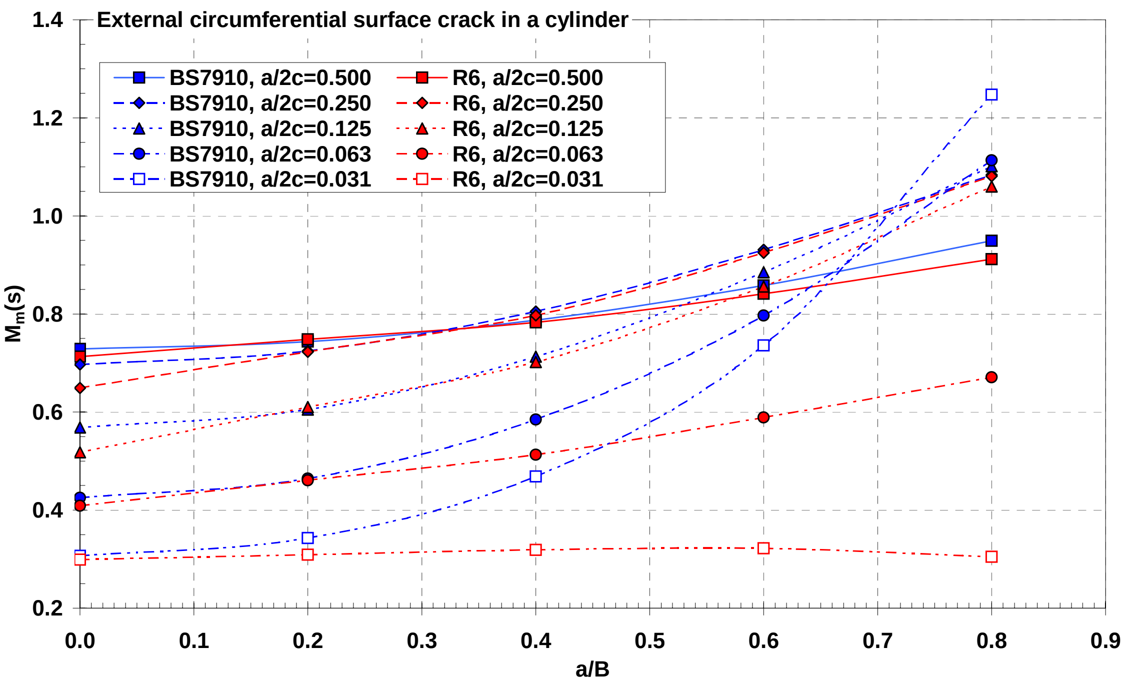
Plots

Figure A.21 compares the BS 7910 and R6 solutions. For the deepest point of the crack, there is little difference between the solutions, except for cracks with low a/2c ratio, eg a/2c=0.031, where the geometry-specific solution may be preferable. Smith [A.13] has demonstrated that the R6 solutions are consistent with those presented in API 579 and a recent solution by Chapuliot [A.40].

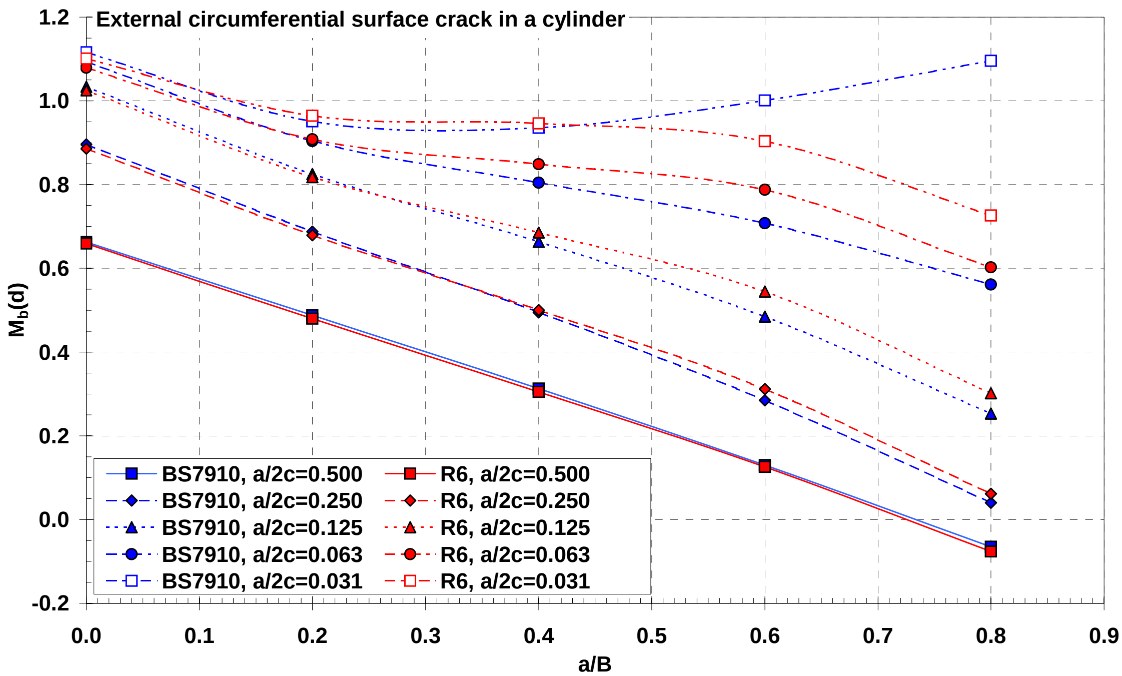
Figure A.21 Normalised K-solution for external circumferential crack in a cylinder



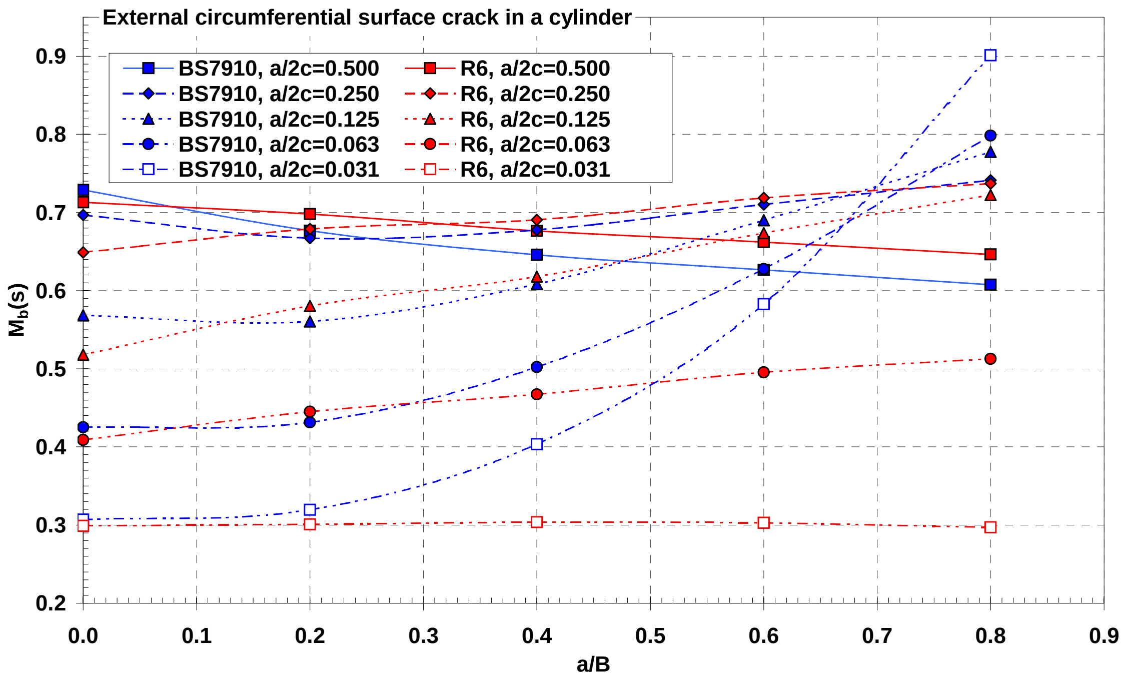
a) Membrane stress, θ=90º, B/r_i=0.1



b) Membrane stress, θ=0º, B/r_i=0.1

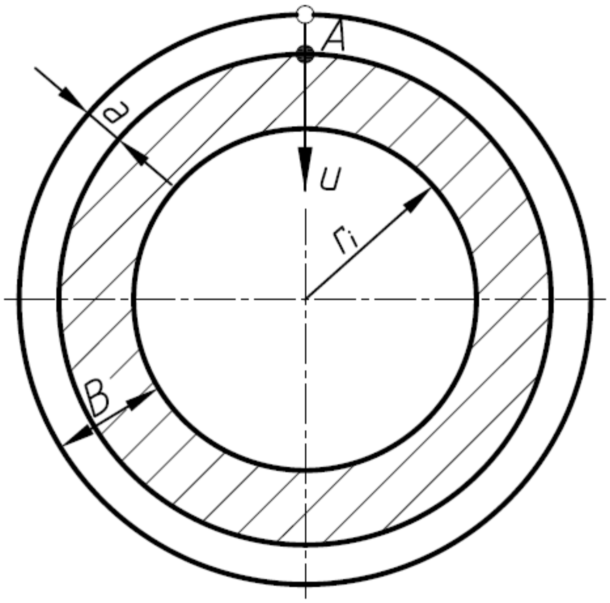


c) Bending stress, θ=90º, B/r_i=0.1



Bending stress, θ=0º, B/r_i=0.1

b) Extended Crack



BS 7910 Solution [A.3]

Note that this solution is appropriate for membrane loading only; bending stresses P_b and Q_b should be added to P_m and Q_m for assessment purposes. The stress intensity factor solution is calculated from equations (A.1 to (A.6 where

M = f_w = 1;

$$M_b = M_m = \frac{1-\lambda^2}{\left[\left\{1-(1-\lambda)\mu\right\}^2 - \lambda^2\right]}\left[0.8 + \frac{(1-\lambda)\mu}{1-(1-\lambda)\mu}\left\{4 + \frac{1.08\lambda}{(1-\lambda)(1-\mu)}\right\}\right]^{-0.5} \quad \text{(A.77)}$$

where:

$\lambda = r_i / r_o$

$\mu = a / B$

Validity limits:

None given

R6 Solution [A.19]

The stress intensity factor K_I is given by

$$K_I = \frac{1}{\sqrt{2\pi a}} \int_0^a P(u) \sum_{i=1}^{i=3} f_i\left(a/B, B/r_i\right)\left(1-\frac{u}{a}\right)^{i-\frac{3}{2}} du \qquad \text{(A.78)}$$

The stress distribution P = P(u) is to be taken normal to the prospective crack plane in an uncracked cylinder. The co-ordinate u is the distance from the outer surface of the cylinder as shown above.

The geometry functions f_i (i = 1 to 3) are given in Table A.24 for the deepest point of the crack.

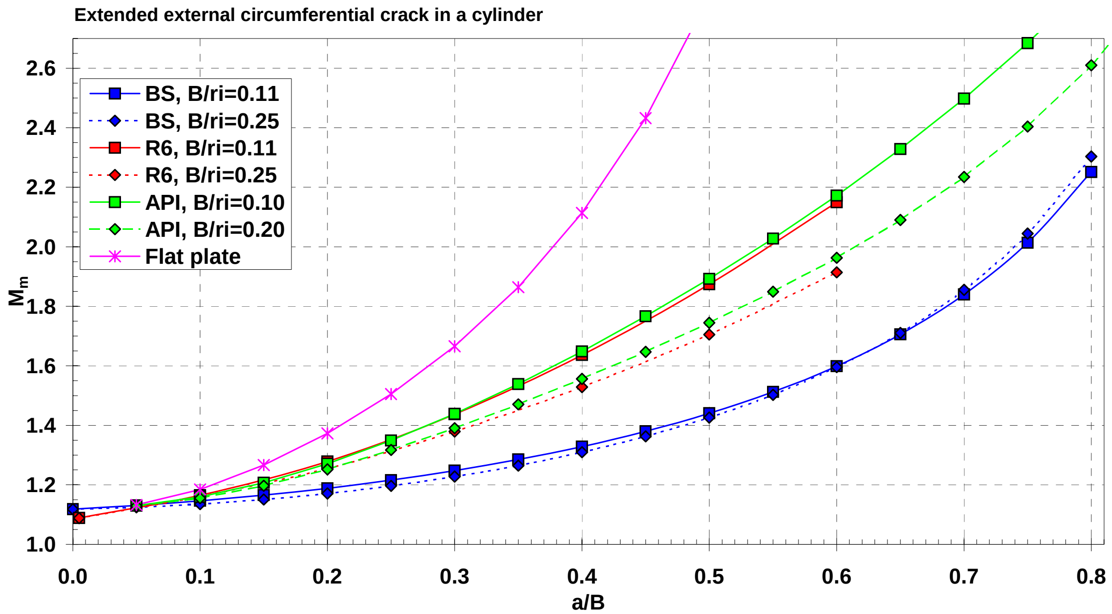
Table A.24 R6 geometry functions for an extended external circumferential crack in a cylinder.

a/*B*	B/r_i = 0.43		
0	2.000	1.359	0.220
0.1	2.000	1.642	0.236
0.2	2.000	2.127	0.307
0.3	2.000	2.727	0.447
0.4	2.000	3.431	0.668
0.5	2.000	4.271	0.951
0.6	2.000	5.406	1.183
a/*B*	B/r_i = 0.25		
0	2.000	1.362	0.221
0.1	2.000	1.659	0.221
0.2	2.000	2.220	0.303
0.3	2.000	2.904	0.535
0.4	2.000	3.701	0.857
0.5	2.000	4.603	1.311
0.6	2.000	5.671	1.851
a/*B*	B/r_i = 0.11		
0	2.000	1.364	0.220
0.1	2.000	1.694	0.211
0.2	2.000	2.375	0.310
0.3	2.000	3.236	0.630
0.4	2.000	4.252	1.136
0.5	2.000	5.334	1.972
0.6	2.000	6.606	2.902

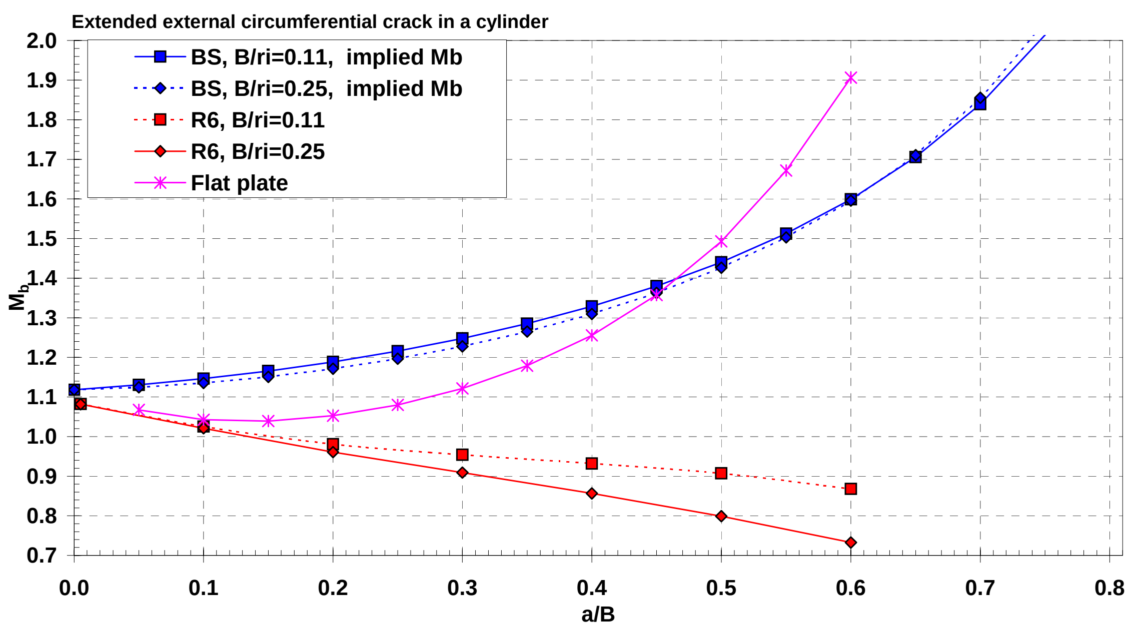
Plot

See Figure A.22 for details; this compares the R6, BS 7910 and API solutions. The BS 7910 solution for an extended surface crack in a plate is also shown for information. Note that the BS 7910 solution falls below the solutions given in R6 and API 579, and is also lower than the BS 7910 solution for an extended **internal** circumferential crack in a cylinder (cf Figure A.20). This is anomalous, and the R6 solution is recommended.

Figure A.22 Normalised K-solution for extended external circumferential crack in a cylinder



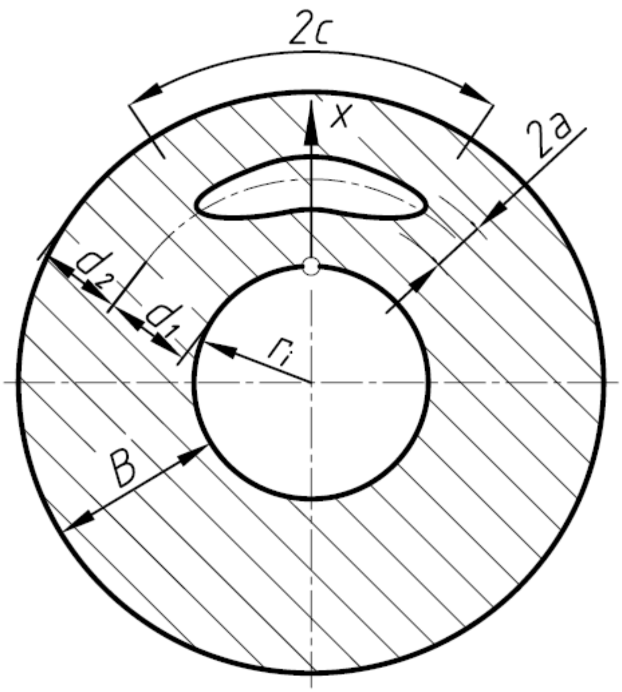
a) Membrane stress



b) Bending stress

A.4.2.3 Embedded crack

a) Finite crack



BS 7910 Solution

The flat plate solution in A.2.2.3 can be applied to embedded cracks in shells.

M = 1

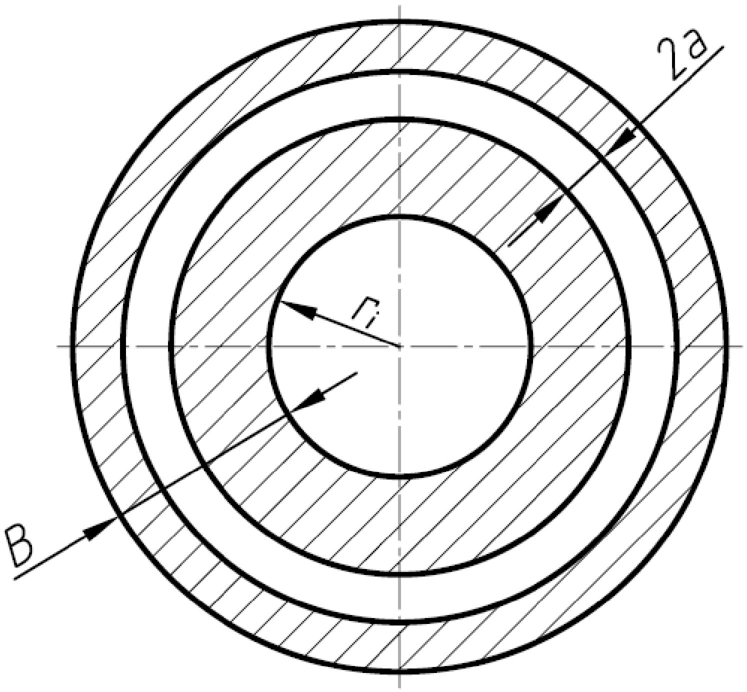
Validity limits:

None given

R6

No solution available

b) Extended Crack



BS 7910, R6

No solution available

API 579 Solution

API 579 recommends the use of the flat plate solution (see A.2.3.2).

A.5 Cracks In Nozzles

BS 7910 solution

The flat plate solution in A.2.7 can be applied to radial internal corner cracks in nozzles, together with appropriate stress concentration factors.

Validity limits:

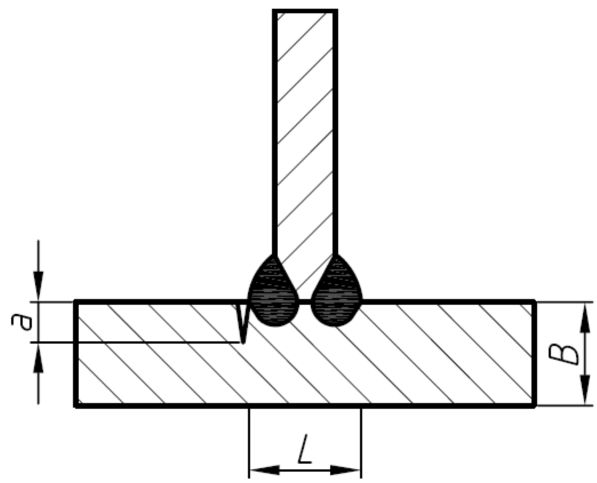
None given

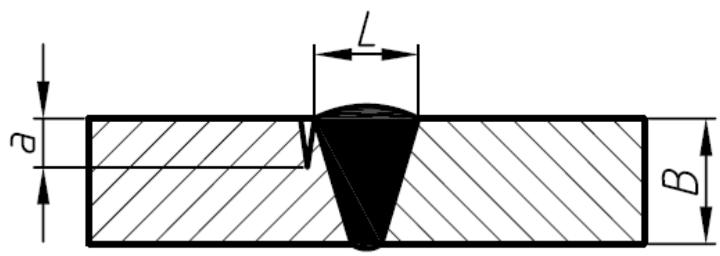
R6

No solution available.

A.6 Welded joints

A.6.1 Butt, Full Penetration and Attachment Welds with Surface Crack at Weld Toe





BS 7910 Solution

BS7910 advises that, when a flaw or crack is situated in a region of local stress concentration such as the weld toe, it is necessary to include the effect of the stress concentration field when calculating K_I. Unless the KI solution being used already incorporates the influence of the stress concentration, it is necessary to introduce the correction factor M_k, which is a function of crack size, geometry and loading. In general, the correction factor, M_k, is the product of the ratio of the K for a crack in material with the stress concentration to the K for the same crack in material without the stress concentration.

Thus, M_k normally decreases with increases in through-thickness distance z from the weld toe to unity at crack heights of typically 30 % of material thickness. For butt welds, T-butt welds, full penetration cruciform joints and members with fillet or butt-welded attachments, M_k has been found to be a function of z, B and L. Here z is the height, measured from the weld toe, and L is the overall length of the attachment measured from weld toe to weld toe, as shown in the figures above.

The M_{km} and M_{kb} stress intensity factor magnification factors, for membrane and bending loading, are required for the general stress intensity factor solutions in equations (A.1 to (A.6. The resulting relationships are given below.

M_k has been calculated by 2-D finite element analysis for profiles representing sections of the welded joint geometry. Thus, M_k is directly applicable to the case of a straight-fronted weld toe surface crack (ie $a/2c = 0$). However, experience indicates that it can also be applied to semi-elliptical cracks ($0 \leq a/2c \leq 0.5$) and other flaw types. The nature of the finite element model used to calculate M_k is such that the solutions produced are not applicable for $z = 0$, and near-surface M_k values should be used ($z = 0.15$ mm) for the intersection of surface flaws with the weld toe, and for through-thickness flaws at weld toes.

The solutions presented apply for 45º weld profiles (α=45º): M_k is slightly lower for lower angles and vice versa.

More accurate solutions based on 3D-stress analysis of semi-elliptical cracks at weld toes are available, and one such solution is presented in Section A.6.1.2.

A.6.1.1 Solutions based on 2D FEA

[A.41]

In general the following solutions apply:

$$M_k = v\left[\frac{z}{B}\right]^w \tag{A.79}$$

down to M_k = 1

where

v and *w* have the values given in Table A.25 for cracks at the toes of full penetration butt or attachment welds.

Table A.25 Values of *v* and *w* for axial and bending loading

Loading mode	*L*/*B*	*z*/*B*	*v*	*w*
Axial	≤ 2	$\le 0.05(L/B)^{0.55}$	$0.51(L/B)^{0.27}$	-0.31
		$>0.05(L/B)^{0.55}$	0.83	$-0.15(L/B)^{0.46}$
	>2	≤ 0.073	0.615	-0.31
		>0.073	0.83	-0.20
Bending	≤ 1	$\le 0.03(L/B)^{0.55}$	$0.45(L/B)^{0.21}$	-0.31
		$>0.03(L/B)^{0.55}$	0.68	$-0.19(L/B)^{0.21}$
	>1	≤ 0.03	0.45	-0.31
		>0.03	0.68	-0.19

A.6.1.2 Solutions based on 3D FEA

[A.46][A.47][A.48]

General

Alternative stress intensity magnification factor solutions (M_k) for the deepest and surface points of a semi-elliptical weld-toe crack are given in this section. The solutions were obtained by curve fitting to individual finite element analyses. They include the weld profile angle α as variables, but the following simplified solutions are valid for 45º weld profiles with sharp radii (less than 0.1*B*) and for the following parametric ranges:

$0.005 < a/B < 1.0$

$0.05 \le a/2c \le 0.5$

$0.5 \le L/B \le 2.75$

(for *L*/*B* > 2.75, use the value for *L*/*B* = 2.75)

Deepest point

a) *Axial*

$$M_{km} = f_1\left(\frac{a}{B},\frac{a}{c}\right) + f_2\left(\frac{a}{B}\right) + f_3\left(\frac{a}{B},\frac{L}{B}\right) \quad \text{(A.80)}$$

where:

$$f_1\left(\frac{a}{B},\frac{a}{c}\right) = 0.433\,58(a/B)^{\left[g_1+\{g_2(a/B)\}^{g_3}\right]} + 0.931\,63\exp\left\{(a/B)^{-0.050\,966}\right\} + g_4$$

where:

$$g_1 = -1.034\,3(a/c)^2 - 0.156\,57(a/c) + 1.340\,9\,;$$

$$g_2 = 1.321\,8(a/c)^{-0.611\,53}\,;$$

$$g_3 = -0.872\,38(a/c) + 1.278\,8\,;$$

$$g_4 = -0.461\,90(a/c)^3 + 0.670\,90(a/c)^2 - 0.375\,71(a/c) + 4.651\,1\,.$$

and where:

$$f_2\left(\frac{a}{B}\right) = -0.215\,21\{1-(a/B)\}^{176.419\,9} + 2.814\,1(a/B)^{-0.107\,40(a/B)}\,;$$

$$f_3\left(\frac{a}{B},\frac{L}{B}\right) = 0.339\,94(a/B)^{(g_5)} + 1.949\,3(a/B)^{0.230\,03} + \left\{g_6(a/B)^2 + g_7(a/B) + g_8\right\}\,;$$

where:

$$g_5 = -0.015\,647(L/B)^3 + 0.090\,889(L/B)^2 - 0.171\,80(L/B) - 0.245\,87\,;$$

$$g_6 = -0.201\,36(L/B)^2 + 0.933\,11(L/B) - 0.414\,96\,;$$

$$g_7 = 0.201\,88(L/B)^2 - 0.978\,57(L/B) + 0.068\,225\,;$$

$$g_8 = -0.027\,338(L/B)^2 + 0.125\,51(L/B) - 11.218\,.$$

NOTE If equation (A.80 gives a value of $M_k < 1.0$, assume that $M_k = 1.0$.

b) *Bending*

If 0.005 ≤ a/B ≤ 0.5, then the following expression applies:

$$M_{\mathrm{kb}} = f_1\left(\frac{a}{B},\frac{a}{c}\right) + f_2\left(\frac{a}{B}\right) + f_3\left(\frac{a}{B},\frac{L}{B}\right) \quad \text{(A.81)}$$

where:

$$f_1\left(\frac{a}{B},\frac{a}{c}\right) = 0.065\,916(a/B)^{\left[g_1+\left\{g_2(a/B)\right\}^{g_3}\right]} + 0.520\,86\exp\left\{(a/B)^{-0.103\,64}\right\} + g_4\,;$$

Where:

$$g_1 = -0.014\,992(a/c)^2 - 0.021\,401(a/c) - 0.238\,51\,;$$

$$g_2 = 0.617\,75(a/c)^{-1.027\,8}\,;$$

$$g_3 = 0.000\,132\,42(a/c) - 1.474\,4\,;$$

$$g_4 = -0.287\,83(a/c)^3 + 0.587\,06(a/c)^2 - 0.371\,98(a/c) - 0.898\,87$$

and where:

$$f_2\left(\frac{a}{B}\right) = -0.219\,950(1 - a/B)^{2.808\,6} + 0.021\,403(a/B)^{g_5}\,;$$

where:

$$g_5 = -17.195(a/B)^2 + 12.468(a/B) - 0.516\,62\,;$$

and where:

$$f_3\left(\frac{a}{B},\frac{L}{B}\right) = 0.233\,44(a/B)^{(g_6)} - 0.148\,27(a/B)^{-0.200\,77} + \left\{g_7(a/B)^2 + g_8(a/B) + g_9\right\};$$

where

$$g_6 = -0.059\,798(L/B)^3 + 0.380\,91(L/B)^2 - 0.802\,20(L/B) + 0.319\,06\,;$$

$$g_7 = -0.358\,48(L/B)^2 + 1.397\,5(L/B) - 1.753\,5\,;$$

$$g_8 = 0.312\,88(L/B)^2 - 1.359\,9(L/B) + 1.661\,1\,;$$

$$g_9 = -0.001\,470\,1(L/B)^2 - 0.002\,507\,4(L/B) - 0.008\,984\,6\,.$$

NOTE If equation (A.81 gives a value of $M_k < 1.0$, assume that $M_k = 1.0$.

Surface point

a) *Axial*

$$M_{km} = f_1\left(\frac{a}{B},\frac{c}{a},\frac{L}{B}\right) \times f_2\left(\frac{a}{T},\frac{a}{c}\right) \times f_3\left(\frac{a}{B},\frac{a}{c},\frac{L}{B}\right) \quad \text{(A.82)}$$

where:

$$f_1\left(\frac{a}{B},\frac{c}{a},\frac{L}{B}\right) = g_1\,(a/B)^{\left\{g_2\left(\frac{c}{a}\right)^2 + g_3\left(\frac{c}{a}\right) + g_4\right\}} + g_5\,\{1-(a/B)\}^{\left\{g_6\left(\frac{c}{a}\right)^2 + g_7\left(\frac{c}{a}\right) + g_8\right\}}$$

where:

$$g_1 = 0.007\,815\,7(c/a)^2 - 0.070\,664(c/a) + 1.850\,8\,;$$

$$g_2 = -0.000\,054\,546(L/B)^2 + 0.000\,136\,51(L/B) - 0.000\,478\,44\,;$$

$$g_3 = 0.000\,491\,92(L/B)^2 - 0.001\,359\,5(L/B) + 0.011\,400\,;$$

$$g_4 = 0.007\,165\,4(L/B)^2 - 0.033\,399(L/B) - 0.250\,64\,;$$

$$g_5 = -0.018\,640(c/a)^2 + 0.243\,11(c/a) - 1.764\,4\,;$$

$$g_6 = -0.00\,167\,13(L/B)^2 + 0.009\,062\,0(L/B) - 0.016\,479\,;$$

$$g_7 = -0.003\,161\,5(L/B)^2 - 0.010\,944(L/B) + 0.139\,67\,;$$

$$g_8 = -0.045\,206(L/B)^3 + 0.323\,80(L/B)^2 - 0.689\,35(L/B) + 1.495\,4\,.$$

and where:

$$f_2\left(\frac{a}{B},\frac{a}{c}\right) = \left\{-0.286\,39(a/c)^2 + 0.354\,11(a/c) + 1.643\,0\right\}(a/B)^{g_9} + 0.274\,49\{1-(a/B)\}^{g_{10}}\,;$$

where:

$$g_9 = -0.254\,73(a/c)^2 + 0.409\,28(a/c) + 0.002\,189\,2\,;$$

$$g_{10} = 37.423(a/c)^2 - 15.741(a/c) + 64.903 .$$

and where:

$$f_3\left(\frac{a}{B},\frac{a}{c},\frac{L}{B}\right) = g_{11}(a/B)^{0.754\,29} + g_{12}\exp\left\{(a/B)^{g_{13}}\right\};$$

where:

$$g_{11} = -0.105\,53(L/B)^3 + 0.598\,94(L/B)^2 - 1.094\,2(L/B) - 1.265\,0 ;$$

$$g_{12} = 0.043\,891(L/B)^3 - 0.248\,98(L/B)^2 + 0.447\,32(L/B) + 0.601\,36 ;$$

$$g_{13} = -0.011\,411(a/c)^2 + 0.004\,369(a/c) + 0.517\,32 .$$

b) Bending

$$M_{\mathrm{kb}} = f_1\left(\frac{a}{B},\frac{c}{a},\frac{L}{B}\right) \times f_2\left(\frac{a}{B},\frac{a}{c}\right) \times f_3\left(\frac{a}{B},\frac{a}{c},\frac{L}{B}\right) \qquad \text{(A.83)}$$

where:

$$f_1\left(\frac{a}{B},\frac{c}{a},\frac{L}{B}\right) = g_1(a/B)^{\left\{g_2\left(\frac{c}{a}\right)^2 + g_3\left(\frac{c}{a}\right) + g_4\right\}} + g_5\left\{1-(a/B)\right\}^{\left\{g_6\left(\frac{c}{a}\right)^2 + g_7\left(\frac{c}{a}\right) + g_8\right\}} + g_9 ;$$

where:

$$g_1 = 0.002\,323\,2(c/a)^2 - 0.000\,371\,56(c/a) + 4.598\,5 ;$$

$$g_2 = -0.000\,044\,010(L/B)^2 + 0.000\,144\,25(L/B) - 0.000\,867\,06 ;$$

$$g_3 = 0.000\,399\,51(L/B)^2 - 0.001\,371\,5(L/B) + 0.014\,251 ;$$

$$g_4 = 0.004\,616\,9(L/B)^2 - 0.017\,917(L/B) - 0.163\,35 ;$$

$$g_5 = -0.018\,524(c/a)^2 + 0.278\,10(c/a) - 5.425\,3 ;$$

$$g_6 = -0.000\,379\,81(L/B)^2 + 0.002\,507\,8(L/B) + 0.000\,146\,93 ;$$

$$g_7 = -0.003\,850\,8(L/B)^2 + 0.002\,321\,2(L/B) - 0.026\,862 ;$$

$$g_8 = -0.011\,911(L/B)^3 + 0.082\,625(L/B)^2 - 0.160\,86(L/B) + 1.230\,2 ;$$

$$g_9 = 0.277\,98(a/B)^3 - 1.214\,4(a/B)^2 - 2.468\,0(a/B) + 0.099\,981 .$$

and where:

$$f_2\left(\frac{a}{B},\frac{a}{c}\right)=\left\{-0.350\,06\left(a/c\right)^2+0.407\,68\left(a/c\right)+1.705\,3\right\}\left(a/B\right)^{g_{10}}+0.249\,88\left\{1-\left(a/B\right)\right\}^{g_{11}}$$

where:

$$g_{10}=-0.259\,22\left(a/c\right)^2+0.395\,66\left(a/c\right)+0.011\,759\,;$$

$$g_{11}=6.597\,4\left(a/c\right)^2+55.787\left(a/c\right)+37.053\,.$$

and where:

$$f_3\left(\frac{a}{B},\frac{a}{c},\frac{L}{B}\right)=g_{12}\left(a/B\right)^{0.947\,61}+g_{13}\exp\left\{\left(a/B\right)^{g_{14}}\right\};$$

where:

$$g_{12}=-0.148\,95\left(L/B\right)^3+0.815\,26\left(L/B\right)^2-1.479\,5\left(L/B\right)-0.898\,08\,;$$

$$g_{13}=0.055\,459\left(L/B\right)^3-0.301\,80\left(L/B\right)^2+0.541\,54\left(L/B\right)+0.534\,33\,;$$

$$g_{14}=-0.013\,43\left(a/c\right)^2+0.006\,670\,2\left(a/c\right)+0.759\,39\,.$$

Validity limits:

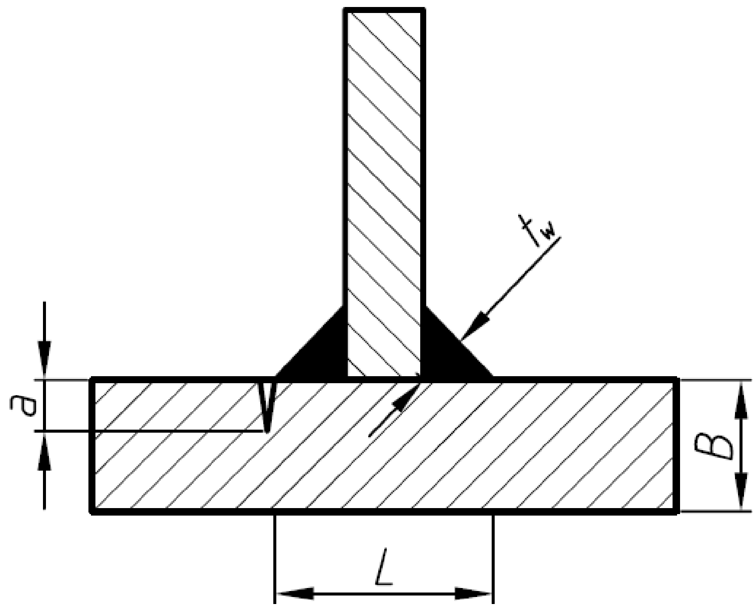
valid for 45º weld profiles with sharp radii (less than 0.1*B*) and for the following parametric ranges:

$0.005 < a/B \leq 1.0$
$0.05 < a/2c < 0.5$
$0.5 \leq L/B \leq 2.75$
(for $L/B > 2.75$, use the value for $L/B = 2.75$)

R6

No solution available.

A.6.2 Load Carrying Fillet or Partial Penetration Weld with Surface Crack at Weld Toe [A.49]



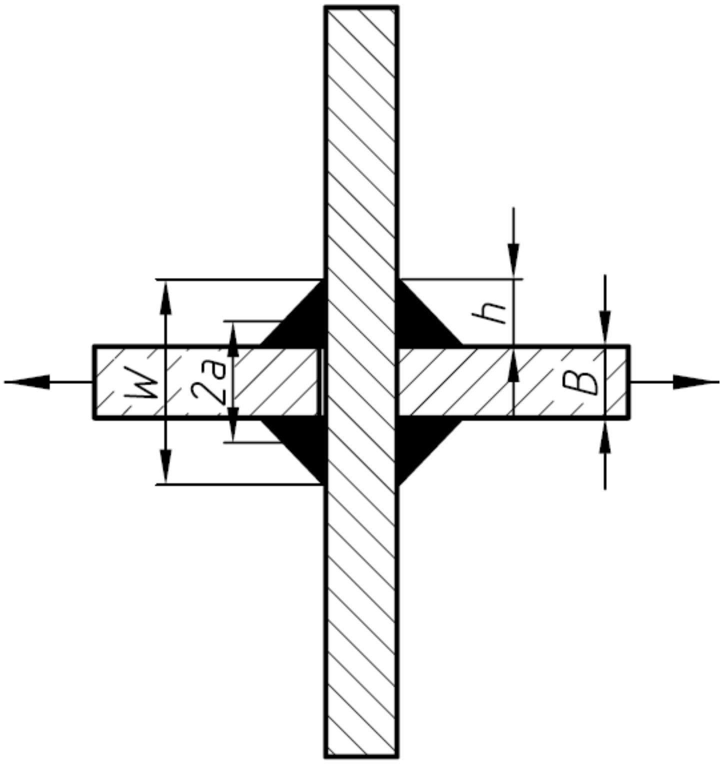
BS 7910 Solution

For cracks at the toes of fillet or partial penetration load-carrying welds, the 2-D approach described in Section A.6.1.1 may be used, where the values of *v* and *w* are those corresponding to L/B >2 for axial loading or $L/B \geq 1$ for bending. The resulting value of *v* is then multiplied by $(B/t_w)^{0.5}$.

R6

No solution available

A.6.3 Root Cracks in Cruciform Joints



BS 7910 Solution [A.49]

Note that this refers only to straight fronted cracks [$a/2c = 0$]. The stress intensity solution is calculated from equations (A.1 to (A.6 where:

$M = M_m = M_b = 1$;
σ is the stress in the loaded member.

The influence of joint geometry on stress intensity factors for root cracks in fillet and partial penetration welds is accounted for by the application of modified finite width correction and stress intensity factor magnification factors, f_{wm}, f_{wb}, M_{km} and M_{kb}, for membrane and bending loading.

Membrane loading

$$f_{wm} = \left(\sec\left[\frac{\pi}{2} \left(\frac{2a}{W} \right) \right] \right)^{0.5} \tag{A.84}$$

$$M_{km} = \lambda_o + \lambda_1 (2a/W) + \lambda_2 (2a/W)^2 \tag{A.85}$$

where:

$$\lambda_0 = 0.956 - 0.343(h/B)$$
$$\lambda_1 = -1.219 + 6.210(h/B) - 12.220(h/B)^2 + 9.704(h/B)^3 - 2.741(h/B)^4$$
$$\lambda_2 = 1.954 - 7.938(h/B) + 13.299(h/B)^2 - 9.541(h/B)^3 + 2.513(h/B)^4$$

Range of application: $0.1 \le 2a/W \le 0.7;$ $0.2 \le h/B \le 1.2.$

Bending loading

$$f_{wb} = \frac{\alpha}{2} \frac{(1-\alpha)^{0.5}}{1-\alpha^3}\left[1 + \frac{1}{2}\alpha + \frac{3}{8}\alpha^2 - \frac{11}{16}\alpha^3 + 0.464\alpha^4\right] \tag{A.86}$$

where:

$\alpha = 2a/W$

$$M_{kb} = \exp(\lambda_0)(2a/W)^{\lambda_1}(2a/W)^{\lambda_2 \ln(2a/W)} \tag{A.87}$$

for 0.2 ≤ h/B ≤ 0.7

$M_{kb} = \exp(\mu_0)(2a/W)^{\mu_1}$

for 0.7 ≤ h/B ≤ h/B ≤ 0.7

where:

λ_0 = 0.792 - 3.560(h/B) + 1.276$(h/B)^2$

λ_1 = 1.064 - 4.898(h/B) + 3.670$(h/B)^2$

λ_2 = 0.496 - 1.328(h/B) + 1.012$(h/B)^2$

μ_0 = 0.285$(h/B)^2$ – 1.866(h/B)

μ_1 = 0.028(h/B) – 0.761

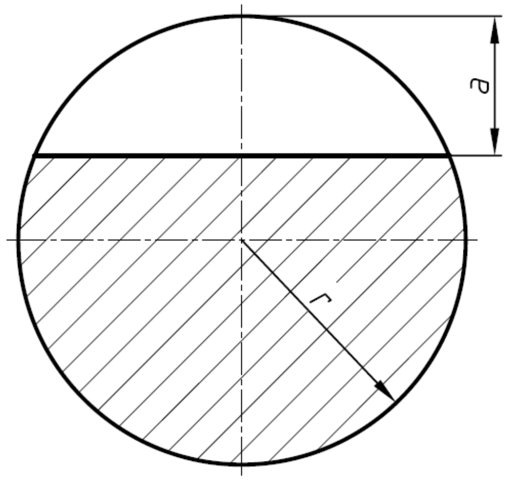
Range of application: 0.1 ≤ 2a/W ≤ 0.7.

R6

No solution available

A.7 Round Bars and Bolts

A.7.1 Straight-fronted Crack in a Bar



BS 7910 Solution [A.50]

The stress intensity factor is calculated from equations (A.1 to (A.6:

Where:

$M = M_{km} = M_{kb} = f_w = 1$, with M_m and M_b as follows:

$$M_m = 0.926 - 1.771\left(\frac{a}{2r}\right) - 26.421\left(\frac{1}{2r}\right)^2 - 78.481\left(\frac{1}{2r}\right)^3 + 87.911\left(\frac{a}{2r}\right)^4 \quad \text{(A.88)}$$

$$M_b = 1.04 - 3.64\left(\frac{a}{2r}\right) + 16.86\left(\frac{a}{2r}\right)^2 - 32.59\left(\frac{a}{2r}\right)^3 + 28.41\left(\frac{a}{2r}\right)^4 \quad \text{(A.89)}$$

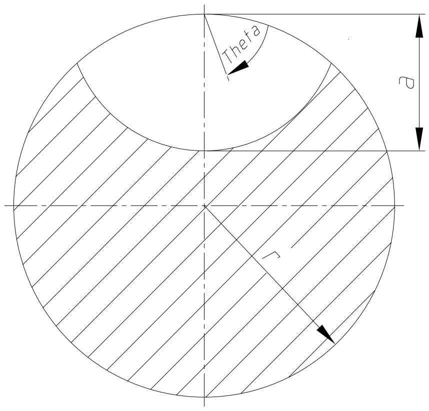
Validity limits:

Range of application: $0.062\,5 \leq a/2r \leq 0.625$.

R6

No solution available.

A.7.2 Semi-circular Surface Crack in a Bar



BS 7910 Solution [A.50]

The stress intensity factor solution is calculated from equations (A.1 to (A.6

Where:

$M = M_{km} = M_{kb} = f_w = 1$, with M_m and M_b as follows:

$$M_m = g\left[0.752 + 2.02\left(\frac{a}{2r}\right) + 0.37\left\{1 - \sin\left(\frac{\pi a}{4r}\right)\right\}^3\right] \tag{A.90}$$

$$M_b = g\left[0.923 + 0.199\left\{1 - \sin\left(\frac{\pi a}{4r}\right)\right\}^4\right] \tag{A.91}$$

where:

$$g = \frac{\frac{1.84}{\pi}\left\{\tan\left(\frac{\pi a}{4r}\right)\Big/\left(\frac{\pi a}{4r}\right)\right\}^{0.5}}{\cos\left(\frac{\pi a}{4r}\right)}$$

NOTE K solutions for straight fronted cracks in round bars (Section A.7.1) are generally conservative compared to semi-circular surface cracks.

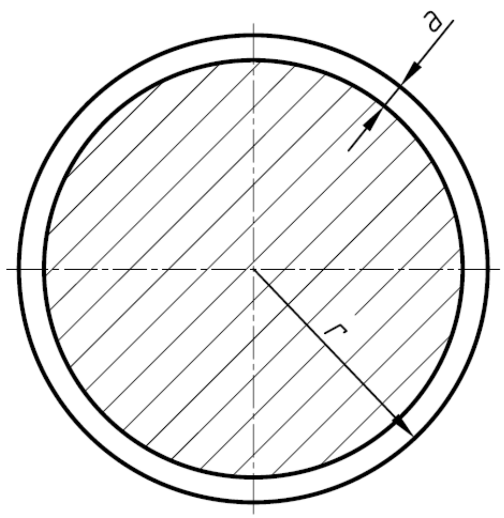
Validity limits:

Range of application: $a/2r < 0.6$.

R6

No solution available.

A.7.3 Fully Circumferential Crack in a Bar



BS 7910 Solution [A.2]

The stress intensity factor solution is calculated from equations (A.1 to (A.6

Where:

$M = M_{km} = M_{kb} = f_w = 1$, with M_m and M_b as follows:

$$M_m = \frac{r^{1.5}}{2(r-a)^{1.5}}\left\{1+0.5\left(\frac{r-a}{r}\right)+0.375\left(\frac{r-a}{r}\right)^2-0.363\left(\frac{r-a}{r}\right)^3+0.731\left(\frac{r-a}{r}\right)^4\right\} \quad \text{(A.92)}$$

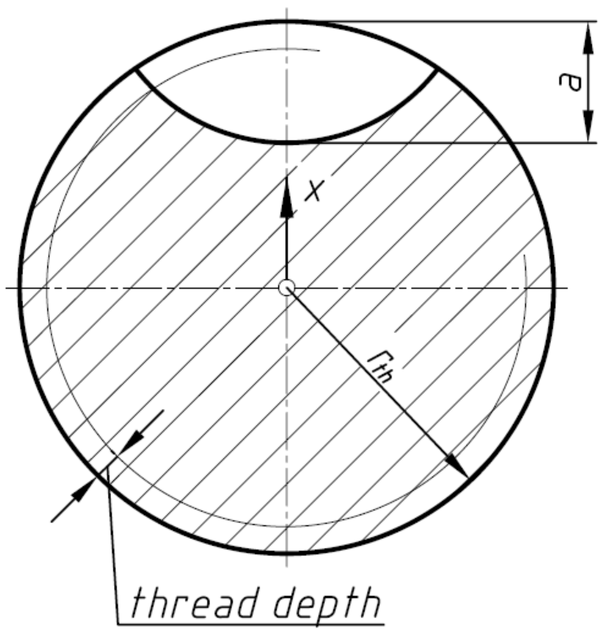
$$M_b = \frac{0.375r^{2.5}}{(r-a)^{2.5}}\left\{1+0.5\left(\frac{r-a}{r}\right)+0.375\left(\frac{r-a}{r}\right)^2-0.313\left(\frac{r-a}{r}\right)^3+0.273\left(\frac{r-a}{r}\right)^4+0.537\left(\frac{r-a}{r}\right)^5\right\} \quad \text{(A.93)}$$

Validity limits:

None given

R6
No solution available

A.7.4 Semi-circular Crack in a Bolt



BS 7910 Solution [A.50]

This solution has been developed for the ISO M8 × 1.0 bolt geometry. The stress intensity factor is calculated from equations (A.1 to (A.6

Where:

$M = M_{km} = M_{kb} = f_w = 1$, with M_m and M_b as follows:

$$M_m = M_b = \lambda_0 + \lambda_1 (a/2r) + \lambda_2 (a/2r)^2 \quad \text{(A.94)}$$

where the following apply for the conditions indicated:

a) Tension loading (M_m), at the deepest point in the crack:

λ_0 = 1.015 5 - 0.237 5(a/c);

λ_1 = -0.584 + 0.015(a/c);

λ_2 = 6.455 75 - 3.348 75(a/c).

Intersection of the crack with the free surface:

λ_0 = 0.469 5 + 0.822 5(a/c);

λ_1 = 0.377 75 - 1.478 75(a/c);

λ_2 = -0.160 25 + 2.946 25(a/c).

b) Bending loading (M_b), at the deepest point in the crack:

λ_0 = 0.893 75 - 0.363 75(*a*/*c*);

λ_1 = -0.559 25 + 0.366 25(*a*/*c*);

λ_2 = 2.379 - 1.88(*a*/*c*).

Intersection of the crack with the free surface:

λ_0 = 0.653 5 - 0.092(a/c);

λ_1 = -1.148 75 + 1.558 75(*a*/*c*);

λ_2 = 3.028 - 1.855(*a*/*c*).

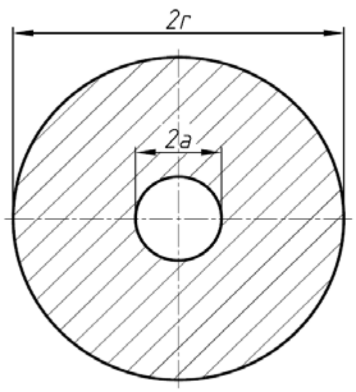
Validity limits:

$0.2 \leq a/c \leq 1$;
$0.1 \leq a/2r \leq 0.5$.

R6

No solution available.

A.7.5 Embedded Crack in a Bar



BS 7910, R6, API 579, SINTAP

No solution available.

FKM solution [A.51]

A solution for this geometry is given in the FKM procedure. The bar may be under tension (P denotes the tensile load), bending (M denotes bending moment) and torsion (T) as follows:

$$K_I = \left[F_p P + F_M \frac{4Ma}{\left(r^2 + a^2\right)} \right] \cdot \frac{\sqrt{\frac{(r-a)}{r}}}{\pi \left(r^2 - a^2\right)} \sqrt{\pi a} \qquad \text{(A.95)}$$

where:

$$F_p = \frac{2}{\pi}\left[1 + \frac{1}{2}\lambda - \frac{5}{8}\lambda^2\right] + 0.268\lambda^2$$

$$\lambda = \frac{a}{r}$$

$$F_M = \frac{4}{3\pi}\left[1 + \frac{1}{2}\lambda + \frac{3}{8}\lambda^2 + \frac{5}{16}\lambda^3 - \frac{93}{128}\lambda^4 + 0.483\lambda^5\right]$$

$$K_{III} = F_T \frac{2Ta\sqrt{\frac{(r-a)}{r}}}{\pi\left(r^4 - a^4\right)} \sqrt{\pi a} \qquad \text{(A.96)}$$

where:

$$F_r = \frac{4}{3\pi}\left[1 + \frac{1}{2}\lambda + \frac{3}{8}\lambda^2 + \frac{5}{16}\lambda^3 - \frac{93}{128}\lambda^4 + 0.038\lambda^5\right]$$

A.8 Tubular Joints

BS 7910 Solution

BS 7910 does not provide any specific SIF solutions. It cites papers [A.51][A.53][A.54][A.55] where SIFs were obtained using numerical methods.

R6, API 579

No solution available.

SINTAP [A.54] [Ree et al]

Section AIII.5 of SINTAP covers the following geometries and loading modes:

- Surface crack at the saddle point of T-joints
  - axial loading: deepest and surface points
  - in-plane bending: deepest and surface points
  - out-of-plane bending: deepest and surface points

- Surface crack at the saddle point of Y-joints,
  - axial loading: deepest point

A.9 Bibliography

[A.1] Murakami Y: 'Stress intensity factors handbook', Pergamon Press, Oxford, 1987, pp.1356-1358.

[A.2] Tada H, Paris P C and Irwin G: 'The stress analysis of cracks handbook', Third Edition, ASME, New York, 2000.

[A.3] Rooke D P and Cartwright D J: 'Compendium of stress intensity factors', HM Stationery Office, London, 1976.

[A.4] Sih G C, Paris P C and Erdogan F: 'Stress intensity factors for plane extension and plate bending problems', J Appl Mech 29, 1962, pp 306-312.

[A.5] BS 7910, 'Guide on methods for assessing the acceptability of flaws in metallic structures', BSI, 2005.

[A.6] R6: 'Assessment of the integrity of structures containing defects', Rev 4, British Energy, 2001.

[A.7] API Recommended Practice 579, Fitness-for-Service, API Publishing Services, First edition January 2000.

[A.8] SINTAP (Structural Integrity Procedures for European Industry), Final Procedure, November 1999.

[A.9] FKM, 'Bruchmechanischer Festigkeitsnachweis', 2004, also available in English as 'Fracture mechanics proof of strength for engineering components', 2004.

[A.10] INSTA Technical Report, 1991: 'Assessment of structures containing discontinuities', Materials Standards Institution, Stockholm.

[A.11] RSE-M: Feb 2000: Regles de surveillance en exploitation des materials mecaniques des ilots nucleaires R.E.P.

[A.12] IIW: 'IIW guidance on assessment of the fitness for purpose of welded structures', Draft for Development, 1990, IIW/IIS-SST-1157-90

[A.13] Smith, M C: 'A comparison of R6 Code SIF solutions for defects in cylinder with other solutions', British Energy Generation Report E/REP/ATEC/0006/GEN/01, 2001.

[A.14] Fett T and Munz D: 'Stress intensity factors and weight functions', Computational Mechanics Publications, Southampton, 1997.

[A.15] Newman J C and Raju I S (1984): 'Stress intensity factor equation for cracks in three-dimensional finite bodies subjected to tension and bending loads', Report NASA-TM-85793, National Aeronautics and Space Administration, Langley Research Centre, Hampton, VA. 1984.

[A.16] Sih G C: 'Mechanics of Fracture 3, Plates and shells with cracks', Noordhoff International Publishing Leiden, The Netherlands, 1977.

[A.17] Raju I S and Newman JC (1979): 'Stress intensity factors for a wide range of semi-elliptical surface cracks in finite thickness plates', Engineering Fracture Mechanics, **11** (4) pp 817-829. ISSN 0013-7944. 1979.

[A.18] T Fett, D Munz and J Neumann, Local stress intensity factors for surface cracks in plates under power-shaped stress distributions, Engng Fract Mech **36**, 647-651 (1990).

[A.19] Wu X R and Carlsson A J: 'Weight functions and stress intensity factor solutions', Pergamon Press, Oxford, 1991.

[A.20] Zvezdin Y I: 'Handbook - Stress intensity and reduction factors calculation', Central Research Institute for Technology of Machinery Report MR 125-01-90, Moscow, 1990.

[A.21] Fuhrey M and Osage D A: 'Stress intensity factor solutions for long surface cracks in flat plates, cylinders and spheres'. To be published.

[A.22] Merkblatt DVS 2401: 'Bruchmechanische Bewertung von Fehlern in Schweissverbindungen', Deutscher Verband fűr Schweisstechnik, 1982.

[A.23] Chell G C: 'The stress intensity factors and crack profiles for centre and edge cracks in plates subject to arbitrary stresses', Int J Fracture, **12**, 1976, pp33-46.

[A.24] Newman J C and Raju I S (1983): 'Stress intensity factor equation for cracks in three-dimensional finite bodies', ASTM STP 791, 1983, ppI238 – I265.

[A.25] Zahoor A: 'Closed form expressions for fracture mechanics analysis of cracked pipes', J Pressure Vessel Technology, **107**, N2, 1985, pp203-205.

[A.26] Chell G C: 'ADISC: A computer program for assessing defects in spheres and cylinders', CEGB Report TPRD/L/MT0237/M84, 1984.

[A.27] Delale F and Ergodan F: 'Effect of transverse shear and material orthotropy in a cracked spherical cap', Int J Solids Structures, 15, 1979, pp907-926.

[A.28] France C C, Green D, Sharples J K and Chivers T C: 'New stress intensity factors and crack opening area solutions for through-wall cracks in pipes and cylinders', ASME PVP Conference Fatigue and Fracture 1997, PVP-Vol. 350, pp. 143-193.

[A.29] Newman J C and Raju I S (1981): 'An empirical stress intensity factor equation for the surface crack', Engineering Fracture Mechanics, **15**, 1981, pp185-192.

[A.30] Raju I S and Newman J C (1982): 'Stress intensity factor influence coefficients for internal and external surface crack in cylindrical vessels', In: Palusamy S S and Sampath S G, eds., 'Aspects of fracture mechanics in pressure vessels and piping'. Presented at pressure vessels and piping conference, Orlando, FL, 27 June – 2 July, 1982. New York: ASME. ASME PVP Vol.-95, 35-48.

[A.31] Raju I S and Newman J C (1978): 'Stress intensity factor influence coefficients for internal and external surface cracks in cylindrical vessels, ASME PVP, 58, 1978, pp37-48.

[A.32] Joseph P F and Erdogan F: 'Surface crack problem in plates', Int J Fracture, **41** (2), 1989, pp.105-131.

[A.33] Andrasic C P and Parker A P: 'Dimensionless stress intensity factors for cracked thick cylinders under polynomial crack face loadings', Engineering Fracture Mechanics, **19** (1), 1984, pp.187-193.

[A.34] Al Laham S: 'Stress intensity factor and limit load handbook', British Energy Generation Report EPD/GEN/REP/0316/98, Issue 2, 1999.

[A.35] D Green and J Knowles, The treatment of residual stress in fracture assessment of pressure vessels, ASME J Pres Ves Tech, 116, 345-352 (1994).

[A.36] Dedhia D D and Harris D O: 'Improved influence functions for part-circumferential cracks in pipes', In: G.M. Wilkowsky, ed. Circumferential cracks in pressure vessels and piping. Presented at the 4th National Congress on Pressure Vessel and Piping Technology, Portland, Oregon, 19-24 June 1983, Vol.2, ASME PVP Vol. 95, New-York, 1984, pp35-48.

[A.37] Bergman M and Brickstad B: 'Stress-intensity factors for circumferential cracks in pipes by line-spring calculations', Swedish Plant Inspectorate, SA/FoU Report 90/05, Stockholm, 1990.

[A.38] M. Bergman, Stress intensity factors for circumferential surface cracks in pipes, Fatigue Fract Engng Mater Struct 18, 1155-1172 (1995).

[A.39] Toribio J: 'Effect of crack shape and loading conditions on the stress intensity factor for a cracked bolt', In: Barbas et al., eds. Proc. 11th international conference on offshore mechanics and arctic engineering, (OMAE'92), Calgary, Canada 7-12 June 1992, Vol. III-B, Materials Engineering, New York, ASME, 1992, pp363-370.

[A.40] Chapuliot S: 'Formulaire de K1 pour les tubes comportant un defaut de surface semi-elliptique longitudinal, ou circomferentiel, interne ou externe', CEA-R-5900, CEA Saclay, Gif-sur-Yvette, France, March 2000.

[A.41] Maddox S J. and Andrews R M, 1990: 'Stress intensity factors for weld toe cracks'. *In:* M.H. ALIABADI, C.A. BREBBIA and D.J. CARTWRIGHT, eds., *Localized damage computer-aided assessment and control. Proc. 1st international conference on computer-aided assessment and control of localized damage,* Portsmouth, UK, 26-28 June 1990. Vol.2. Southampton: Computational Mechanics Pubs. 329-342. ISBN 1853120685.

[A.42] Kiefner J F, Maxey W A, Eiber R J and Duffy A R: 'Failure stress levels of flaws in pressurized cylinders', Proc Conf, Philadelphia, 28-30 Aug 1972, ASTM STP 536, pp461-481.

[A.43] Willoughby A A and Davey T G: 'Plastic collapse at part wall flaws in plates', In: R.P. WEI, ed. Fracture mechanics: perspectives and directions. Proc. 20th national symposium, Bethlehem, PA, 23-25 June 1981, ASTM STP 1020, pp390-409.

[A.44] France C C, Green D and Sharples J K: 'New stress intensity factor and crack opening area solutions for through wall cracks in pipes and cylinders', AEAT-0643, 1996.

[A.45] Zang W: 'Stress intensity factor solutions for axial and circumferential through-wall cracks in cylinders', SAQ Report SINTAP/SAQ/02, 1997.

[A.46] Bowness D and M M M Lee: 'Prediction of weld toe magnification factors for semi-elliptical cracks in T-butt joints', Int J of Fatigue, 22 (5), 2000, pp. 389-396.

[A.47] Thurlbeck S D: 'A fracture mechanics based methodology for the assessment of weld toe cracks in tubular offshore joints', PhD Thesis, Department of Civil and Construction Engineering, University of Manchester Institute of Science and Technology, UK, 1991.

[A.48] Cheaitani M J, Thurlbeck S D and Burdekin F M: 'Fatigue, fracture and plastic collapse of offshore tubular joints using BSI PD 6493:1991. In: Salama et al, Eds. Proc 14th International Conference on Offshore Mechanics and Arctic Engineering, (OMAE 1995), Copenhagen, Denmark 18-22 June 1995, Vol. 3, New York, ASME, 1995, pp41-54.

[A.49] Noblett J E and Andrews R A: 'A stress intensity factor solution for root defects in fillet and partial penetration welds', TWI Research Report 575/1996, TWI, Abington, Cambridge, UK, 1996.

[A.50] James L A and Mills W J: 'Review and synthesis of stress intensity factor solutions applicable to cracks in bolts', Engineering Fracture Mechanics, **30** (5), 1988, pp641-653.

[A.51] Bentham J B and Koiter W T: 'Mechanics of fracture 1. Method of analysis and solutions of crack problems', Ed Sih G C, Noordhoff International Publishing, Leyden, 1973, pp131-178.

[A.52] Haswell J V: 'A general fracture mechanics model for a cracked tubular joint derived from the results of a finite element parametric study', Proc 11th Int Conf Conf Offshore Mechanics and Arctic Engineering Conference (OMAE '92), Calgary 7-12th June 1992, Vol III-B, ASME, New York, 1992, pp267-274.

[A.53] Ree H C, Han S and Gibson G S: 'Reliability of solution method and empirical formulas of stress intensity factors for weld toe cracks of tubular joints', Proc 10th Int Conf Offshore Mechanics and Arctic Engineering Conference (OMAE '91), Vol III-B, ASME, New York, 1991, pp441-452.

[A.54] Ho C M and Zwerneman F J: ‘Assessment of simplified methods’, Joint Industry Project Fracture Mechanics Investigation of Tubular Joints – Phase Two, Oklahoma State University, Stillwater, OK, Jan 1995.

[A.55] Burdekin F M and Cowling M J: ‘Defect assessment in offshore structures, a procedure, CMPT Publications, Aberdeen, 1998.

[A.56] R-Code: ‘Software for assessing the integrity of structures containing defects’, Version 4.1, British Energy Generation Ltd, 2001.