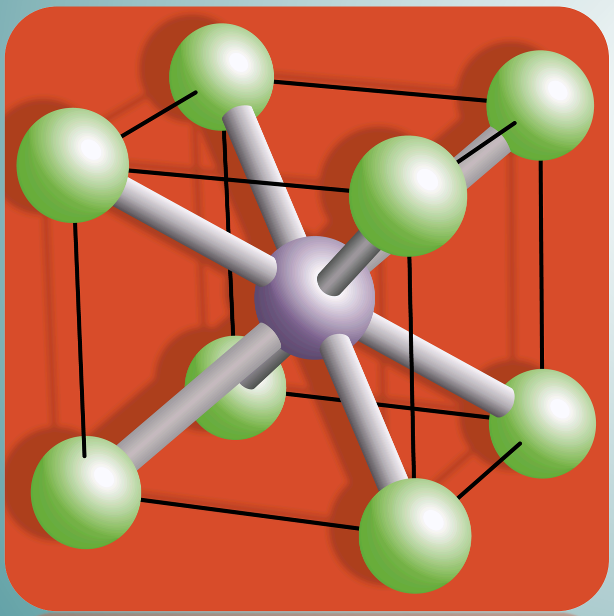
Materials

Topic 2. Hooke's Law



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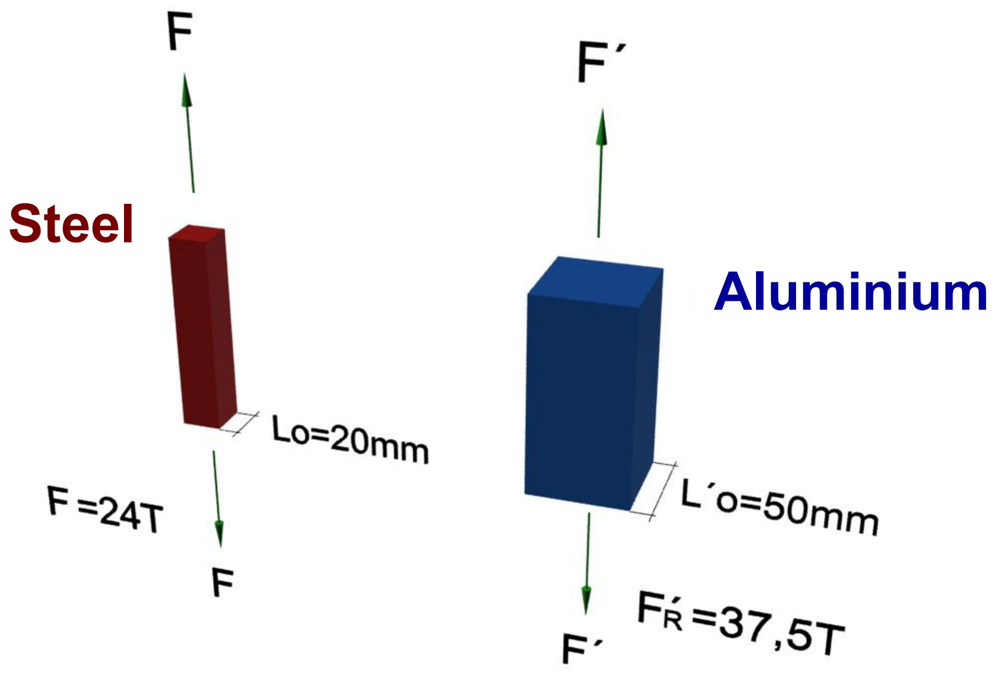
2.1. TENSION

- **Comparison of the tensile strength of two different materials:**



Which of the two materials is more resistant?

- Engineering stress (s): relationship between acting force and the surface of the initial section on which it acts.

$$\sigma = \frac{F}{A_0}$$

F: represents the load applied (N).

A_0: represents the initial cross section (m^2).

Dimensional stress formula: $F\,L^{-2}$ → International, or Metric, System of measurements

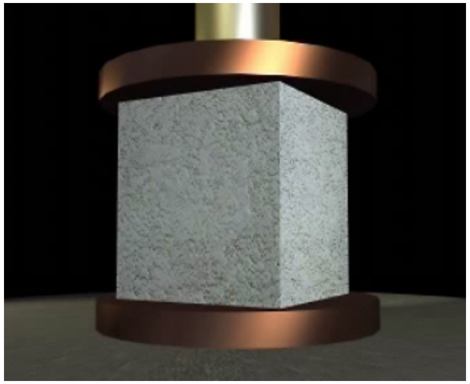
$N/m^2 = Pa$

(usual multiple: 1 MPa = 10^6 Pa = 1 N/mm^2).

- Type of effort:

Static

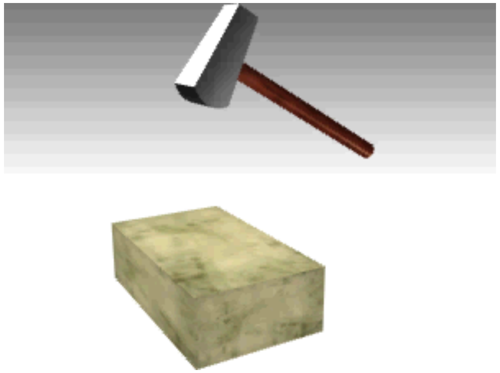
Constant or slowly changes



Dynamic

Impact

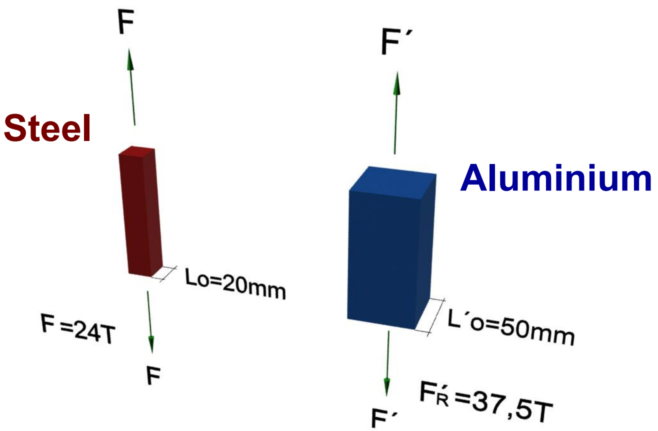
Shock between two bodies



Cyclic

fluctuates between two limits

- **In the proposed example:**



STEEL

$A_0 = L_0 \times L_0 = 20\ mm \times 20\ mm = 400\ mm^2$

$F_R = 24\ T = 24.000\ kg = 240.000\ N$

$$\sigma_R = \frac{F_R}{A_0} = \frac{240.000\ N}{400\ mm^2} = 600\ MPa$$

ALUMINIUM

$A'_0 = L'_0 \times L'_0 = 50\ mm \times 50\ mm = 2500\ mm^2$

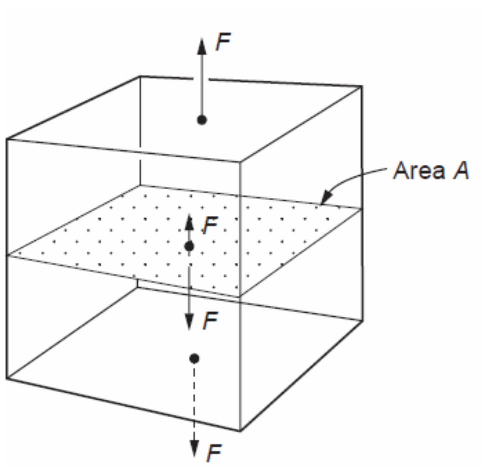
$F'_R = 37.5\ T = 37.500\ kg = 375.000\ N$

$$\sigma'_R = \frac{F'_R}{A'_0} = \frac{375.000\ N}{2.500\ mm^2} = 150\ MPa$$

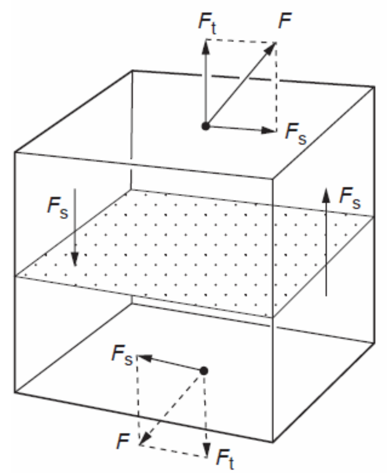
$$\sigma_R > \sigma'_R$$

STEEL IS MORE RESISTANT THAN ALUMINUM

- **Common states of stress**:
  - **Tensile stress, σ** — **Shear stress, τ**



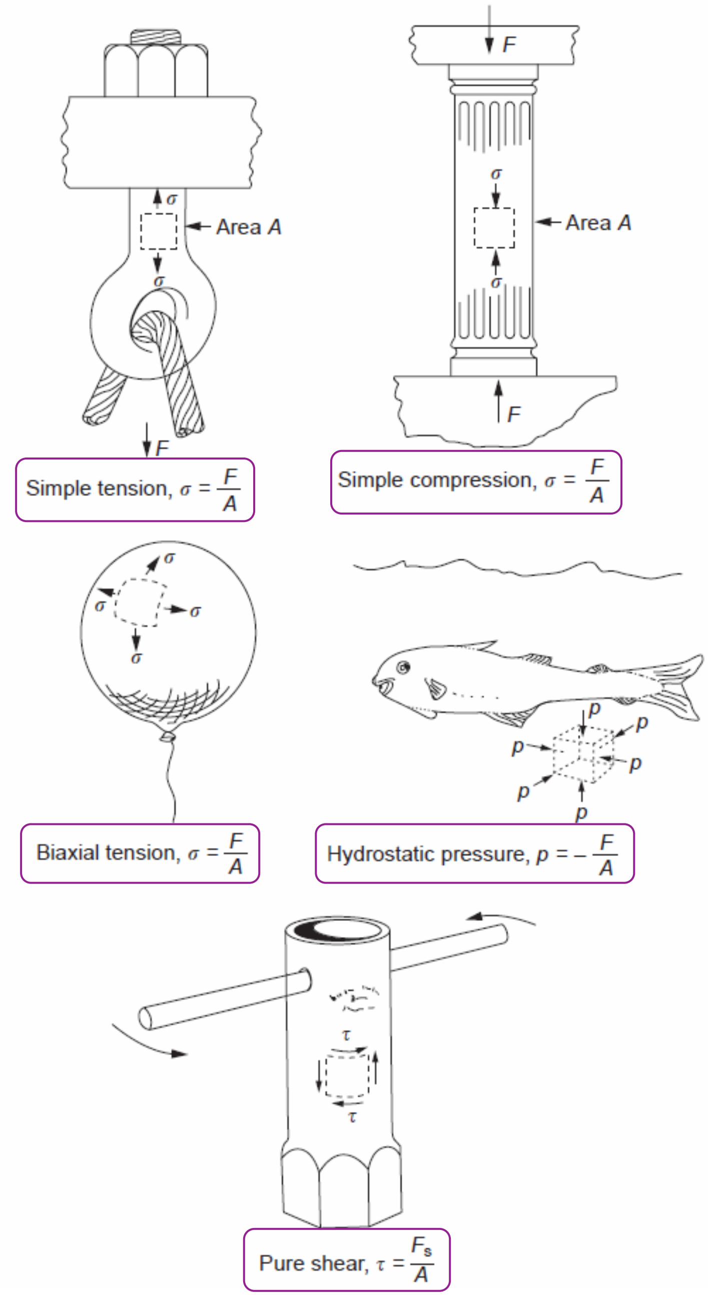
Tensile stress $\sigma = \frac{F}{A}$



Shear stress $\tau = \frac{F_s}{A}$

Tensile stress $\sigma = \frac{F_t}{A}$

Balancing shear required for equilibrium as shown



2.2. STRAIN

- Change of shape or dimensions produced by the action of efforts.

Engineering strain (ε): is defined as:

$$\varepsilon = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

(dimensionless)

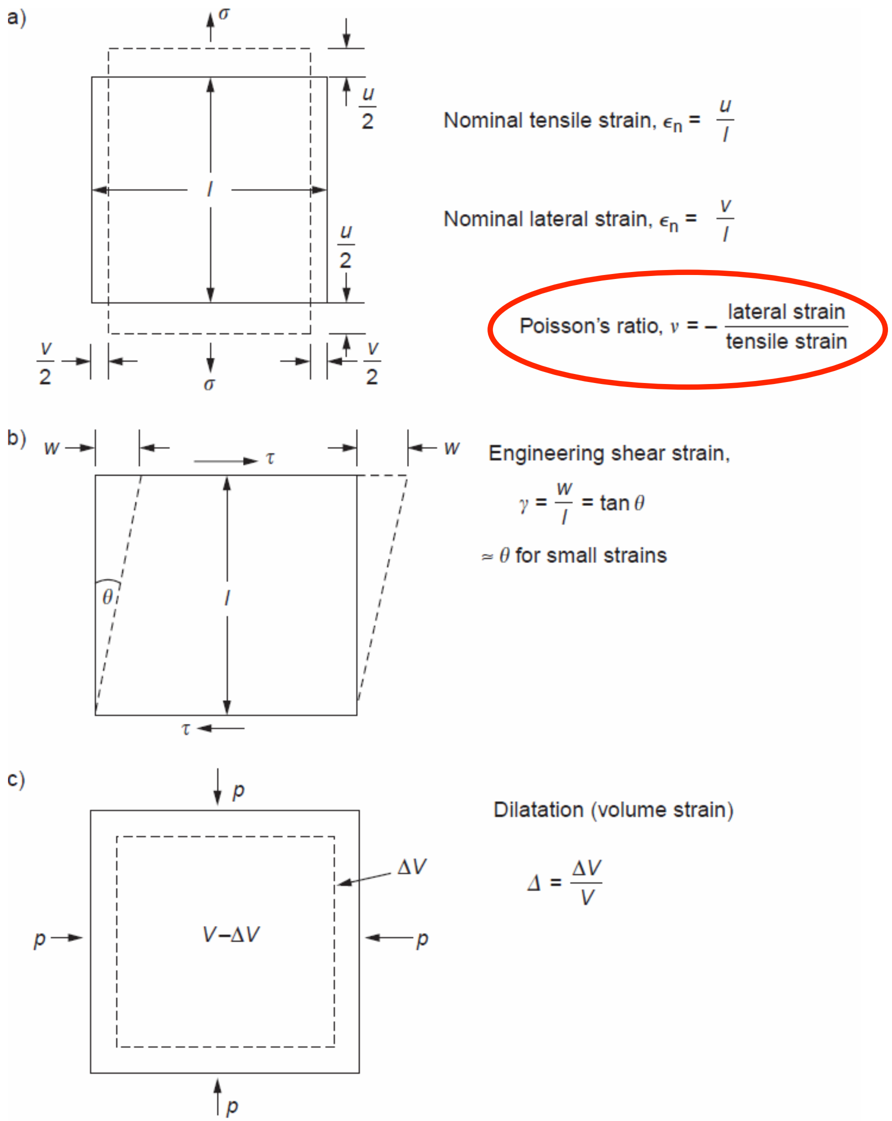
Where l is the reference length corresponding to a given load and l_0 is the initial reference length (gauge) corresponding to a zero stress value.

The reference length under a given load is:

$$l = l_0 + \Delta l$$

Where Δl represents the elongation corresponding to that load.

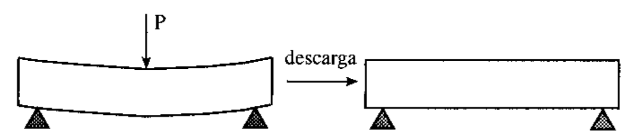
- **Tensile strain €, shear strain (γ) and dilatation (Δ).**



Elastic strain

It is a non-permanent deformation, which recovers completely when removing the load.

Elasticity is the property that solid bodies present to recover shape and dimensions when efforts are eliminated.



Plastic strain

It is a permanent deformation, which is not recovered by removing the load. However it is possible recover a small component of elastic deformation.

2.3. RELATIONSHIP BETWEEN STRESS AND STRAIN: HOOKE´s LAW

- **For small elastic deformations (~ 0.1%), there is a direct proportionality between the applied stress and the strain produced.**

$$\sigma = E \cdot \varepsilon$$

HOOKE´s Law

- ***E* represents the YOUNG´s modulus, parameter that measures the resistance of a material to elastic deformation.**

International, or Metric, System of measurements: N/m^2 = Pa, (usual multiple: 1 GPa = 10^3 MPa = 10^9 Pa).

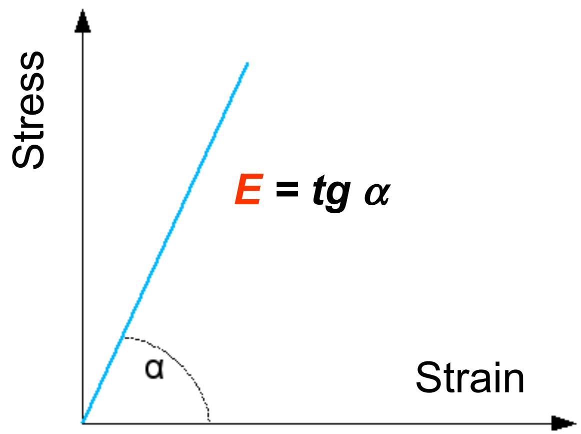
For other stress states:

$\tau = G \cdot \gamma$ **G (Shear modulus).**

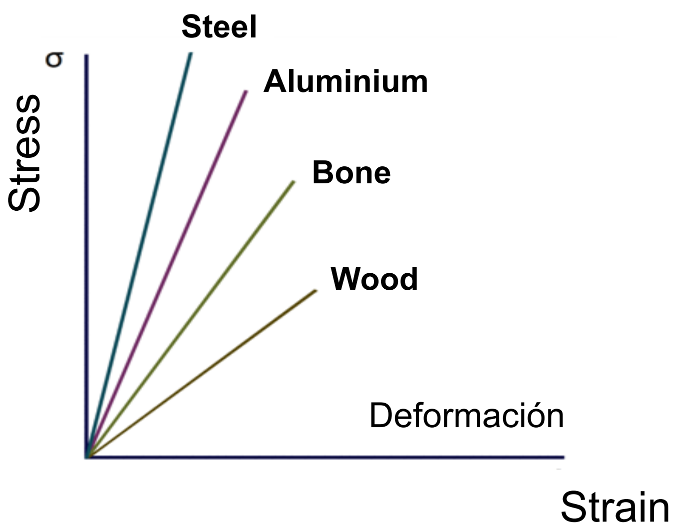
$p = -K \cdot \Delta$ **K (Bulk modulus).**

Low modulus of elasticity	**High modulus of elasticity**
FLEXIBILITY	**RIGIDITY**

- **HOOKE´s Law represents the equation of a line of slope E that passes through the origin of coordinates.**



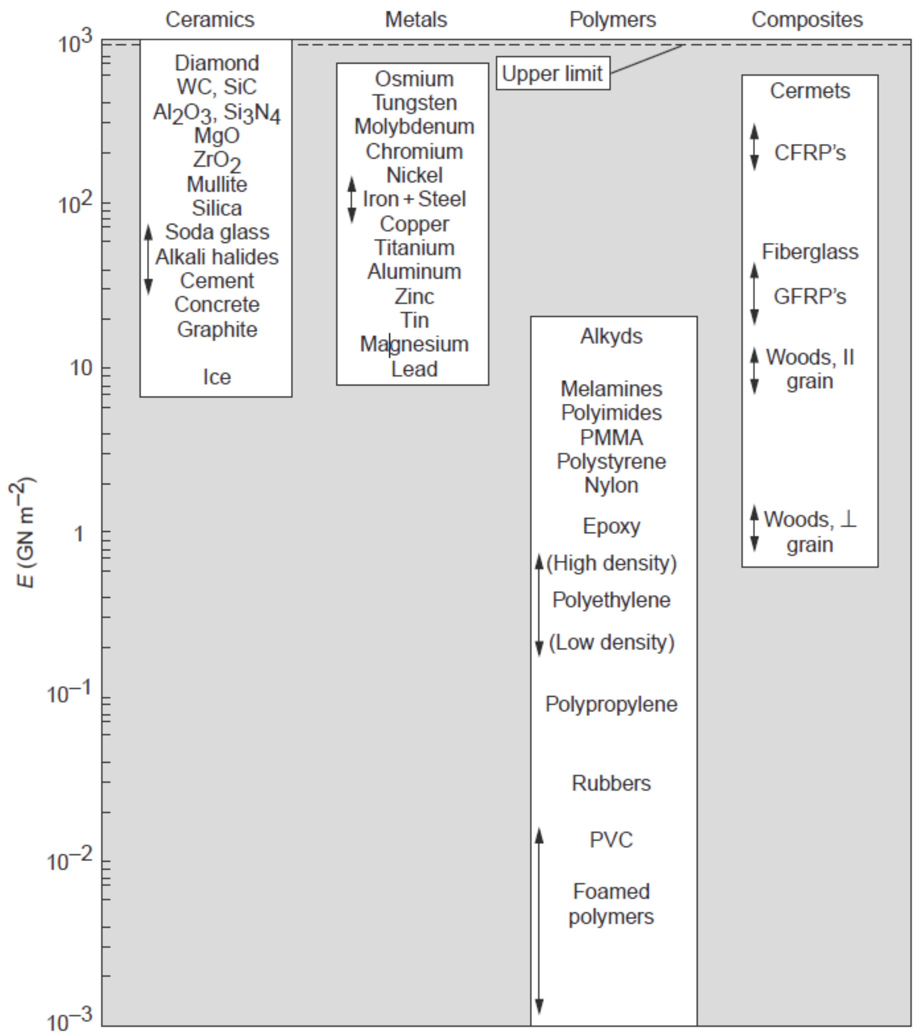
The loading and unloading stages coincide.



$E_{steel} > E_{Aluminium} > E_{bone} > E_{wood}$

- **The parameter yield strength σ_Y of a material represents the maximum stress it supports without suffering permanent (plastic) deformations.**

2.4. DATA FOR YOUNG´S MODULUS, E



Bar chart of data for Young's modulus, E.

2.4. DATA FOR YOUNG´S MODULUS, E

Material	*E* (GN m^{-2})
Diamond	1000
Tungsten carbide, WC	450–650
Osmium	551
Cobalt/tungsten carbide cermets	400–530
Borides of Ti, Zr, Hf	450–500
Silicon carbide, SiC	430–445
Boron	441
Tungsten and alloys	380–411
Alumina, Al_2O_3	385–392
Beryllia, BeO	375–385
Titanium carbide, TiC	370–380
Tantalum carbide, TaC	360–375
Molybdenum and alloys	320–365
Niobium carbide, NbC	320–340
Silicon nitride, Si_3N_4	280–310
Beryllium and alloys	290–318
Chromium	285–290
Magnesia, MgO	240–275
Cobalt and alloys	200–248
Zirconia, ZrO_2	160–241
Nickel	214
Nickel alloys	130–234
CFRP	70–200
Iron	196
Iron-based super-alloys	193–214
Ferritic steels, low-alloy steels	196–207
Stainless austenitic steels	190–200
Mild steel	200
Cast irons	170–190
Tantalum and alloys	150–186
Platinum	172
Uranium	172
Boron/epoxy composites	80–160
Copper	124
Copper alloys	120–150
Mullite	145
Vanadium	130
Titanium	116
Titanium alloys	80–130
Palladium	124
Brasses and bronzes	103–124
Niobium and alloys	80–110
Silicon	107
Zirconium and alloys	96

(*Continued*)

Material	*E* (GN m^{-2})
Silica glass, SiO_2 (quartz)	94
Zinc and alloys	43–96
Gold	82
Calcite (marble, limestone)	70–82
Aluminum	69
Aluminum and alloys	69–79
Silver	76
Soda glass	69
Alkali halides (NaCl, LiF, etc.)	15–68
Granite (Westerly granite)	62
Tin and alloys	41–53
Concrete, cement	30–50
Fiberglass (glass-fiber/epoxy)	35–45
Magnesium and alloys	41–45
GFRP	7–45
Calcite (marble, limestone)	31
Graphite	27
Shale (oil shale)	18
Common woods, $\parallel$ to grain	9–16
Lead and alloys	16–18
Alkyds	14–17
Ice, H_2O	9.1
Melamines	6–7
Polyimides	3–5
Polyesters	1.8–3.5
Acrylics	1.6–3.4
Nylon	2–4
PMMA	3.4
Polystyrene	3–3.4
Epoxies	2.6–3
Polycarbonate	2.6
Common woods, $\perp$ to grain	0.6–1.0
Polypropylene	0.9
PVC	0.2–0.8
Polyethylene, high density	0.7
Polyethylene, low density	0.2
Rubbers	0.01–0.1
Cork	0.01–0.03
Foamed polymers	0.001–0.01

Values expressed in GPa (1 GPa = 10^9 Pa)

HIGH

Diamond	1000
Tungsten carbide, WC	550
Silicon carbide, SiC	450
Alumina, Al_2O_3	390

MEDIUM

Chromium	290
Niquel	215
Iron, steels	200
Cast irons	180

LOW

Gold	80
Siver	75
Aluminium	70
Granite	60
Concrete	50
Wood $\parallel$ fiber	15
Wood $\perp$ fiber	1

Very LOW

Nylon	3
Polyethylene HD	0.7
Polyethylene LD	0.2
Rubber	0.05
Foams	0.005

Materials of practical application in engineering:

E: **10^3 – 10^{-3} GPa**