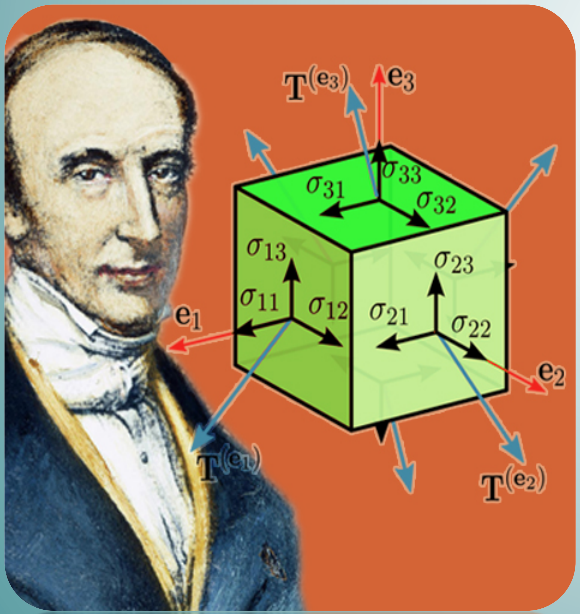
Mechanical Properties of Materials, Processing and Design

Topic 1. Elastic behaviour



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1.1. INTRODUCTION

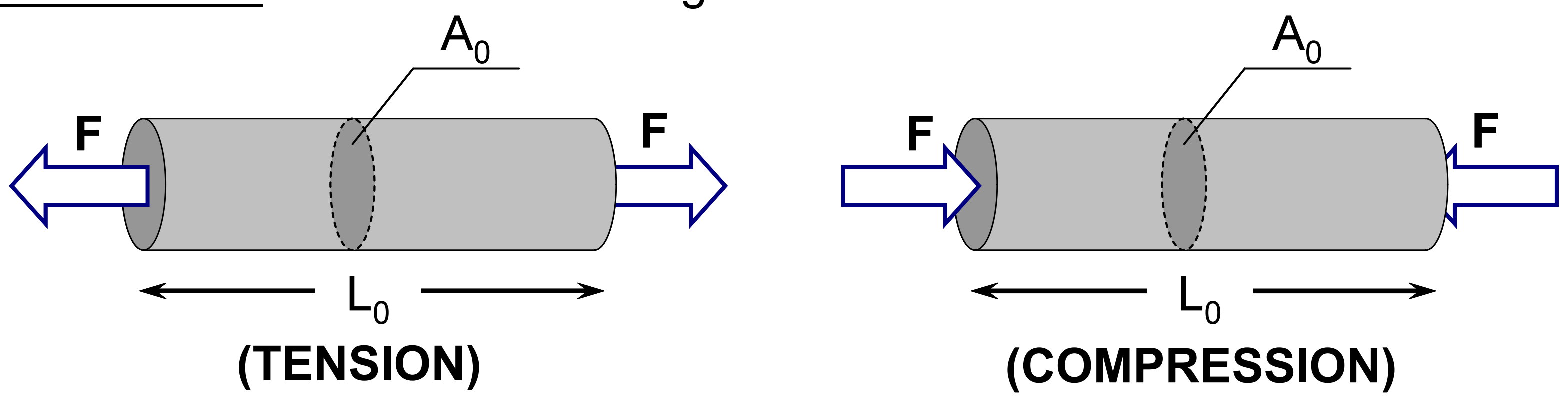
ELASTICITY:

- It is the ability of a material to **recover its original geometric shape** after experimenting a mechanical action.
- **It is not an absolute property:** it depends on the material and on the range of loads and stresses applied to it.

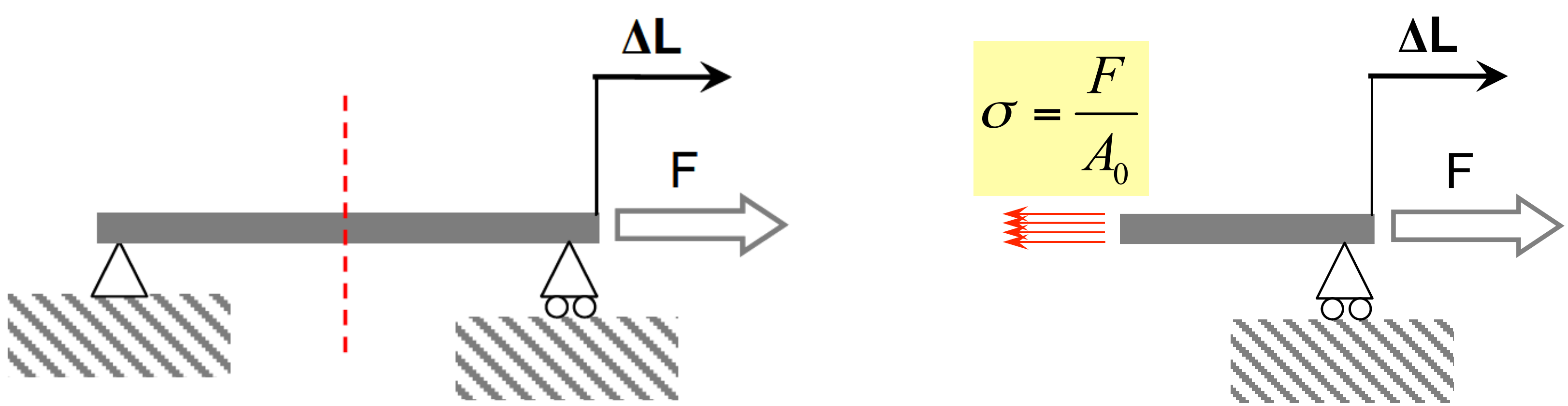
1.1. INTRODUCTION

STRESS STATE: INTRODUCTION

- **EXTENSION**: Uniaxial load acting on a linear element:



- It is assumed that the loads applied on the cross section are uniform and normal to that surface.
- Under equilibrium conditions, each and every part of a body (either real or imaginary) must be in equilibrium too.

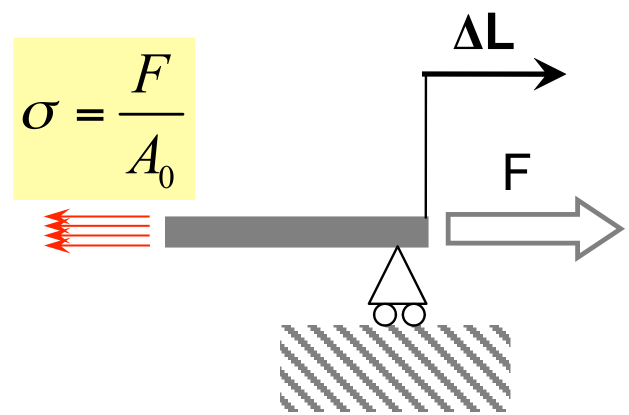


$$\sigma = \frac{F}{A_0}$$

1.1. INTRODUCTION

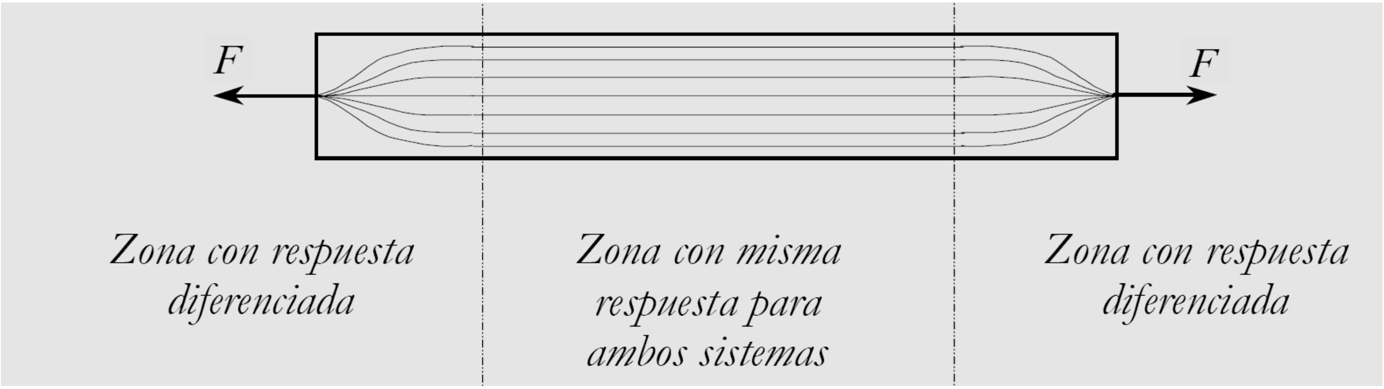
STRESS STATE: INTRODUCTION

- **Note**: when stress is defined this way (using the original geometry of the cross section) it is called **engineering stress**.
- The difference between engineering and true variables (stress and strain) will be explained in Lesson 2.



SAINT VENANT'S PRINCIPLE:

- Empirical principle (Check: Oliver and Agelet, UPC, 2000).
- Beam under the action of a punctual force F. It is extremely complicated to solve this elastic problem analytically (usually, numerical solution).



1.1. INTRODUCTION

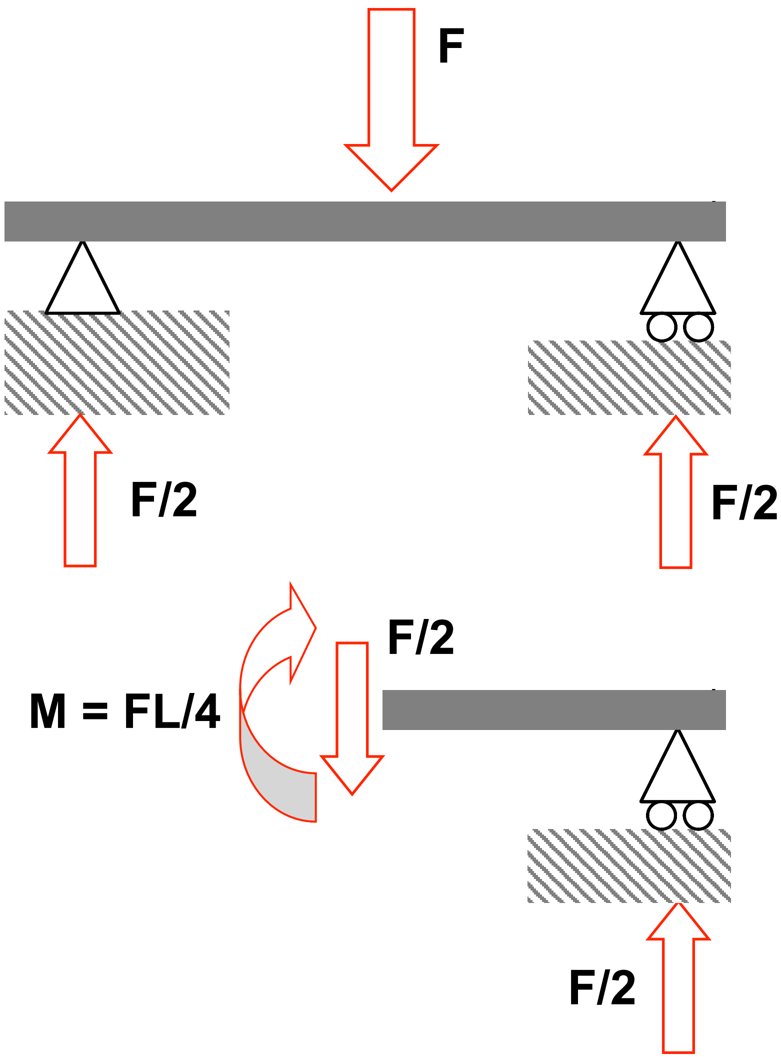
STRESS STATE: INTRODUCTION

SAINT VENANT'S PRINCIPLE:

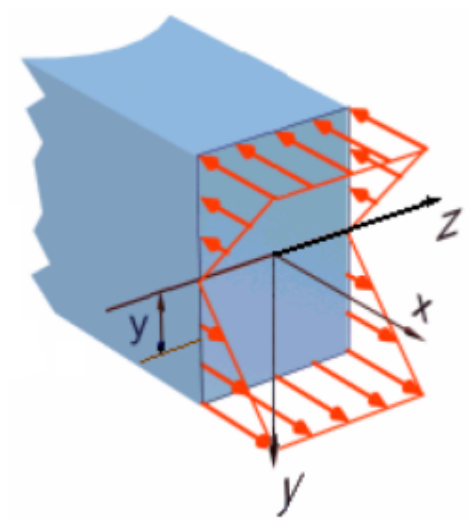
- Force F is replaced by a statically equivalent system of uniformly distributed stresses in the extreme section. The elastic analysis of this new problem is straightforward.
- Saint Venant's Principle allows to approximate the stress state of (I) by (II) provided the point to be analyzed is far enough to the punctual load (one or two times the width of the beam).

1.1. INTRODUCTION STRESS STATE: INTRODUCTION

MORE ABOUT NORMAL STRESS. EXAMPLES:



- Bending moment on a beam: internal stresses must balance the external moments and forces (equilibrium condition).
- There is a non-uniform distribution of normal (and shear) stresses through the cross section. According to Strength of Materials:

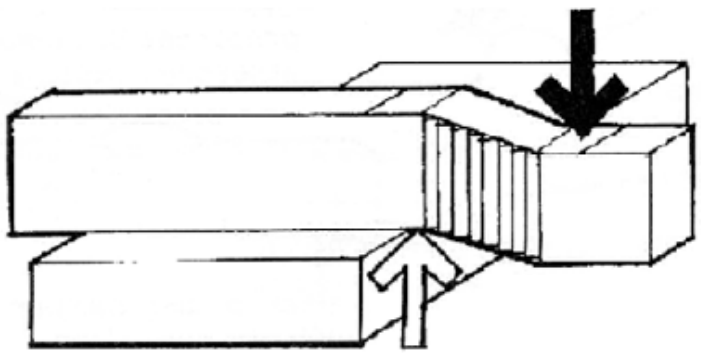


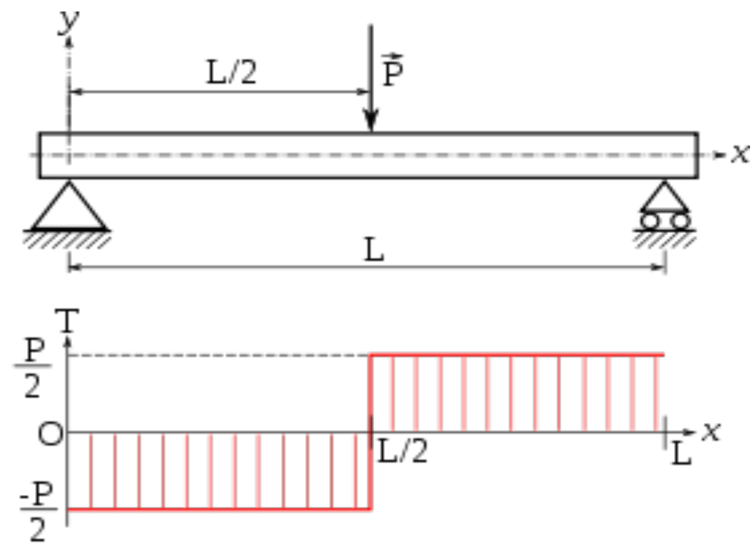
$$\sigma_x = \frac{M}{I_z} y$$

1.1. INTRODUCTION

STRESS STATE: INTRODUCTION

SHEAR STRESS:

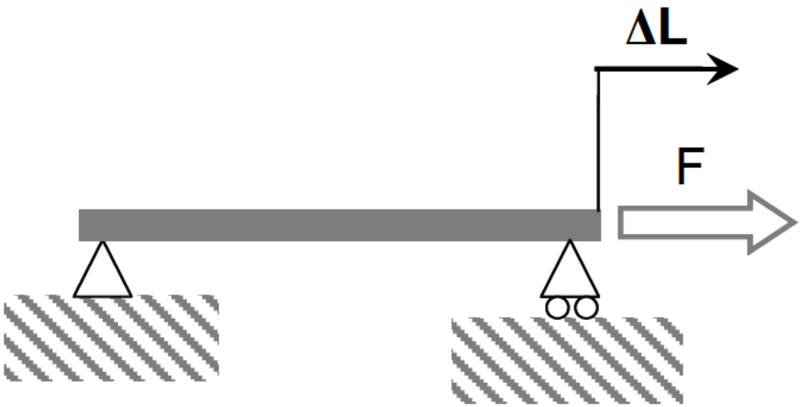




- In **isostatic structures**, stresses (normal, shear) can be directly obtained from axial, bending and torsional forces in the cross section.
- For **hyperstatic structures**, compatibility equations are necessary too.
- This assumption is no longer valid with “general” structures (continuum media). In fact, in such case it makes no sense to talk about axial forces, bending moments and so on.

1.1. INTRODUCTION

STRESS STATE: INTRODUCTION



Strain definition (longitudinal):

$$\varepsilon = \frac{\Delta L}{L_0}$$

- This definition is valid as long as L~cte, that is, when ΔL<<L (small strain theory).

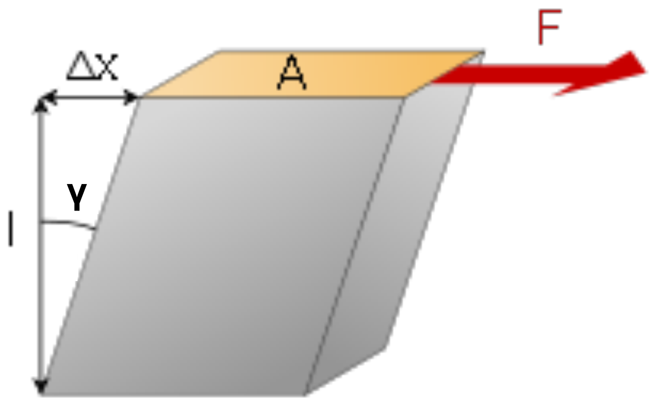
- Otherwise, Cauchy's incremental strain definition should be applied:

$$d\varepsilon = \frac{dL}{L}$$

- **Note**: when studying elastic behavior, we will always assume small strain conditions. Nevertheless, this is no longer valid in situations where large strains occur (for instance, in many cases where plasticity is playing a role).

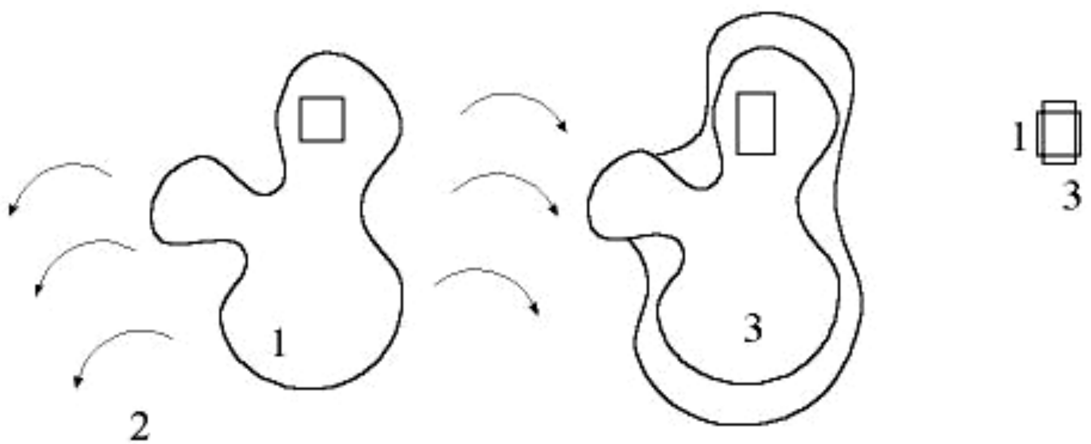
1.1. INTRODUCTION

STRESS STATE: INTRODUCTION



- Along with longitudinal deformations, we can also find distortions, γ.
- The figure (left) shows the distortion (γ) underwent by a small part subjected to a tangential / shear force.

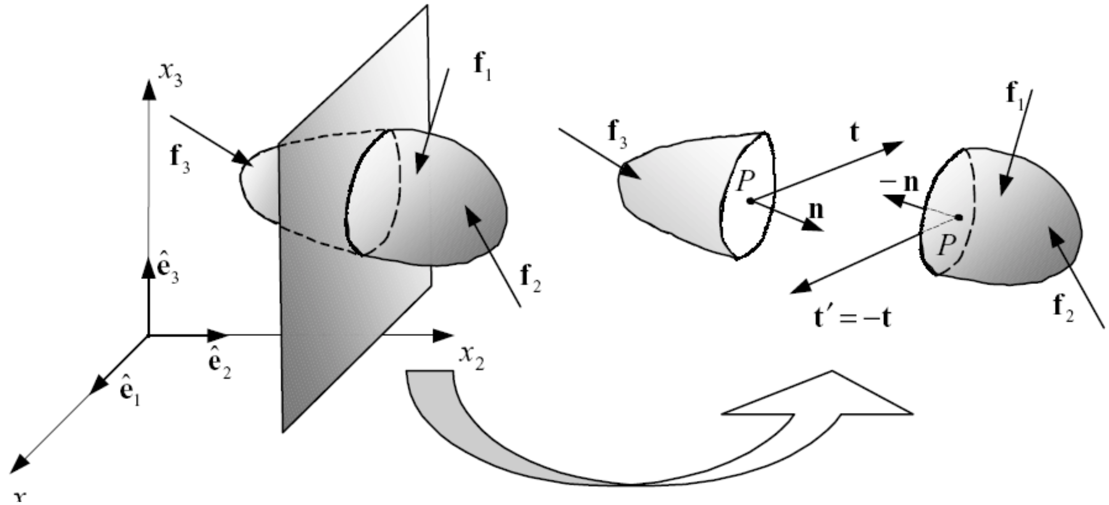
- Deformations must be **compatible** with the location of the material points of the continuum media.



1.1. INTRODUCTION

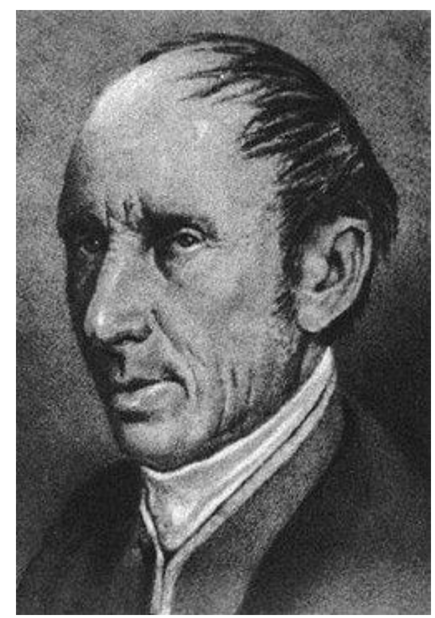
Boundary conditions (forces and displacements)

Stress distribution in every material point.



- 1822: Cauchy’s (1789-1857) stress principle.
- The stress vector depends on the orientation of the plane and on the point:

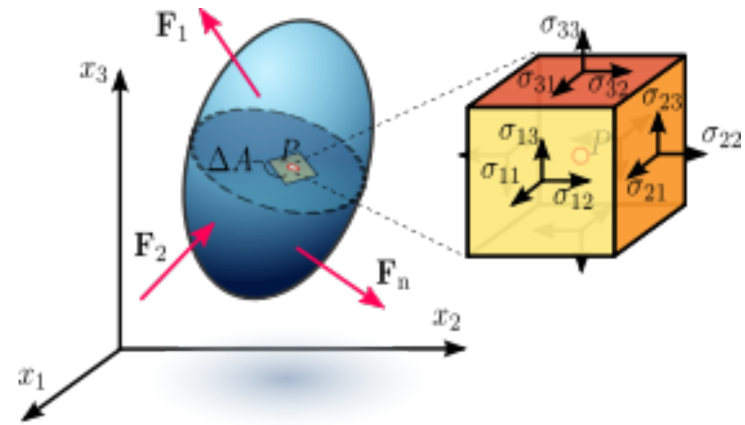
$$\vec{t}\,(P,\vec{n}) = \lim_{\Delta S \to 0} \frac{\Delta \vec{F}}{\Delta S} = \frac{d\vec{F}}{dS}$$



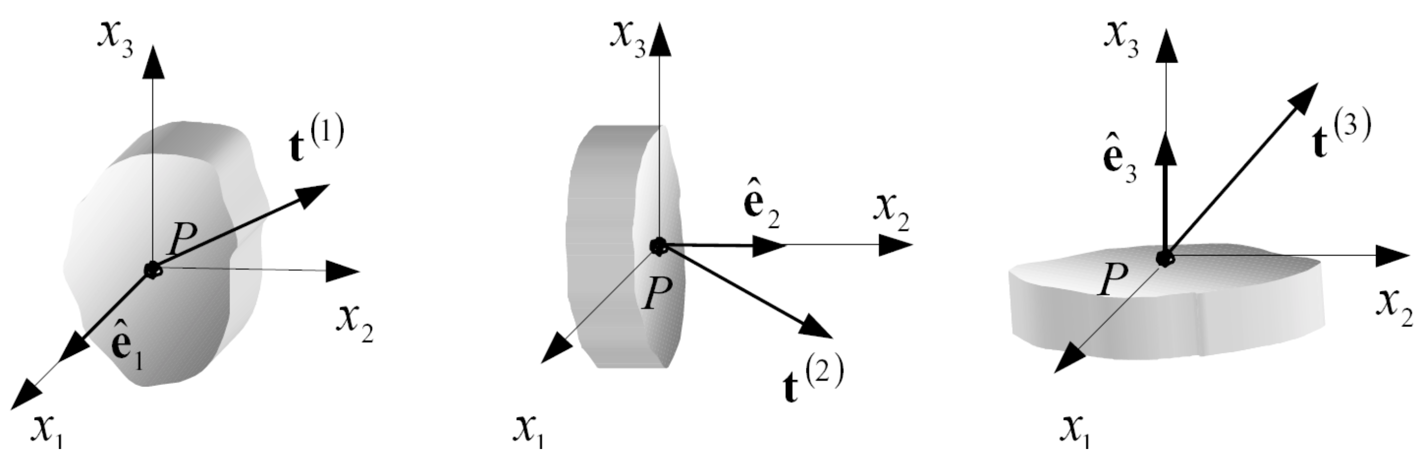
1.1. INTRODUCTION

- **Stress tensor** contains all the information about the stress state of every point of a continuous body.

$$\boxed{\mathbf{t}(P,\mathbf{n}) = \mathbf{n}\cdot\boldsymbol{\sigma}(P)}$$



- The stress tensor ($\boldsymbol{\sigma}$) can be obtained from the coordinates of three stress vectors in three coordinate planes, containing point P:

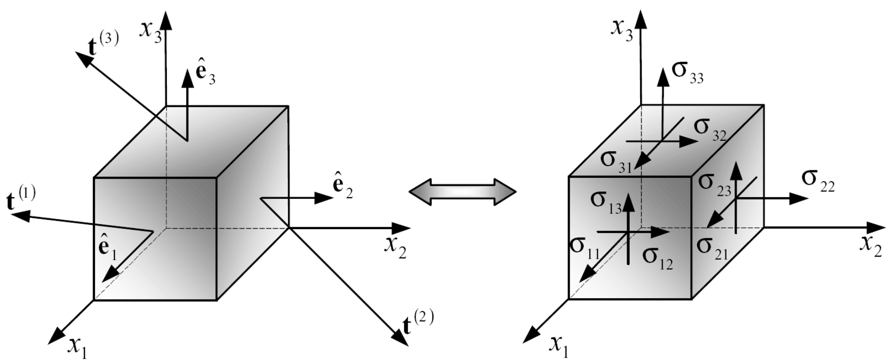


1.1. INTRODUCTION

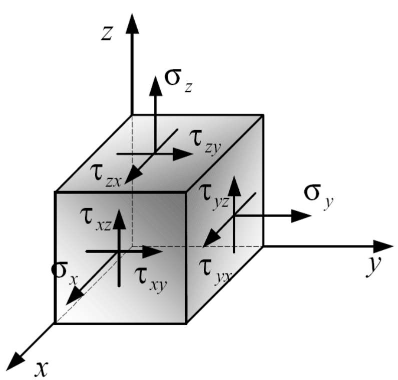
SCIENTIFIC NOTATION FOR THE STRESS TENSOR (MATRIX):

$$\boldsymbol{\sigma} \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$$\sigma_{ij} \rightarrow \begin{cases} \text{índice } i \rightarrow \left| \begin{array}{l} \text{indica el plano de actuación} \\ \text{(plano perpendicular al eje } x_i\text{)} \end{array} \right. \\ \text{índice } j \rightarrow \left| \begin{array}{l} \text{indica la dirección de la tensión} \\ \text{(dirección del eje } x_j\text{)} \end{array} \right. \end{cases}$$



ENGINEERING NOTATION FOR THE STRESS TENSOR (MATRIX):

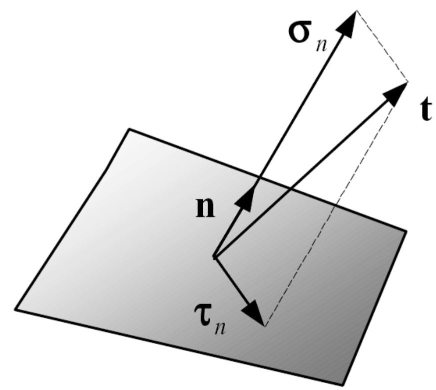


$$\boldsymbol{\sigma} \equiv \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{bmatrix}$$

1.1. INTRODUCTION

SIGN CRITERIA:

- Given a point P and a plane containing the point, the stress vector can be split into its normal (σ) and shear (τ) components.



$$\boldsymbol{\sigma}_n = \sigma\,\mathbf{n}$$

$$\boxed{\sigma = \mathbf{t}\cdot\mathbf{n}\begin{cases} > 0 \text{ tracción} \\ < 0 \text{ compresión}\end{cases}}$$

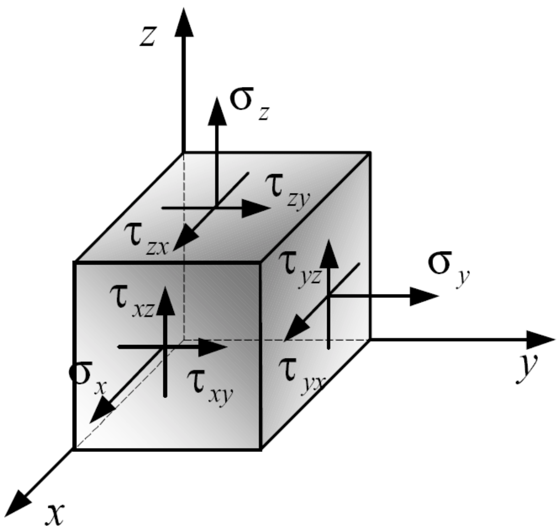
- This criteria can be used for the stress tensor matrix. In the basic parallelepiped we can distinguish between exposed faces (or positive ones) and hidden faces (or negative ones).

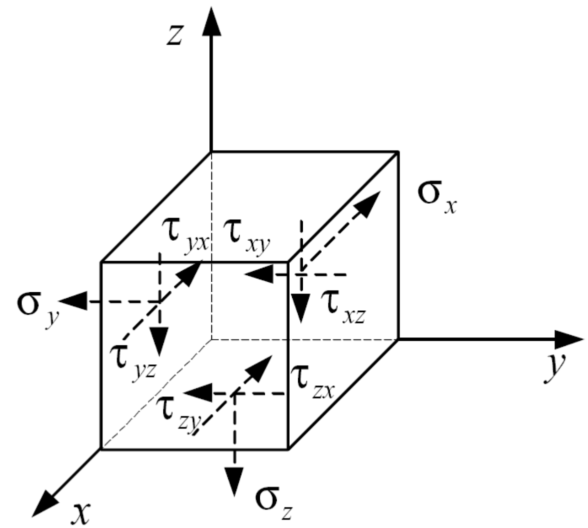
$$\boxed{\sigma_{ij}\ o\ \sigma_a \begin{cases} positivas\ (+) \Rightarrow \text{tracción} \\ negativas\ (-) \Rightarrow \text{compresión}\end{cases}}$$

$$\boxed{\tau_{ab}\begin{cases} positivas\ (+) \Rightarrow \text{sentido del eje b} \\ negativas\ (-) \Rightarrow \text{sentido contrario al eje b}\end{cases}}$$

1.1. INTRODUCTION

SIGN CRITERIA:





$$\sigma_{ij} \ o \ \sigma_a \begin{cases} \textit{positivas} \ (+) \Rightarrow \text{tracción} \\ \textit{negativas} \ (-) \Rightarrow \text{compresión} \end{cases}$$

$$\tau_{ab} \begin{cases} \textit{positivas} \ (+) \Rightarrow \text{sentido del eje b} \\ \textit{negativas} \ (-) \Rightarrow \text{sentido contrario al eje b} \end{cases}$$

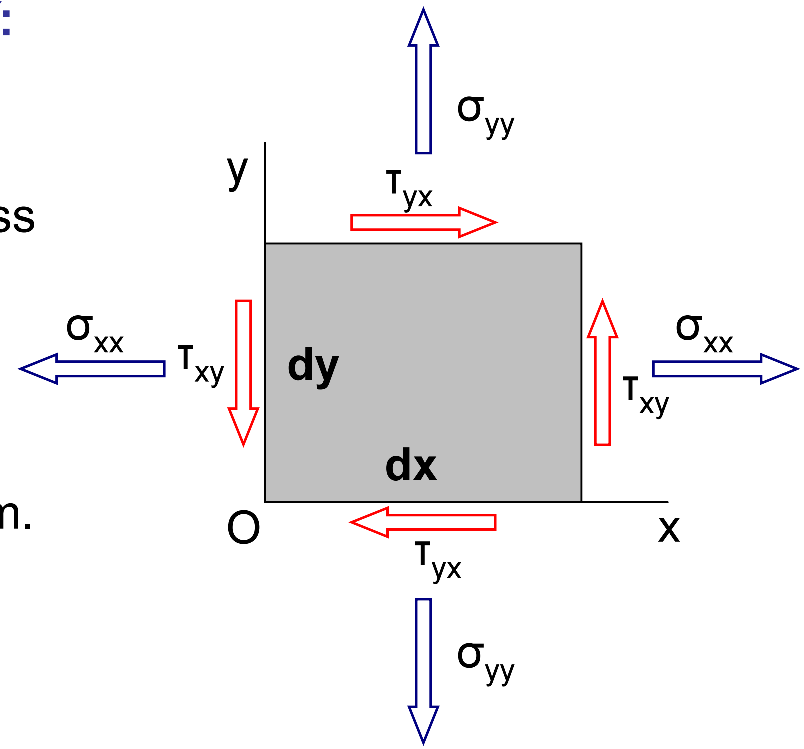
1.1. INTRODUCTION

STRESS TENSOR MATRIX SYMMETRY:

Suggested exercises:

1) Demonstrate the symmetry of the Stress Tensor (Law of the conjugated shear stresses).

- **Tip:** without loss of generality, apply equilibrium conditions on a 2D system.
- Equilibrium conditions:



$$\sum \vec{M}_o = 0$$

$$\tau_{xy}\left(dy \cdot 1\right) \cdot dx - \tau_{yx}\left(dx \cdot 1\right) \cdot dy = 0 \quad \Rightarrow \quad \tau_{xy} = \tau_{yx} \qquad \textbf{(QED)}$$

1.1. INTRODUCTION

PRINCIPAL STRESSES AND DIRECTIONS:

- Tensor algebra guarantees that any second order tensor diagonalizes in an orthonormal base and that its eigenvalues are real numbers.

- Let's consider the stress tensor matrix in an arbitrary Cartesian basis (*x*, *y*, *z*):

$$\boldsymbol{\sigma} \equiv \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}_{(x,y,z)}$$

- In the Cartesian coordinate system (*x'*, *y'*, *z'*) in which σ diagonalizes, the matrix will be:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}_{(x',y',z')}$$

DEFINITIONS:

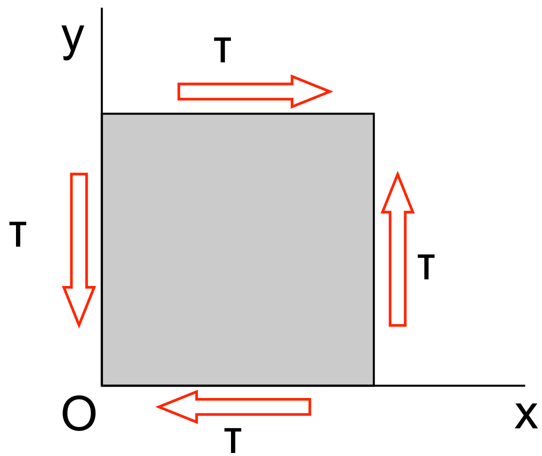
- ***Principal directions (of stresses):*** directions related to the axes (*x'*, *y'*, *z'*) in which the stress tensor matrix is diagonal.

- ***Principal stresses:*** components of the stress tensor (σ_1, σ_2, σ_3) when the basis is changed to a new Cartesian system (x', y', z') so that the shear stress components become zero. **Criterion: $\sigma_1 \geq \sigma_2 \geq \sigma_3$**.

1.1. INTRODUCTION

Simple example:

1) Obtain the stress tensor matrix for a pure 2D shear stress situation. Calculate the principal stresses by diagonalization of the matrix. Analyze its symmetry.



Stress tensor: $$(\sigma) = \begin{pmatrix} 0 & \tau \\ \tau & 0 \end{pmatrix}$$

$$|\sigma - \lambda 1| = 0 \quad \Rightarrow \quad \begin{vmatrix} -\lambda & \tau \\ \tau & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = \pm\tau$$

$$\left.\begin{array}{l} \sigma_1 = \tau : \quad \begin{pmatrix} -\tau & \tau \\ \tau & -\tau \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \Rightarrow \quad a = b \\ \\ \text{Normalization condition:} \quad a^2 + b^2 = 1 \end{array}\right\} \quad \boxed{\vec{v}^{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

1.1. INTRODUCTION

Simple example:

$$\sigma_2 = -\tau : \quad \begin{pmatrix} \tau & \tau \\ \tau & \tau \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \quad \left. \begin{array}{r} a = -b \\ a^2 + b^2 = 1 \end{array} \right\} \quad \boxed{\vec{v}^{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

1.1. INTRODUCTION

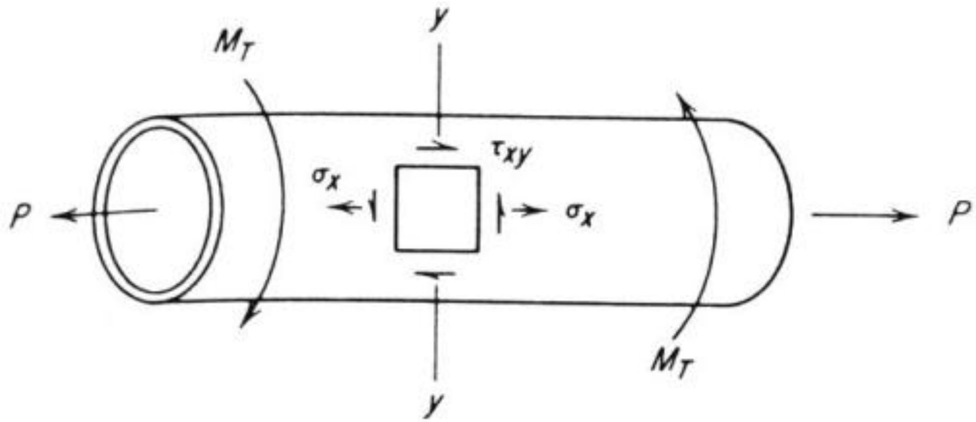
Simple example:

2) Obtain the stress tensor and the principal stresses in this situation: a thin wall pipe with axial stress and a torque moment.

Data:

R = 100 mm; t = 2 mm.

P = 20 kN; M_t = 1 kN·m;



Solution:

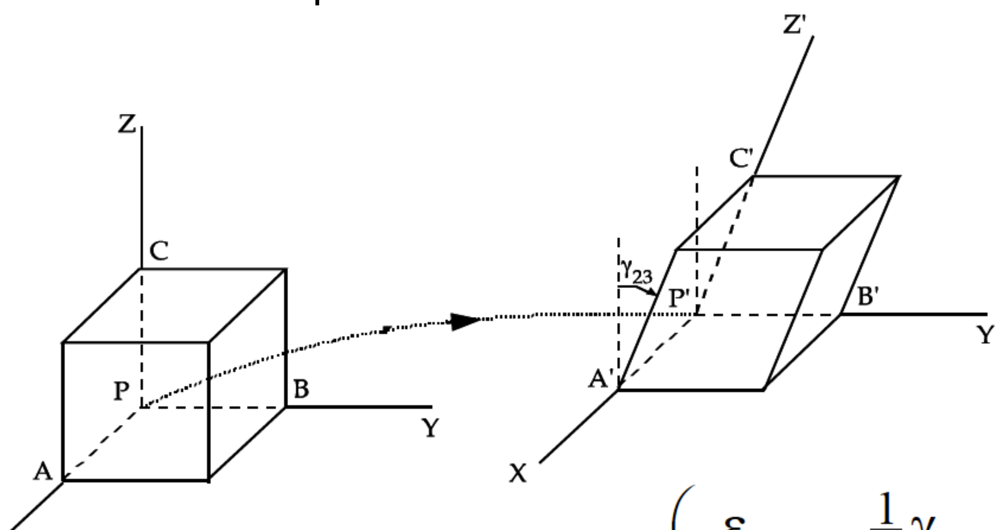
$$\sigma = \begin{pmatrix} 15.9155 & 7.9577 \\ 7.9577 & 0 \end{pmatrix} \qquad \begin{cases} \sigma_1 = 19.2117\ MPa \\ \sigma_2 = -3.2962\ MPa \end{cases}$$

$$\vec{v}^{(1)} = \begin{pmatrix} 0.9239 \\ 0.3827 \end{pmatrix} \qquad \vec{v}^{(2)} = \begin{pmatrix} -0.3827 \\ 0.9239 \end{pmatrix}$$

1.2. STRESS AND STRAIN

The **strain state** in each point of the material of a continuous medium is described by a mathematical object: **(small) Strain Tensor.**

- Interpretation of the components of the Strain Tensor:
  - Principal diagonal components: unitary elongation.
  - Other components: distortions.

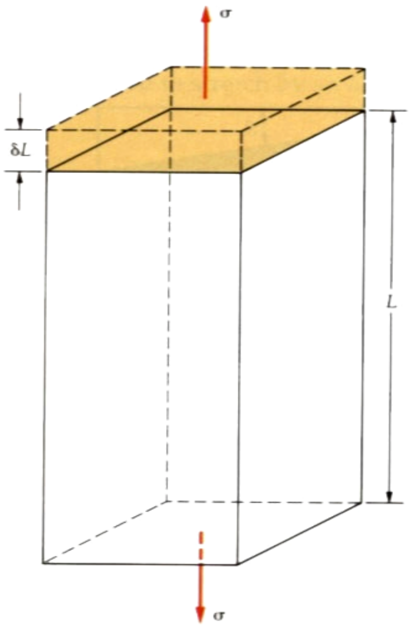


$$\left[\text{Deformation} \quad \varepsilon = \frac{\Delta l}{l}\right]$$

$$\varepsilon(x,y,z);\left[\varepsilon_{ij}\right] = \begin{pmatrix} \varepsilon_{11} & \frac{1}{2}\gamma_{12} & \frac{1}{2}\gamma_{13} \\ \frac{1}{2}\gamma_{21} & \varepsilon_{22} & \frac{1}{2}\gamma_{23} \\ \frac{1}{2}\gamma_{31} & \frac{1}{2}\gamma_{32} & \varepsilon_{33} \end{pmatrix} \equiv \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

1.2. STRESS AND STRAIN

STRAIN ENERGY DENSITY:



Principle of energy conservation:

$$dU = \delta W = FdL = (\sigma \cdot A)(d\varepsilon \cdot L) = (A \cdot L)(\sigma \cdot d\varepsilon)$$

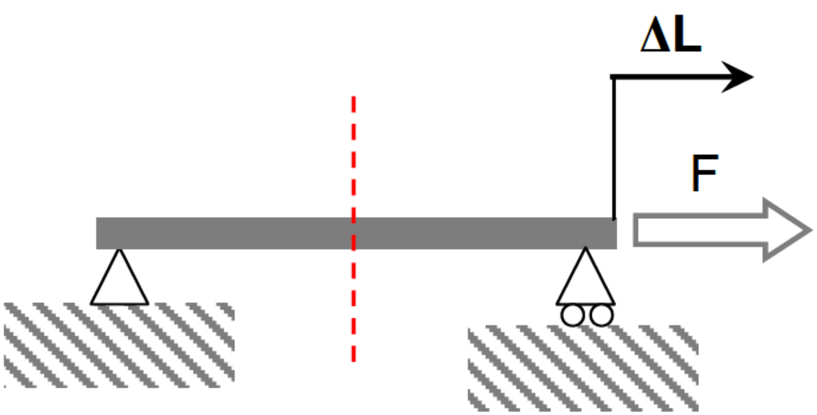
$$du = \frac{dU}{A \cdot L} = \sigma \cdot d\varepsilon \quad \Rightarrow \quad u = \int_1^2 \sigma \cdot d\varepsilon$$

1.3. STRESS-STRAIN RELATIONSHIP

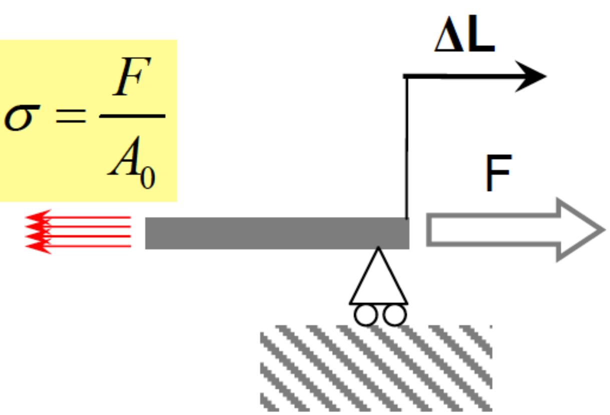
RECALLING: STATIC AND KINEMATIC VARIABLES. EQUILIBRIUM EQUATIONS, CONSTITUTIVE EQUATIONS AND COMPATIBILITY

- **Static variables:** forces, moments, stresses…
- **Kinematic variables:** displacements, deflections, rotations, strains, distortions…

- An **equilibrium equation** is a relationship between static variables.



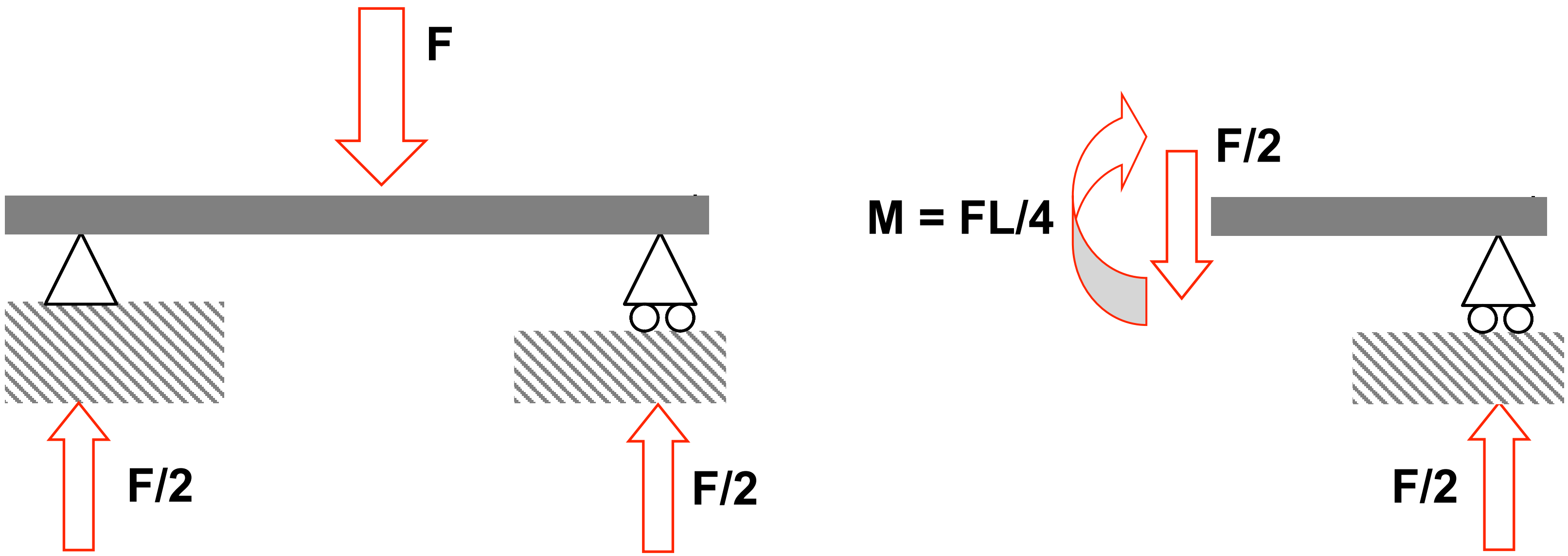
$$\sigma = \frac{F}{A_0}$$



1.3. STRESS-STRAIN RELATIONSHIP

RECALLING: STATIC AND KINEMATIC VARIABLES. EQUILIBRIUM EQUATIONS, CONSTITUTIVE EQUATIONS AND COMPATIBILITY

- An **equilibrium equation** is a relationship between static variables.

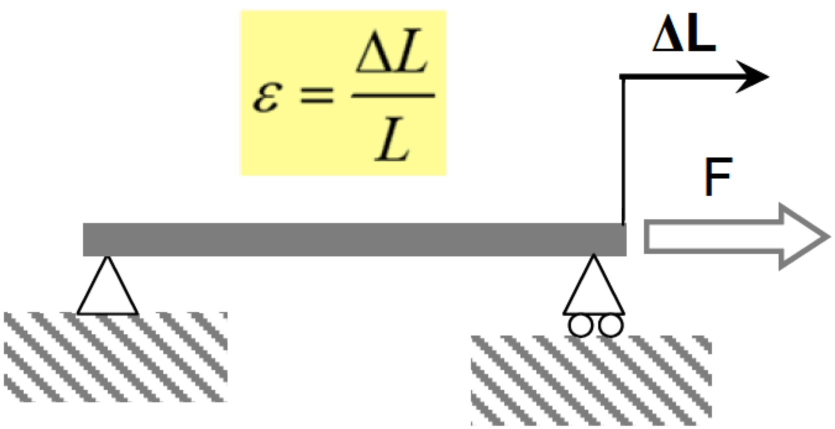


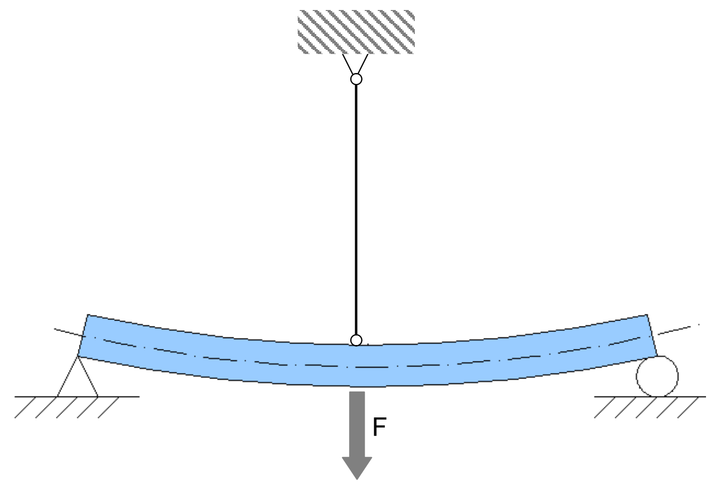
1.3. STRESS-STRAIN RELATIONSHIP

RECALLING: STATIC AND KINEMATIC VARIABLES. EQUILIBRIUM EQUATIONS, CONSTITUTIVE EQUATIONS AND COMPATIBILITY

- A **compatibility equation** is a relationship between kinematic variables.

$$\varepsilon = \frac{\Delta L}{L}$$



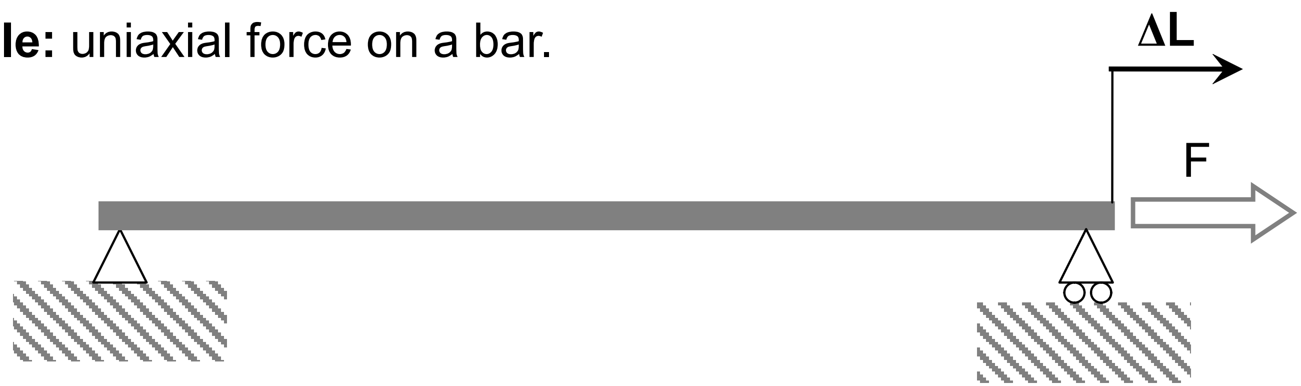


$$\Delta L_{cable} = f_{viga,CL}$$

1.3. STRESS-STRAIN RELATIONSHIP

RECALLING: STATIC AND KINEMATIC VARIABLES. EQUILIBRIUM EQUATIONS, CONSTITUTIVE EQUATIONS AND COMPATIBILITY

- A **constitutive equation** is a relationship between static and kinematic variables.
- **Ejemple:** uniaxial force on a bar.



Hooke's law: $\sigma = E\varepsilon \Rightarrow \frac{F}{A_0} = E\frac{\Delta L}{L_0} \Rightarrow \Delta L = \frac{F\,L_0}{E\,A_0}$

1.3. STRESS-STRAIN RELATIONSHIP

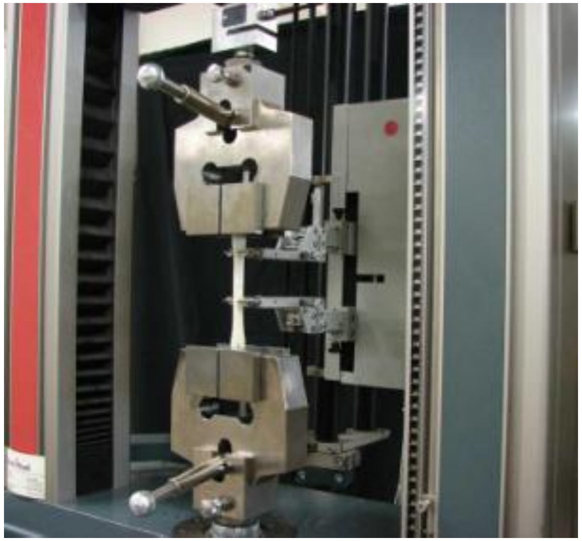
Isotropic medium: is completely defined by two elastic constants.

- There is some freedom in the selection of these two constants. In structural mechanics, E and v are normally used.

  E Elastic / Young's modulus.

  ν Poisson's ratio.

- Tensile test allows both parameters to be obtained.



- Primary variables of this test: force (F) and elongation (ΔL).
- Derived variables: stress and strain.
- In the linear elastic regime:

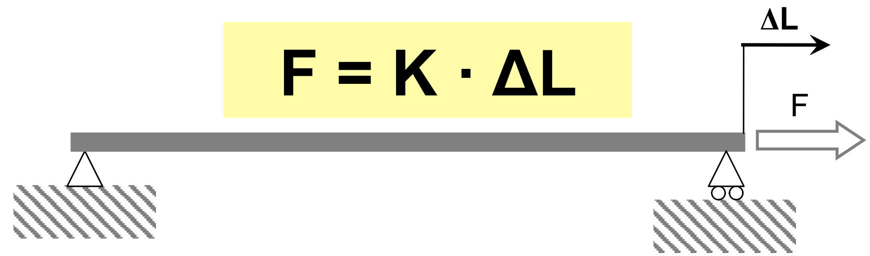
$$\begin{cases} \sigma = E\varepsilon \\ \varepsilon_t = -\nu\varepsilon \end{cases}$$

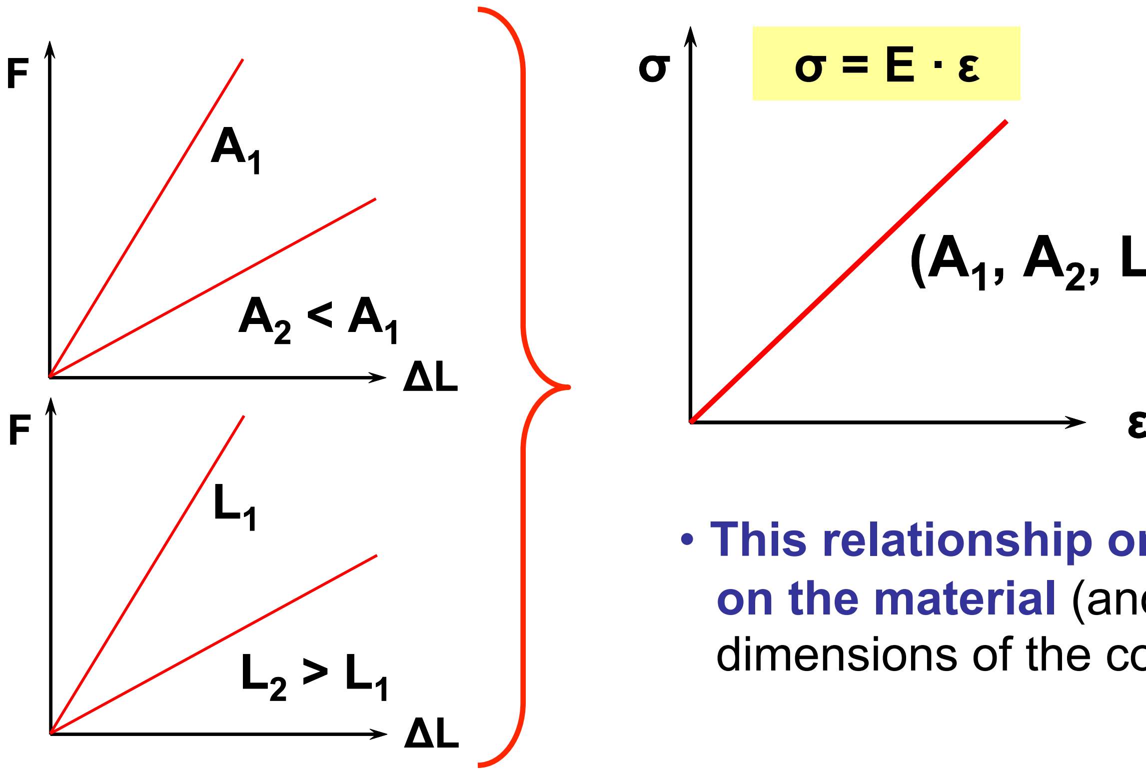
1.3. STRESS-STRAIN RELATIONSHIP

Tensile test:

- Drawback: two bars made out of the same material but with different geometric dimensions.

$$F = K \cdot \Delta L$$





$$\sigma = E \cdot \varepsilon$$

- **This relationship only depends on the material** (and not on the dimensions of the component).

1.3. STRESS-STRAIN RELATIONSHIP

Generalized Hooke's law (triaxial stress, isotropic medium)

$$\varepsilon_i = \frac{1}{E}\left[\sigma_i - \nu\left(\sigma_j + \sigma_k\right)\right] \qquad i, j, k : 1..3, i \neq j \neq k$$

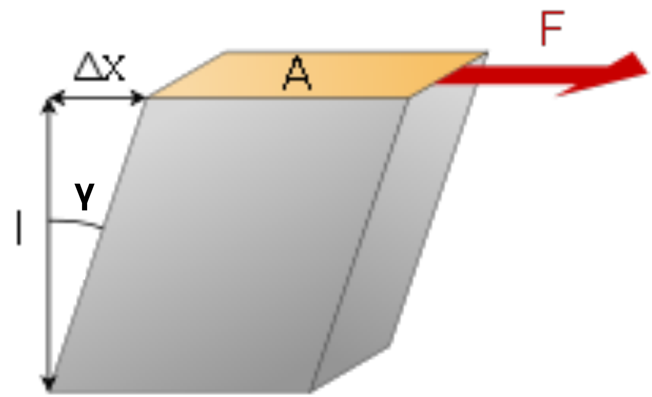
Suggested exercise:

1) Demonstrate Hooke's generalized law (2D) by applying the Superposition Principle on an arbitrary biaxial state. Then, generalize it for a 3D problem.

1.3. STRESS-STRAIN RELATIONSHIP

• Another alternatives: G and K

G Shear modulus; $\tau = G \cdot \gamma$



K Bulk modulus;

$$\sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = K \frac{\Delta V}{V}$$

• Provided the behavior of a continuous and isotropic medium is perfectly defined with two elastic constants, it must be possible to express G and K in terms of E and ν:

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

1.3. STRESS-STRAIN RELATIONSHIP

Suggested exercises:

1) Demonstrate the relationship between K, E and ν (Tip: Analyze the volume change in a parallelepiped (element) under the action of a triaxial stress state).

1.3. STRESS-STRAIN RELATIONSHIP

Suggested exercises:

2) Demonstrate the relationship between G, E and v. To do that, analyze the pure shear stress state described in Figure 1 (stress and strain states), establishing the **equilibrium** equations in the element 1234 (Figure 2) and the **compatibility** equations in line MN (Figure 3).

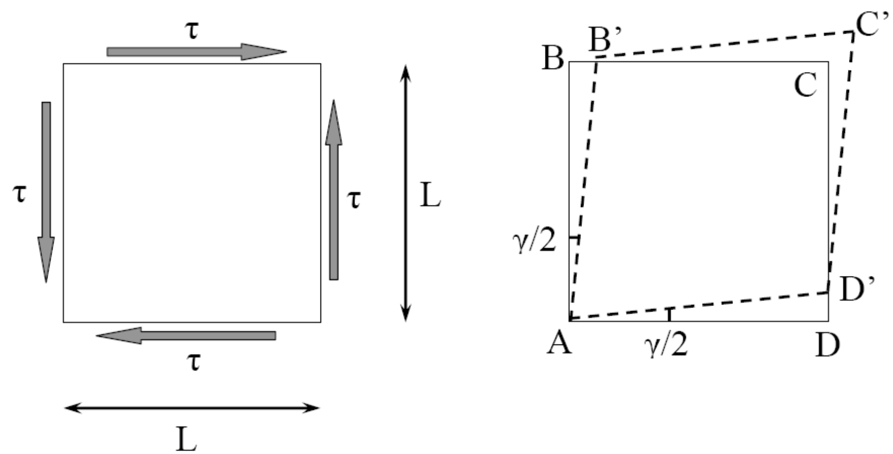


Figura 1

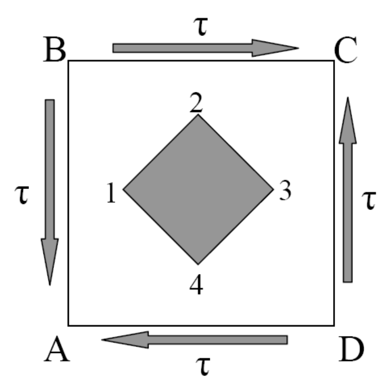


Figura 2

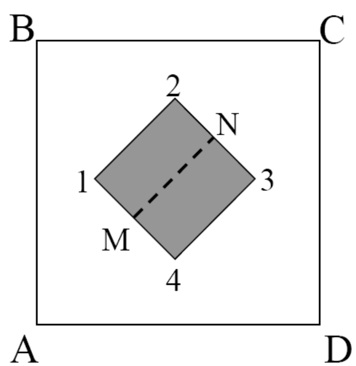


Figura 3

1.3. STRESS-STRAIN RELATIONSHIP

- **Linear medium** definition (Einstein – Grossman's compact notation: repeated indexes represent summatories): **E**: Stiffness tensor

$$\sigma_{ij} = E_{ijkl}\,\varepsilon_{kl}\,; \quad i,j,k,l:1..3$$

- **Question:** how many elastic constants are necessary to describe the most general linear medium?

- Symmetry: $\left(\sigma_{ij} = \sigma_{ji}\right) \wedge \left(\varepsilon_{ij} = \varepsilon_{ji}\right) \Rightarrow E_{ijkl} = E_{jikl} = E_{ijlk} = E_{jilk}$

- Symmetry in stress tensors and small strain tensors helps to simplify the problem (without loss of generality). In matrix notation (stiffness matrix):

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{33} \end{pmatrix} = \begin{pmatrix} E_{1111} & E_{1112} & E_{1113} & E_{1122} & E_{1123} & E_{1133} \\ E_{1211} & E_{1212} & E_{1213} & E_{1222} & E_{1223} & E_{1233} \\ E_{1311} & E_{1312} & E_{1313} & E_{1322} & E_{1323} & E_{1333} \\ E_{2211} & E_{2212} & E_{2213} & E_{2222} & E_{2223} & E_{2233} \\ E_{2311} & E_{2312} & E_{2313} & E_{2322} & E_{2323} & E_{2333} \\ E_{3311} & E_{3312} & E_{3313} & E_{3322} & E_{3323} & E_{3333} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{22} \\ \varepsilon_{23} \\ \varepsilon_{33} \end{pmatrix}$$

- It is enough with 'only' 36 elastic constants.

1.3. STRESS-STRAIN RELATIONSHIP

- It can be demonstrated that such a general relationship implies that energy is not conserved through the process (internal dissipative processes).
- To avoid this drawback, it is necessary to impose symmetry conditions on the stiffness matrix (Green's hyperelastic medium).

$$\sigma_{ij} = \frac{\partial W\left(\varepsilon_{ij}\right)}{\partial \varepsilon_{ij}} \Rightarrow \frac{\partial^2 W}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}} = \frac{\partial^2 W}{\partial \varepsilon_{kl} \partial \varepsilon_{ij}} \Rightarrow \frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}} = \frac{\partial \sigma_{kl}}{\partial \varepsilon_{ij}} \Rightarrow E_{ijkl} = E_{klij}$$

- These kind of materials are called linear hyperelastic and they need <u>21 elastic constants</u>. Stiffness matrix is symmetric.

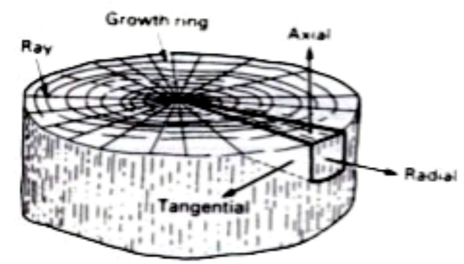
$$\begin{pmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{33} \end{pmatrix} = \begin{pmatrix} E_{1111} & E_{1112} & E_{1113} & E_{1122} & E_{1123} & E_{1133} \\ E_{1211} & E_{1212} & E_{1213} & E_{1222} & E_{1223} & E_{1233} \\ E_{1311} & E_{1312} & E_{1313} & E_{1322} & E_{1323} & E_{1333} \\ E_{2211} & E_{2212} & E_{2213} & E_{2222} & E_{2223} & E_{2233} \\ E_{2311} & E_{2312} & E_{2313} & E_{2322} & E_{2323} & E_{2333} \\ E_{3311} & E_{3312} & E_{3313} & E_{3322} & E_{3323} & E_{3333} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{22} \\ \varepsilon_{23} \\ \varepsilon_{33} \end{pmatrix}$$

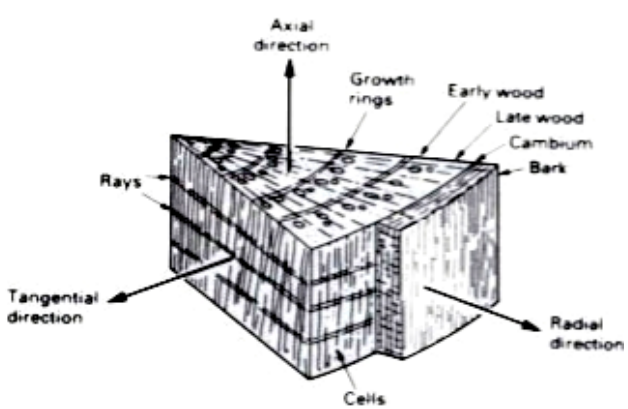
1.3. STRESS-STRAIN RELATIONSHIP

- Materials in nature are rarely that 'sofisticated'. Example: orthotropic behavior.

- The elastic behavior of an **orthotropic** material is defined by **nine independent constants:** 3 longitudinal elasticity modulus (E_x, E_y, E_z), 3 shear modulus (G_{xy}, G_{yz}, G_{zx}) y 3 Poisson's ratios (v_{xy}, v_{yz}, v_{zx}).

- The best example of an orthotropic material is **wood that, due to its structure**, has a different longitudinal elasticity modulus (Young's modulus) along the fiber, tangentially to the growth rings and perpendicularly to the growth rings.

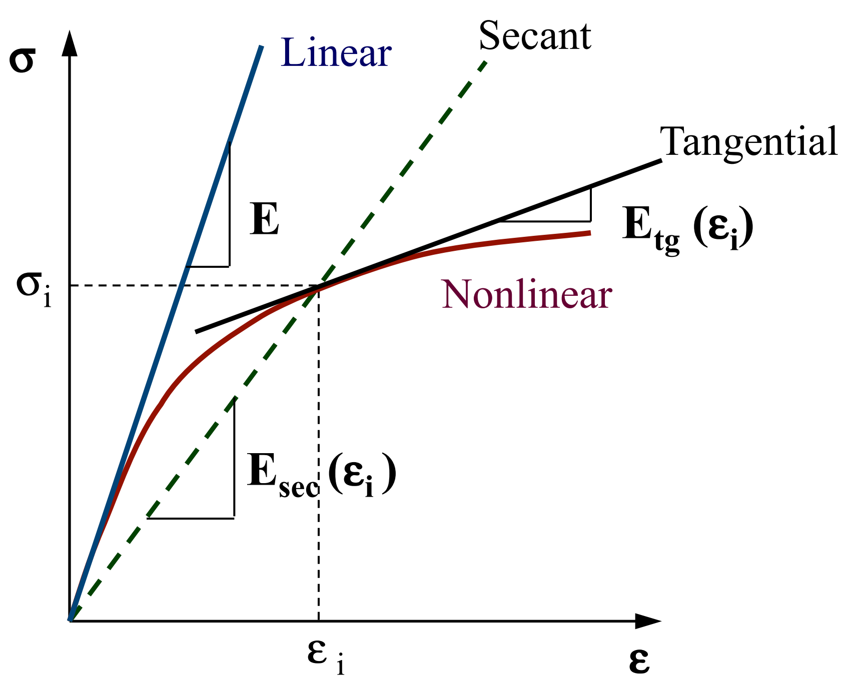




1.4. LINEAR AND NONLINEAR ELASTICITY

- In metallic materials, elasticity (recoverable strain) and linearity (Hooke's law) usually occur simultaneously.
- However this is not true for other materials, where a nonlinear elastic response can take place.

Hooke's law: σ = Eε

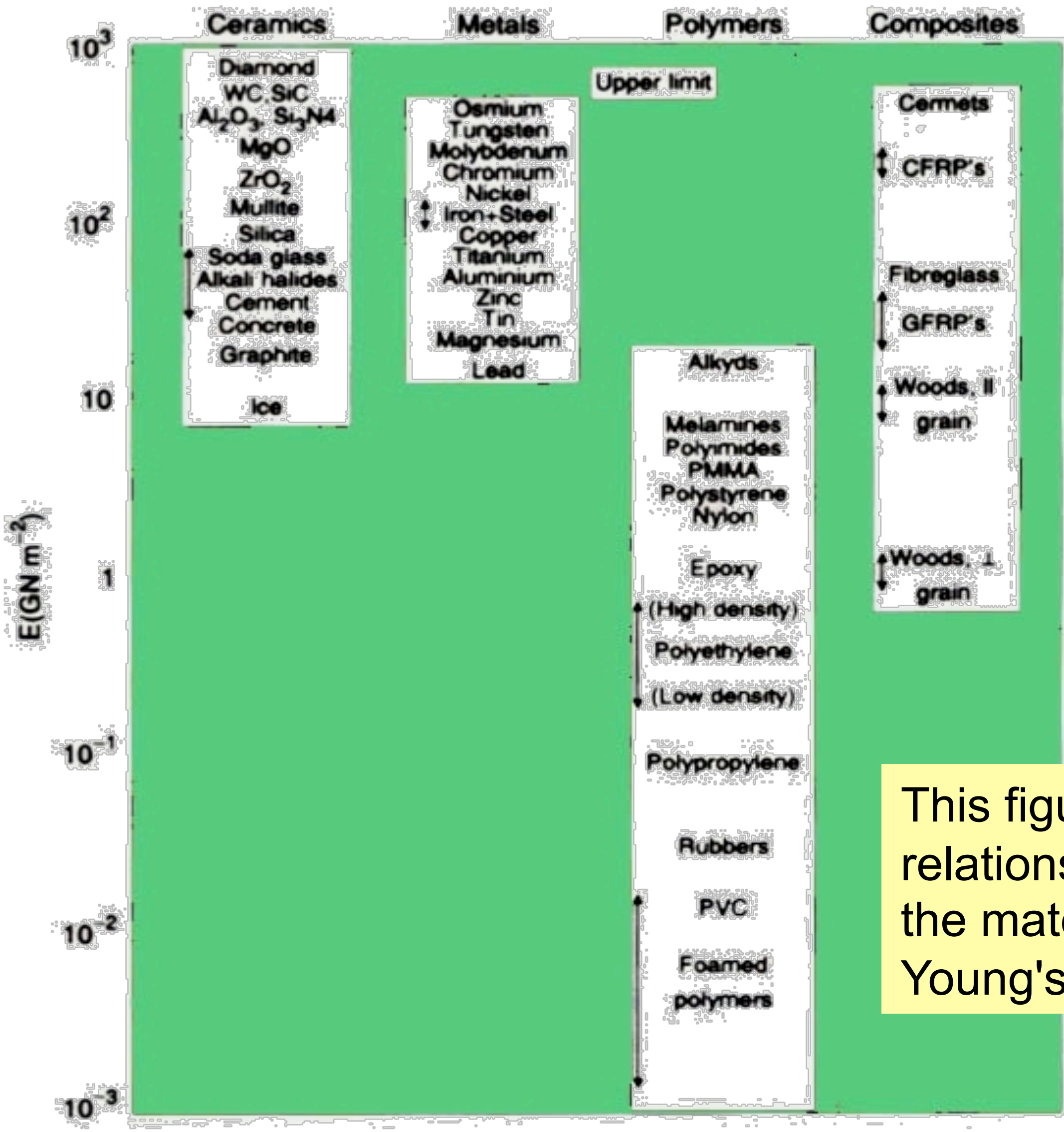


Tangential and secant elastic modulus:

$$E_{tg}(\varepsilon_i) = \left(\frac{d\sigma}{d\varepsilon}\right)_{\varepsilon=\varepsilon_i}$$

$$E_{sec}(\varepsilon_i) = \left(\frac{\sigma_i}{\varepsilon_i}\right)$$

1.5. VALUES OF THE ELASTIC MODULUS



This figure suggests that there is a relationship between the nature of the material and the values of Young's modulus.

1.5. VALUES OF THE ELASTIC MODULUS

Engineering alloys	Young's modulus (GPa)	Poisson's ratio
Aluminum	65–72	0.33–0.34
Copper	100–120	0.34–0.35
Magnesium	45	0.3–0.35
Nickel	200–220	0.31
Steels	200–215	0.27–0.29
Titanium	110–120	0.36
Zinc	105	0.35

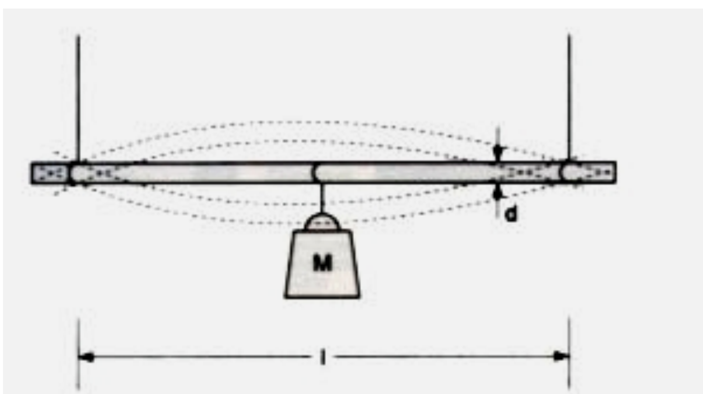
Engineering ceramics and glasses	Young's modulus (GPa)	Poisson's ratio
Titanium diboride, TiB_2	540	0.11
Silicon carbide, SiC	400	0.19
Titanium carbide, TiC	440	0.19
Tungsten carbide, WC	670–710	0.24
Silicon nitride, Si_3N_4	110–325	0.22–0.27
Alumina, Al_2O_3	345–414	0.21–0.27
Beryllium oxide, BeO	300–317	0.26–0.34
Zirconia, ZrO_2	97–207	0.32–0.34
Fused silica	71	0.17
Soda-lime glass	69	0.24
Aluminosilicate glass	88	0.25
Borosilicate glass	63	0.20
High-lead glass	51	0.22

Polymers	Young's modulus (GPa)	Poisson's ratio
Acrylics	2.4–3.1	0.33–0.39
Epoxys	2.6–3.1	0.33–0.37
Polystyrenes	3.1	0.33
Low-density polyethylene	0.1–0.3	0.45
High-density polyethylene	0.4–1.4	0.34
Polypropylene	0.5–1.9	0.36–0.40
PTFE	0.4–1.6	0.40–0.46
Polyurethanes	0.006–0.4	0.49

1.6. DETERMINATION OF THE ELASTIC MODULUS

- Mechanical tests: → In order to obtain the Young's modulus accurately, high precision experimental devices must be used.

- Natural frequency of vibration:



$$f = \frac{1}{2\pi}\left\{\frac{3\pi E d^4}{4 l^3 M}\right\}^{1/2} \qquad E = \frac{16\pi M l^3 f^2}{3 d^4}$$

- Speed of sound propagation: → $v_l = \left(\frac{E}{\rho}\right)^{1/2}$

1.6. DETERMINATION OF THE ELASTIC MODULUS

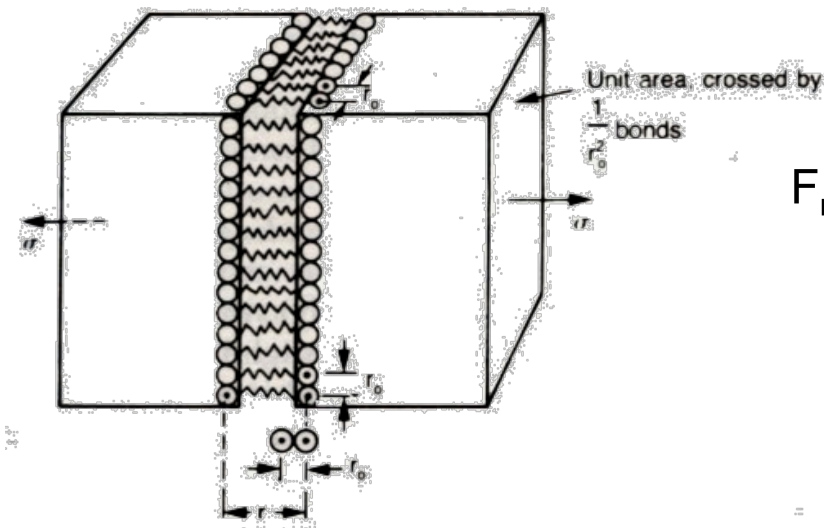
Suggested exercises:

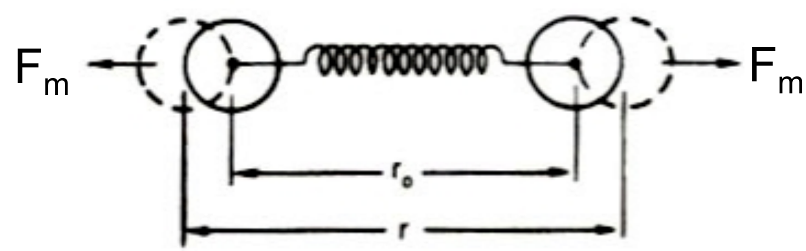
1) Demonstrate the expression to obtain the relation between the Young's modulus and the natural frequency of vibration. It is suggested to obtain the expression of the natural frequency of vibration of a spring with a constant K, with a hanging mass M (harmonic oscillator) and, then, identify the constant K in the flexural stress beam system.

1.7. PHYSICAL BASIS OF THE ELASTIC MODULUS

KEY POINT: microstructural nature justifies macroscopic properties.

- Consequence: in crystalline materials, the nature of the chemical bond (ionic, covalent o metallic) justifies the Young's modulus as well as the linear behavior (in a restricted regime).

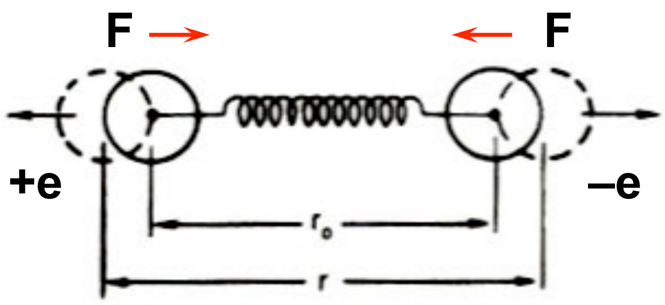




1.7. PHYSICAL BASIS OF THE ELASTIC MODULUS

PARTICULAR CASE: ionic bonding, electrostatic attraction.

Coulomb's law:



$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{(+e)(-e)}{r^2} = \frac{-e^2}{4\pi\varepsilon_0\, r^2}$$

Force-potential energy relationship:

$$\vec{F} = -\vec{\nabla}U = -\frac{dU}{dr} \quad \Rightarrow \quad U = \frac{-e^2}{4\pi\varepsilon_0\, r}$$

A repulsive potential is needed. Phenomenological approach:

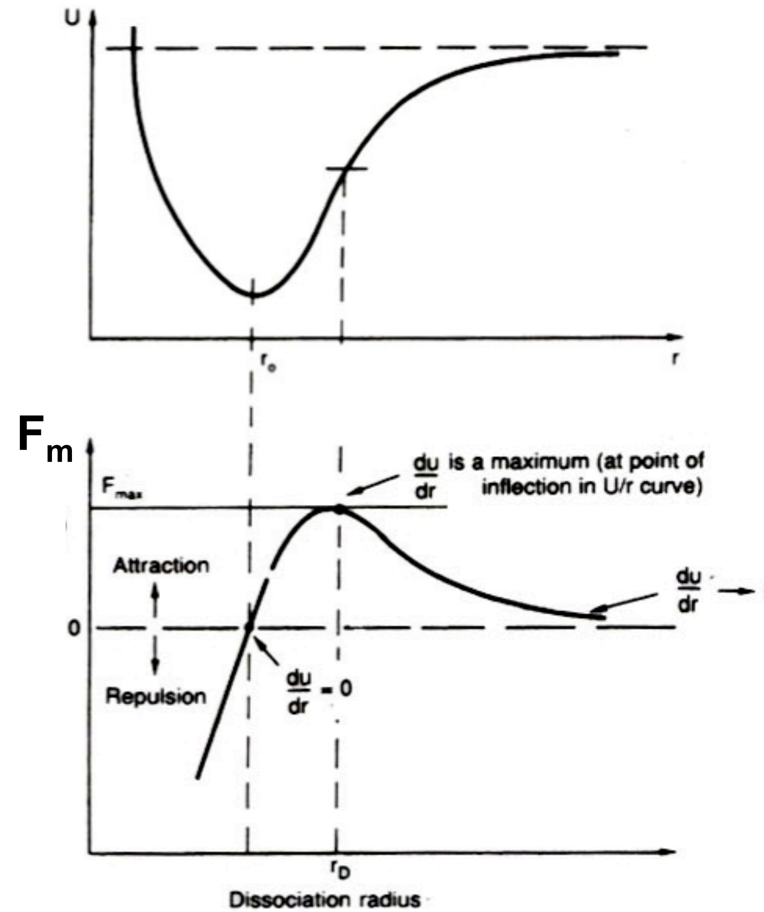
$$\boxed{U = -\frac{e^2}{4\pi\varepsilon_0\, r} + \frac{B}{r^m}}$$

B, m: constants that depend on the bond's nature.

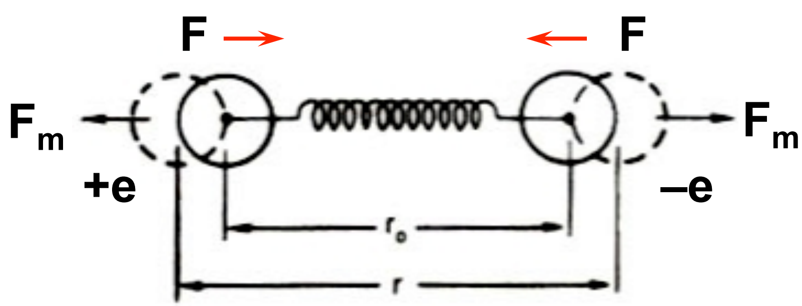
1.7. PHYSICAL BASIS OF THE ELASTIC MODULUS

EQUILIBRIUM CONDITION:

$$\vec{F} = -\left(\frac{dU}{dr}\right)_{r=r_0} = 0$$

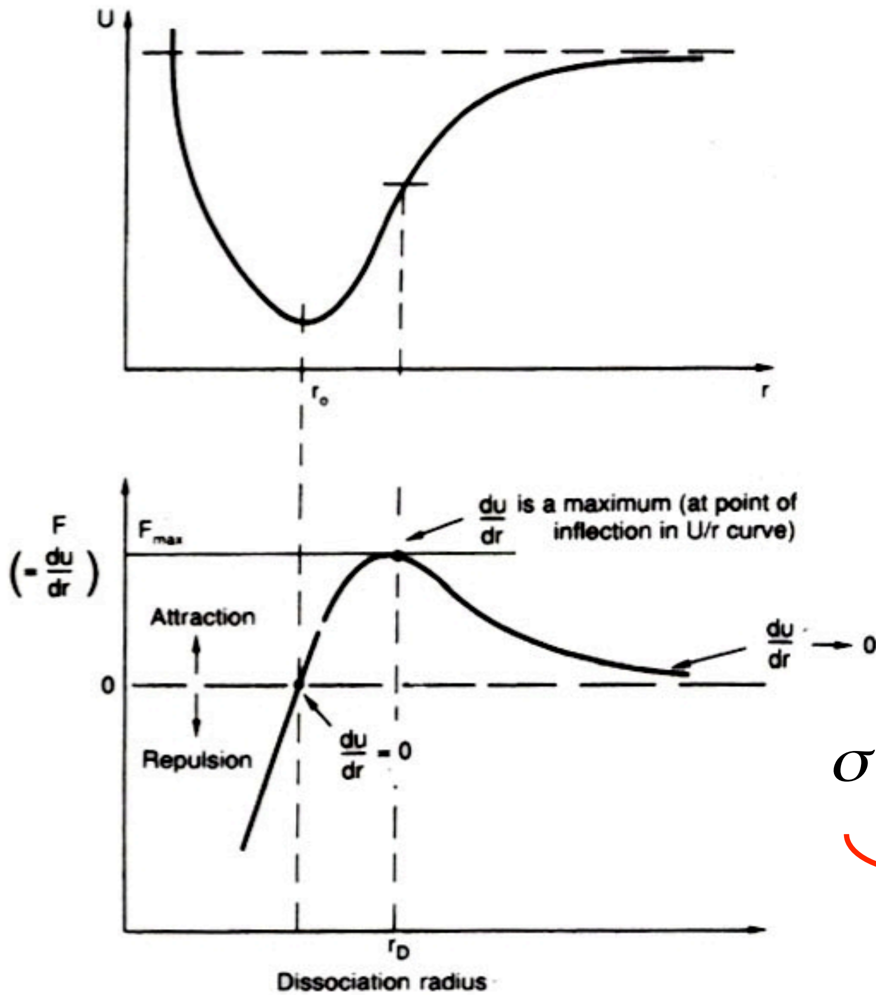


APPLICATION OF FORCES:



$$\vec{F}_m = -\vec{F} = \frac{dU}{dr}$$

1.7. PHYSICAL BASIS OF THE ELASTIC MODULUS



Near the equilibrium position:

$$\vec{F}_m = s_0 \left(r - r_0 \right)$$

Stress and strain:

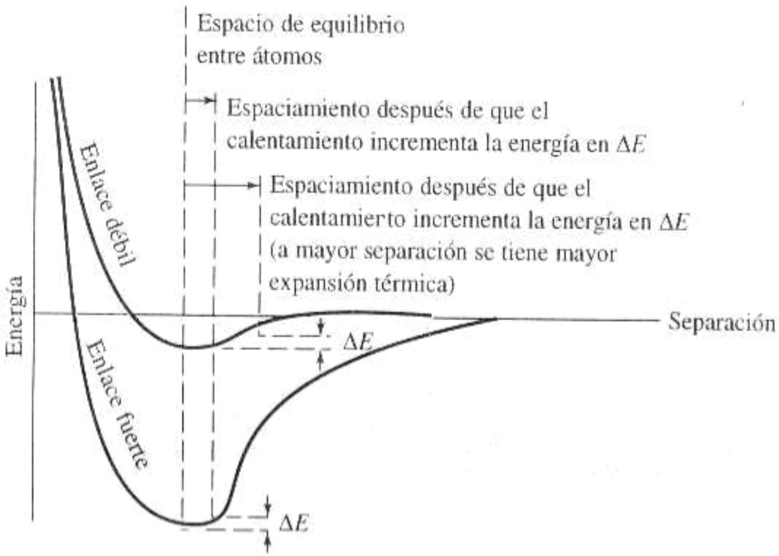
$$\sigma = \frac{F_m}{r_0^2} \qquad \varepsilon = \frac{r - r_0}{r_0}$$

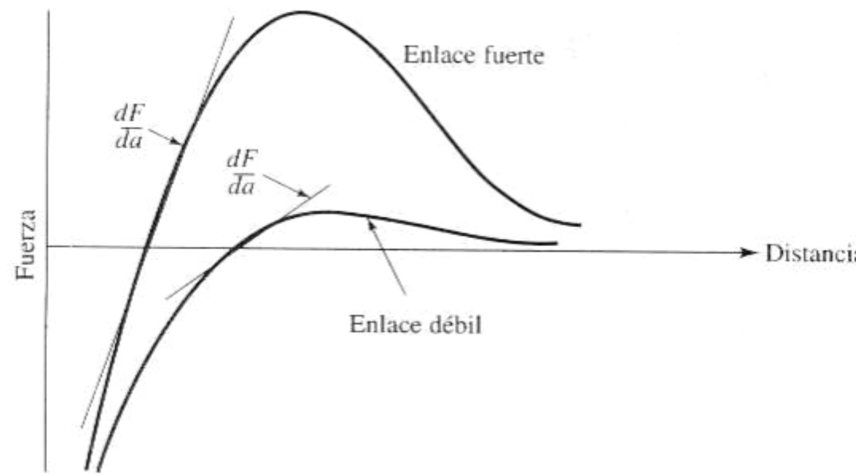
$$\sigma = \frac{F_m}{r_0^2} = \frac{s_0 \left(r - r_0 \right)}{r_0^2} = \frac{s_0}{r_0} \frac{r - r_0}{r_0} = \frac{s_0}{r_0} \varepsilon$$

$$\boxed{E = \frac{s_0}{r_0}}$$

1.7. PHYSICAL BASIS OF THE ELASTIC MODULUS

Bond energy and elastic behavior:

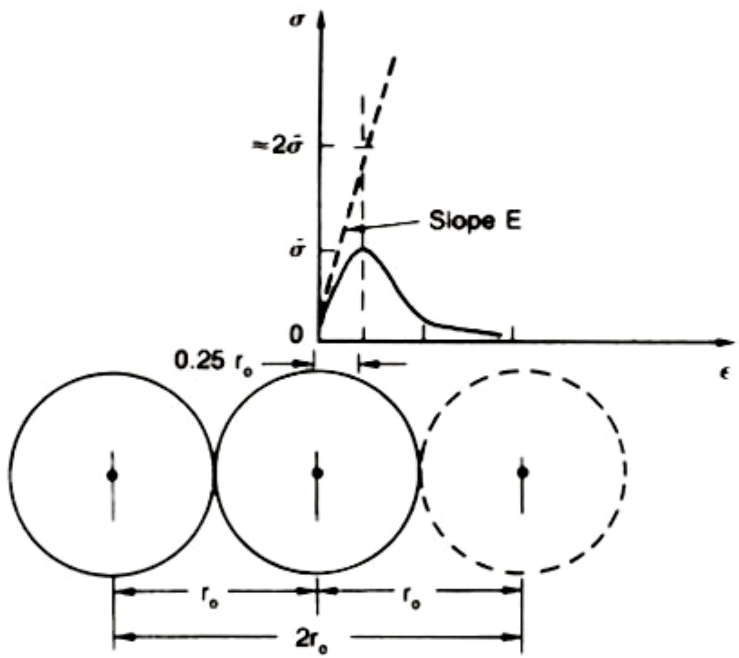




RELATIONSHIP BETWEEN PROPERTIES AND BOND TYPES				
Property	**Bond type**			
	Metallic	**Ionic**	**Covalent**	**Secundary**
Bond energy	High (3)	Really high(1)	Really high(2)	Really small
Melting point	High (3)	High (1,2)	High (1,2)	Really low (4)
Elastic modulus	High (2)	High (1)	High (1)	Really small

1.8. ELASTIC BEHAVIOUR LIMITS

Elastic behavior limit: DEFECTS



$$\boxed{\sigma_{ultimate} \approx E / 10}$$

But, before that:

Brittle fracture

Plastic deformation

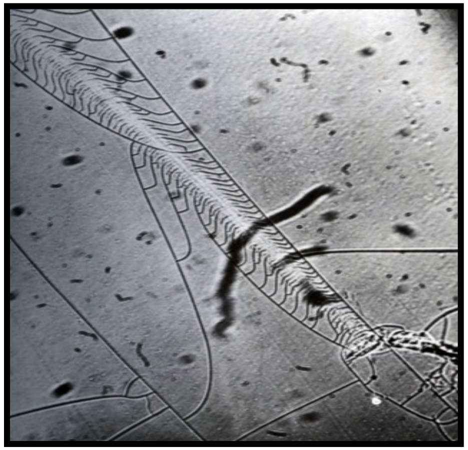
1.8. ELASTIC BEHAVIOUR LIMITS

Elastic behavior limit

Actually: $\sigma_{E.L.} << E/10$

I: Brittle fracture

Due to discontinuities in the material.



Inherent in the material (brittle behavior).

II: Plastic behavior

Some materials flow from a certain tensional state (Yield stress, s_y) leading to permanent deformations even before the material breaks (ductile behavior).