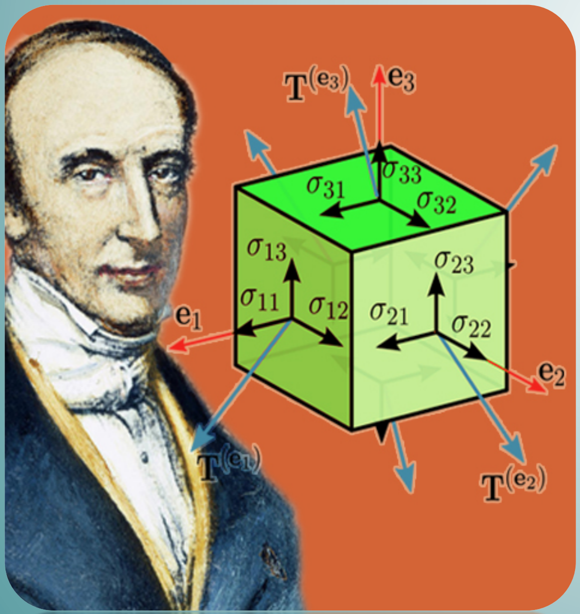
Mechanical Properties of Materials, Processing and Design

Topic 4. Fracture mechanics



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4.1. INTRODUCTION

WHY IS FRACTURE MECHANICS IMPORTANT?

- Because things break! (and, normally, no one wants this to occur).
- Estimated costs associated with fracture of components and structures in the USA is of about **4% of its GDP**.
- Our knowledge on fracture processes has advanced significantly since World War II and, particularly, since the mid-70s.

WHY DO STRUCTURES / COMPONENTS BREAK?

a) **Negligence** during the design phase, construction or operating conditions.

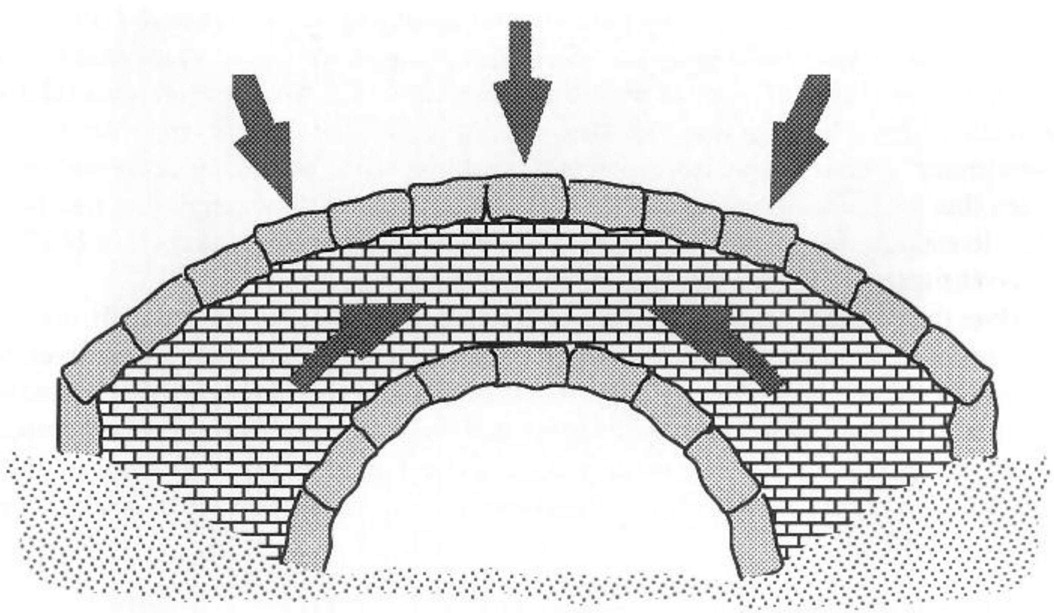
b) Application of a **new design**, of a new manufacturing technique or a new material that produces an unexpected (and unwanted) result.

- The current state of knowledge can help sorting out situation **a)** and address situation **b)**.

4.1. INTRODUCTION

HISTORICAL PERSPECTIVE:

- **Stone** materials: risk of breaking in structures under tensile / bending stress.
- Solution: to design looking for compressive stress states (**antifunicularity**).
- Metals (**steel** in particular) show a good behavior under tensile conditions; nevertheless, they can experience fracture.



4.1. INTRODUCTION

LIBERTY SHIPS (1943):

- In the beginning of World War II, German navy was able to destroy British war ships three times faster than these were built.

- USA collaborated with Great Britain by supplying the so called Liberty war ships. Unlike traditional boats, made with riveted plates, Liberty ships were monolithic structures completely welded.

4.1. INTRODUCTION

LIBERTY SHIPS (1943):

- Everything worked well until December 1943, when a Liberty ship was sailing between Siberia and Alaska…
- 400 ships, out of 2400, experienced fractures of various kinds, 90 of them very serious. 20 of the ships halved spontaneously.
- All this was due to a ‘slight’ change in the manufacturing process.
- **CAUSES:** brittle welds, with the presence of defects. Embrittlement effect of the low temperatures.

4.1. INTRODUCTION

FLYING ACCIDENT OF THE ALOHA AIRLINES B737 (1988):

- Partial detachment of the fuselage during the flight.
- One of the air hostesses dissapeared.
- **CAUSES:** processes of **fatigue** leading to **fracture** in the **junction of the fuselage**.

4.1. INTRODUCTION

PRESTIGE ACCIDENT (2002):

- 77000 Tm of fuel were spilled.
- **CAUSES:** accumulation of damage due to **thermal and mechanical fatigue** that led to the total fracture of the ship. (Standing Commission for Maritime Accident and Incident Investigations).

4.1. INTRODUCTION

CHALLENGER ACCIDENT(1986):

- Death of the 7 members of the crew. Stopping of the Space Program.
- **CAUSES**: o-rings malfunction due to the low temperatures at the time of launch (Rogers Comision).
- The flight had been delayed up to 7 times, due to the low temperatures.
- According to Feynman, reliable forecast of the NASA were sometimes up to a factor of one thousand different of what engineers estimated.
- ‘For a successful technology, reality must take precedence over public relations, for nature cannot be fooled’.
- Feynman also did a demonstration on TV about the thermal embrittlement of the O – rings.

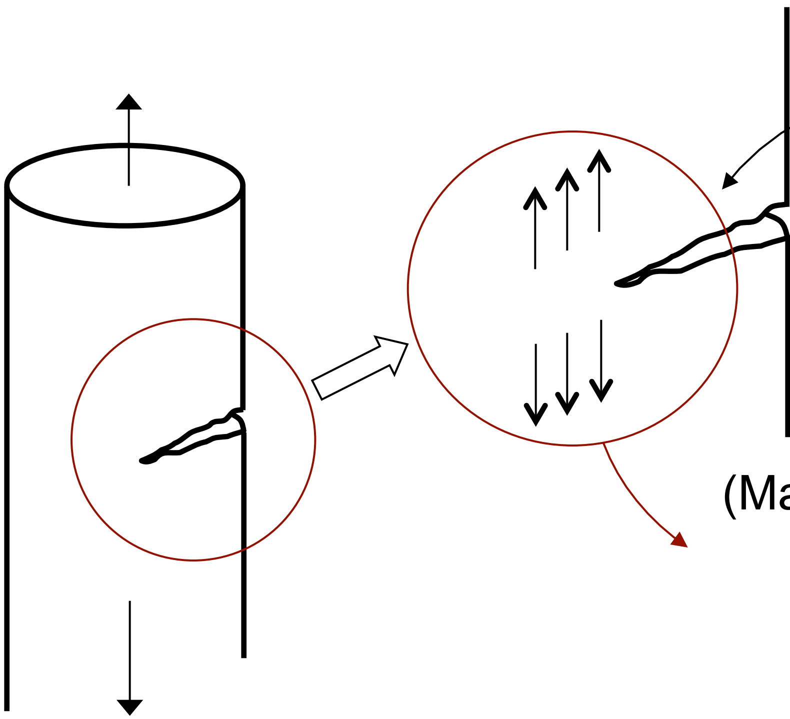
4.1. INTRODUCTION

KEY IDEAS OF FRACTURE MECHANICS:

- Final break (FRACTURE) of materials (or structural components) is associated with the presence of **defects** subjected to a local stress state (from the external solicitation / load).
- Fracture occurs when the combination (**Defect + Local stress state**) reaches a critical condition that depends on the material (**Fracture toughness**).

4.1. INTRODUCTION

KEY IDEAS OF FRACTURE MECHANICS:



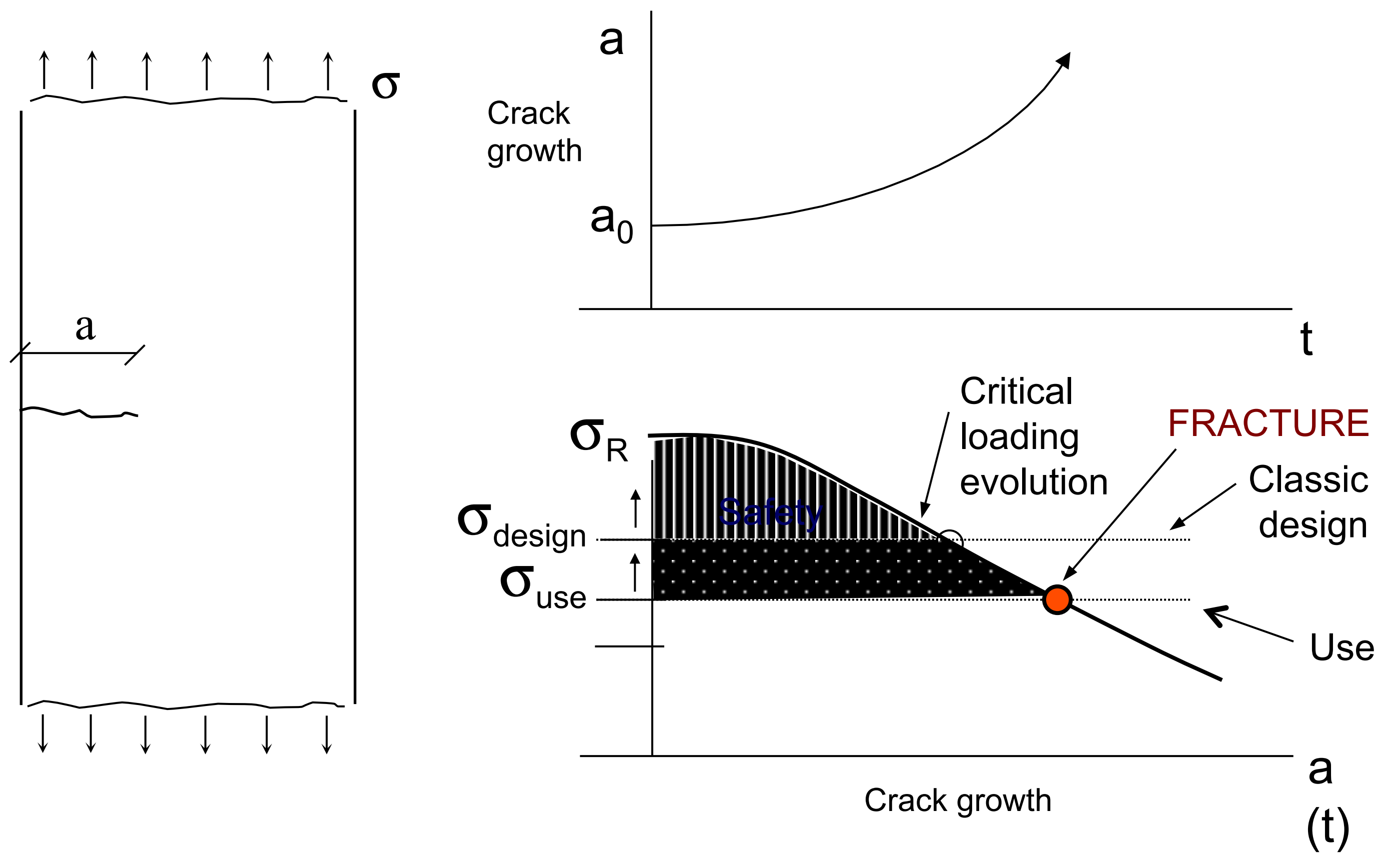
Crack (defect)

- Processing.
- Cracking processes.
- Operation damage.

(Material + defects) + Loading **Vs.** Critical local condition to fracture. (**Toughness**)

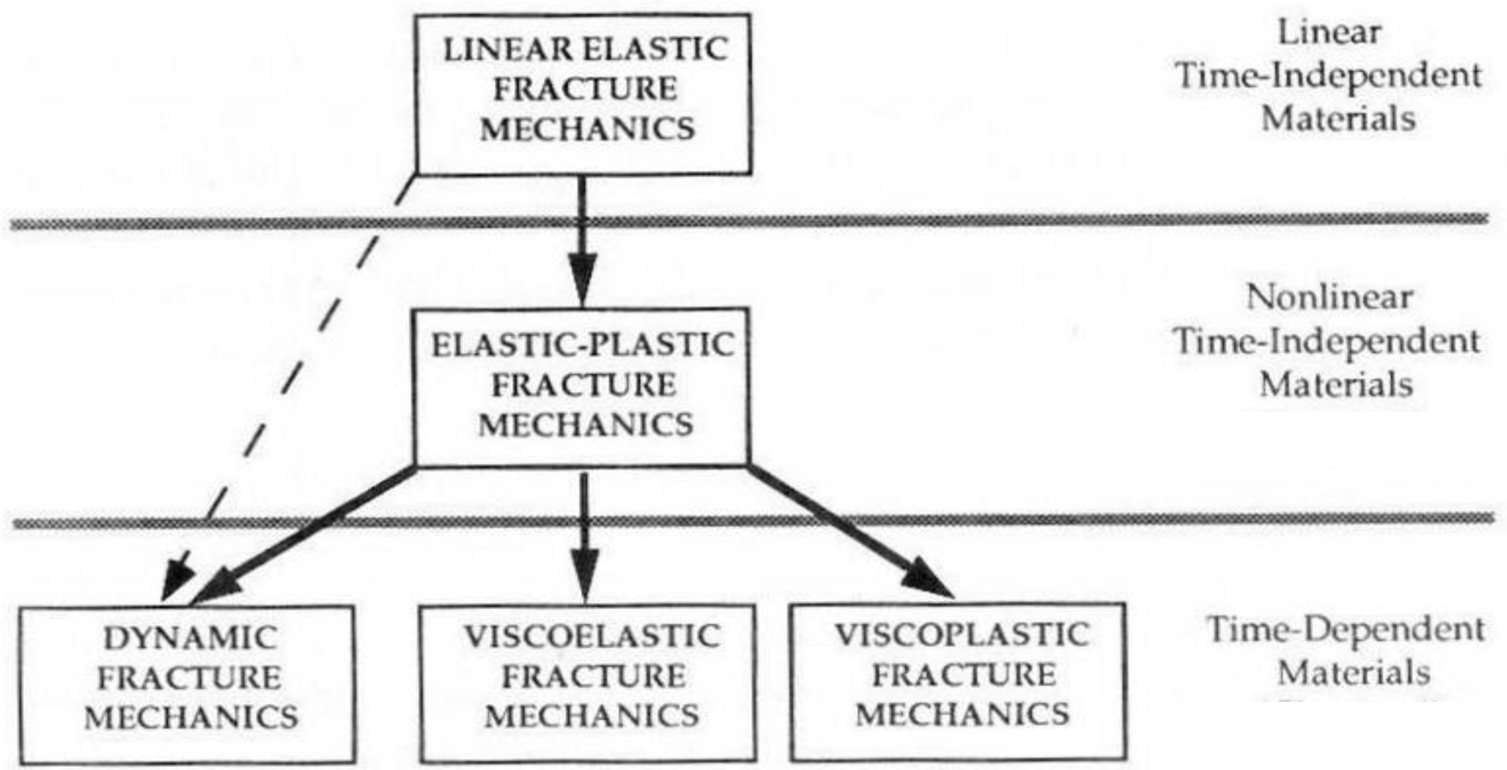
4.1. INTRODUCTION

KEY IDEAS OF FRACTURE MECHANICS:



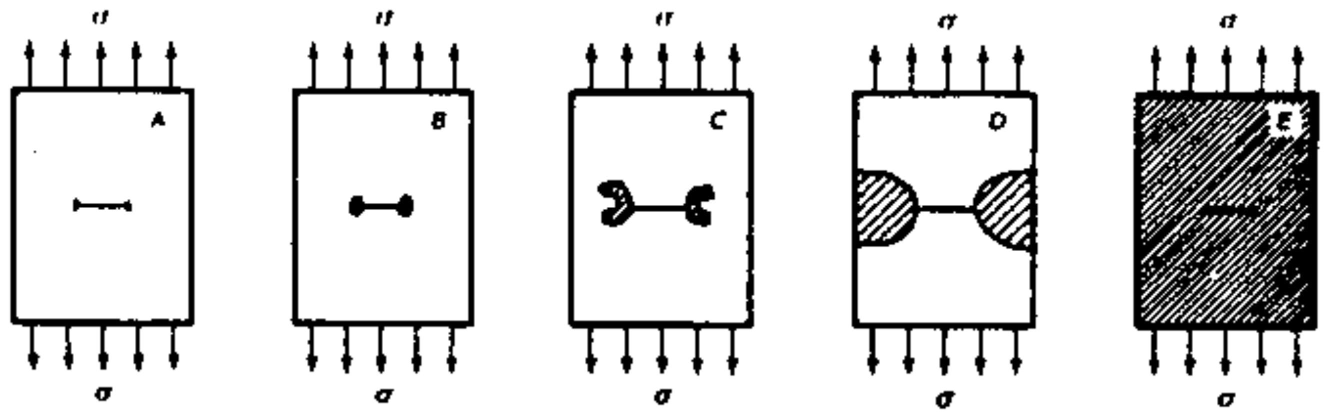
4.1. INTRODUCTION

MECHANICAL STRUCTURE OF FRACTURE MECHANICS:

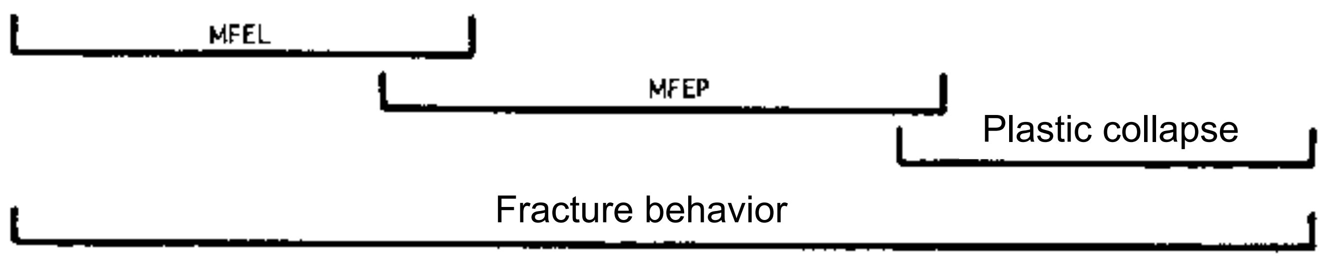


4.1. INTRODUCTION

TYPES OF FRACTURE BEHAVIOR:



Brittle material in plane strain.	Brittle material in plane stress.	More ductile material in plane stress or strain.	Ductile material under large yielding.	Ductile material with completely yielded.



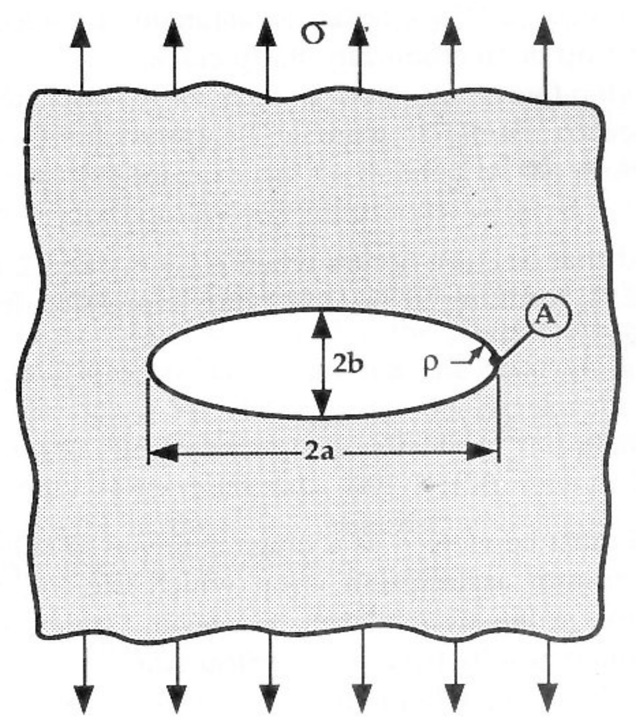
4.2. STRESS STATE IN THE CRACK TIP

- Fracture is a consequence of the stress concentration produced by discontinuities in the material ("defects").
- **Example:** fracture of a paper sheet with a small crack.
- **Inglis (1913)** studied the stress concentration effect of an elliptical drill in an infinite plane plate, in **linear elastic regime**.

$$\left.\begin{aligned} \sigma_A &= \sigma\left(1+\frac{2a}{b}\right) \\ \rho &= \frac{b^2}{a} \end{aligned}\right\} \quad \sigma_A = \sigma\left(1+2\sqrt{\frac{a}{\rho}}\right)$$

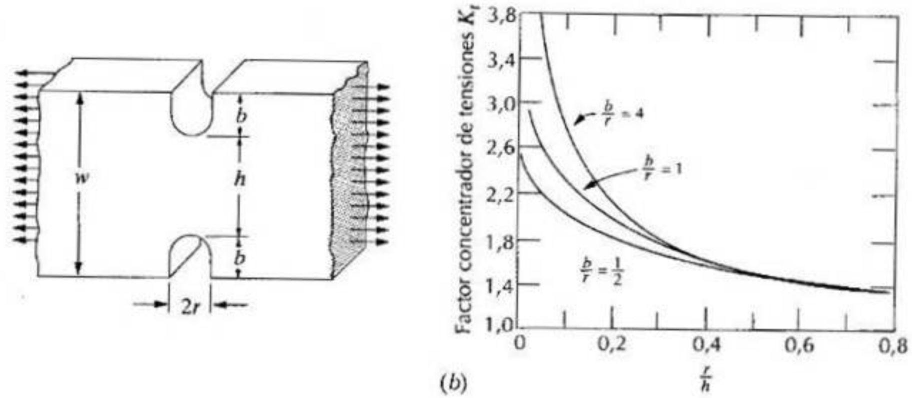
$$a >> \rho \quad \Rightarrow \quad \sigma_A \approx 2\sigma\sqrt{\frac{a}{\rho}}$$

$$(\rho \to 0) \quad \Rightarrow \quad (\sigma_A \to \infty)$$



4.2. STRESS STATE IN THE CRACK TIP

- This fact can be generalized to other geometries.
- Notches represent concentrated stress states (distinguish between **remote stress** and **local stress**).
- Fracture happens when the local stress state in the proximity of a defect exceeds a certain threshold, characteristic of the material (**fracture toughness of the material**).



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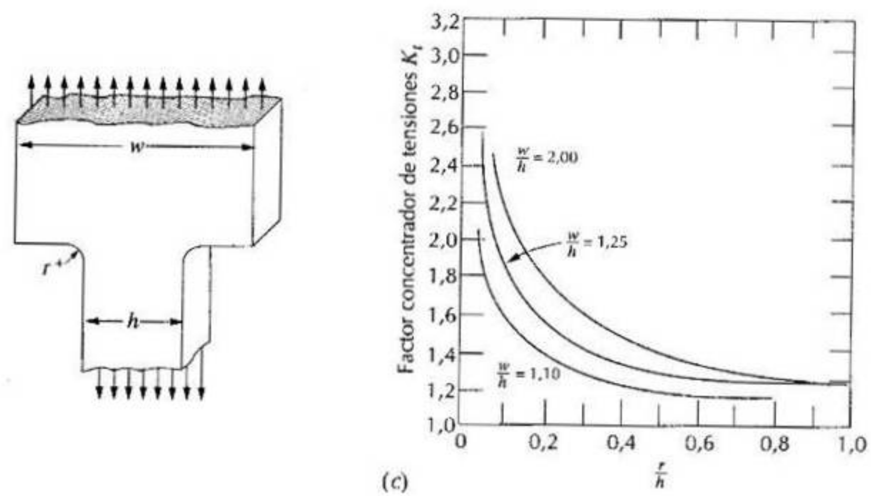


(c)

4.2. STRESS STATE IN THE CRACK TIP

Suggested exercises:

1) A large plate is subjected to a tensile stress of 100 MPa. By applying Inglis model, calculate the stress in the tip of an elliptical defect with a total lengh of 10 mm if the radius of curvature is 10^{-3} mm.

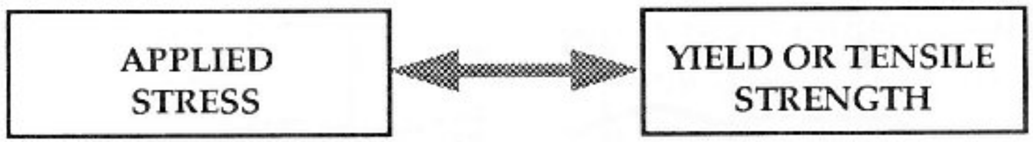
Solution:

$$\sigma_A \approx 2\sigma\sqrt{\frac{a}{\rho}} = 2 \cdot 100\sqrt{\frac{10}{10^{-3}}} = 20000\ MPa$$

Note: ultimate strength of a high strength steel is that of ‘only’ 2000 MPa.

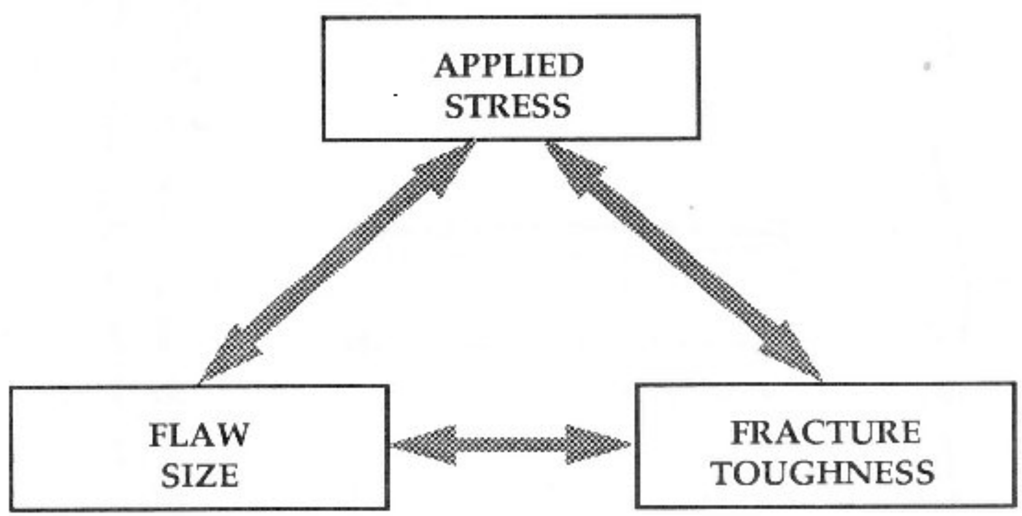
4.3. CLASSIC APPROACH VS. FRACTURE APPROACH

Classic approach (Strength of Materials)



(a) The strength of materials approach.

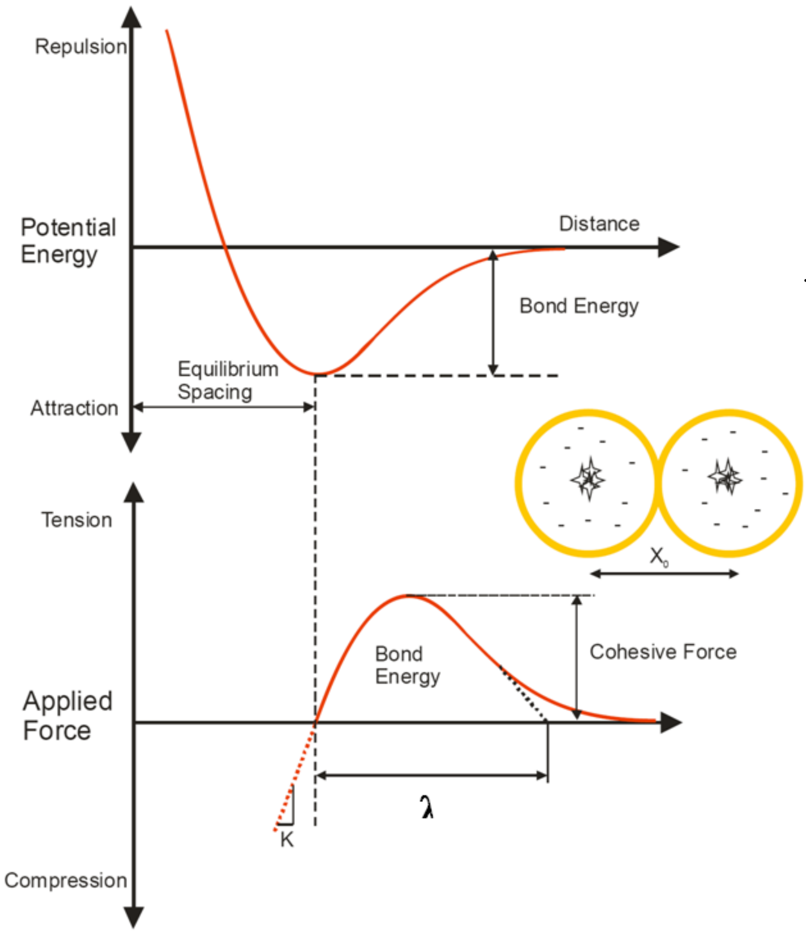
Fracture Mechanics approach



(b) The fracture mechanics approach

4.4. FRACTURE FROM THE ATOMIC POINT OF VIEW

- At atomic level (monocrystal), fracture implies atomic bonding breakage.
- We can get a first estimation of the fracture strength:



$$\approx F_c \, sen\left(\frac{\pi x}{\lambda}\right)$$

$$\left.\frac{dF}{dx}\right|_{x=x_0} = k$$

$$F_c \frac{\pi}{\lambda} = k$$

$$E = \frac{k}{x_0} = \frac{F_c \frac{\pi}{\lambda}}{x_0} = \frac{F_c}{x_0^2} \frac{\pi x_0}{\lambda} = \sigma_c \frac{\pi x_0}{\lambda}$$

$$\sigma_c = \frac{E \lambda}{\pi x_0} \approx \frac{E}{\pi}$$

4.4. FRACTURE FROM THE ATOMIC POINT OF VIEW

Suggested exercises:

1) Estimate the critical stress for a steel, σ_c, using a typical value for the Young's modulus.

Solution:

$$\sigma_c \approx \frac{E}{\pi} = \frac{200\ GPa}{\pi} \approx 70\ GPa$$

- This value is, at least, 25 times the strength of the best steel.
- ***The inevitable existence of many defects is the major reason for this important discrepancy.***
- Although there are defects at all scales (from microdefects to macroscopic discontinuities), Fracture Mechanics term is applied to the study of sharp (macro) cracks.
- Macrocracks are in the material as a result of the **manufacturing process** (e.g. welding) or develop due to **service conditions** (fatigue, creep).

4.5. FRACTURE CRITERIA (LEFM)

Two approaches:

- STRESS APPROACH
- ENERGETIC APPROACH

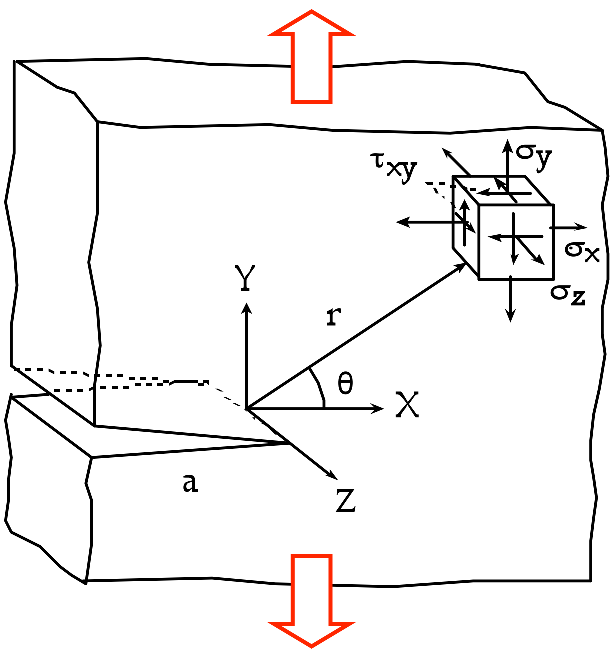
→ EQUIVALENCE IS DEMONSTRATED

Goals:

- To obtain the conditions that determine the final fracture.
- To identify and interpret the material fracture toughness (property that express its resistance to fracture in the presence of defects).

4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

- Perfect linear-elastic material.
- Solid crack (**perfect crack**), subjected to a tensile remote stress state with a polar reference system in the crack tip.



Westergaard, Irwin, Sneddon and Williams:

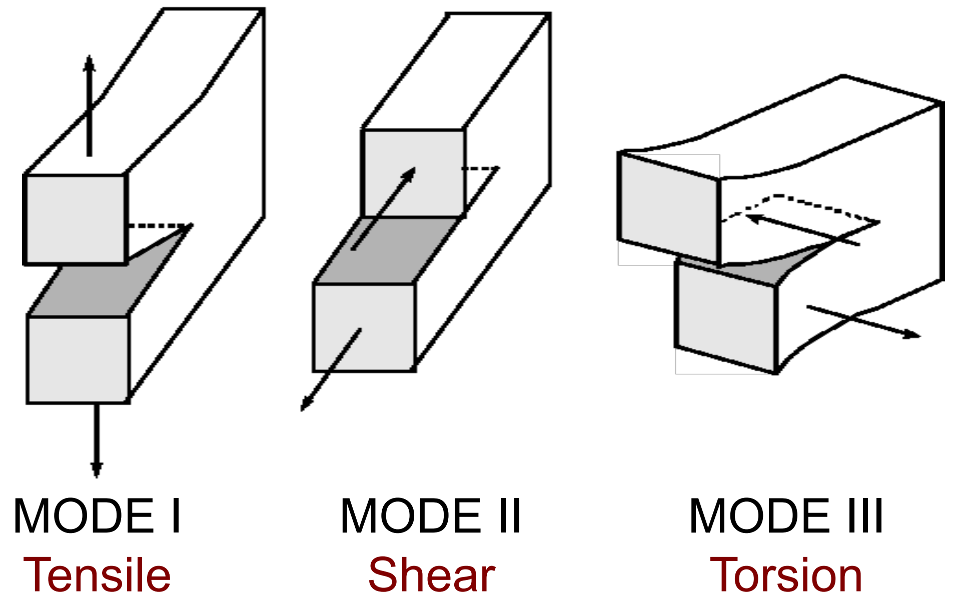
$$\sigma_{ij} = \frac{k}{\sqrt{r}} f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{m/2} g_{ij}^{(m)}(\theta)$$

$$\lim_{r \to 0} \sigma_{ij} = \frac{k}{\sqrt{r}} f_{ij}(\theta) \Rightarrow \infty$$

The model assumes a perfect crack and a perfect linear-elastic material (which implies an idealization of reality).

4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

FRACTURE MODES:



$$\lim_{r \to 0} \sigma_{ij}^{(I)} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{(I)}(\theta)$$

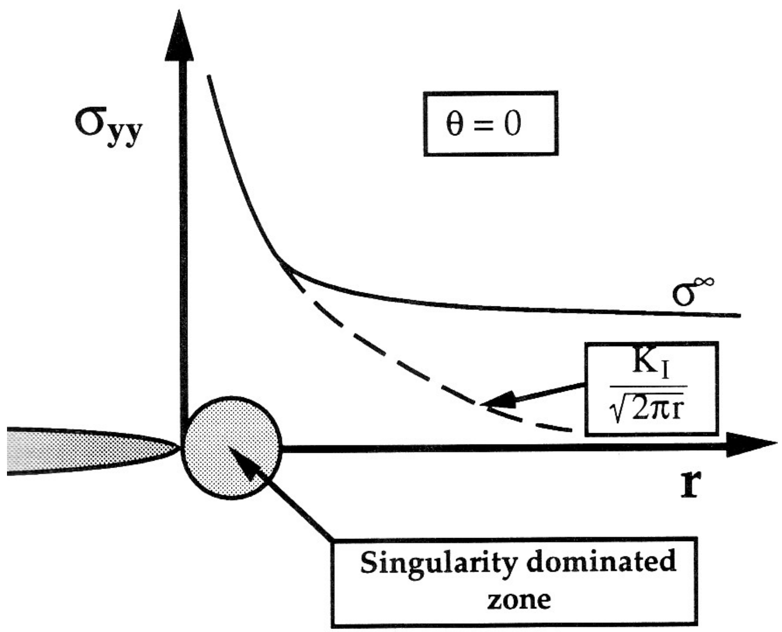
$$\lim_{r \to 0} \sigma_{ij}^{(II)} = \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{(II)}(\theta)$$

$$\lim_{r \to 0} \sigma_{ij}^{(III)} = \frac{K_{III}}{\sqrt{2\pi r}} f_{ij}^{(III)}(\theta)$$

$$\sigma_{ij} = \sigma_{ij}^{(I)} + \sigma_{ij}^{(II)} + \sigma_{ij}^{(III)}$$

- K_I, K_{II}, K_{III}: **Stress Intensity Factors (SIF).**
- SIF determines the stress state in the proximity of the crack front.

4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

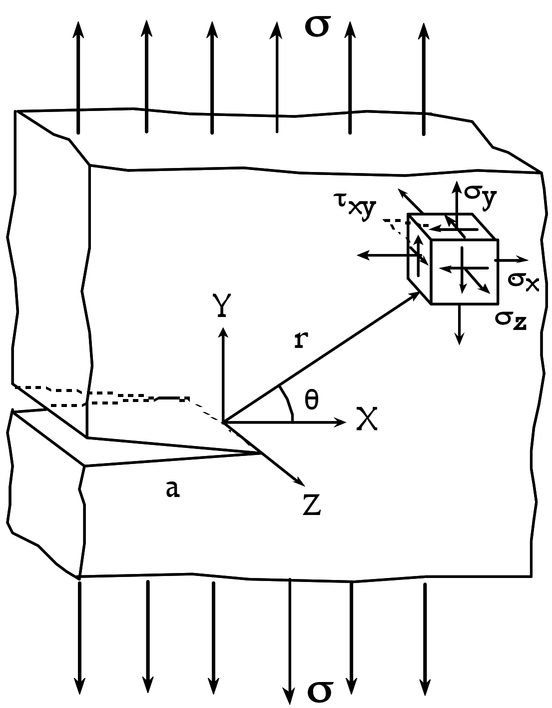


$$\lim_{r \to 0} \sigma_{yy}^{(I)} = \frac{K_I}{\sqrt{2\pi r}}$$

- The singularity r $^{-1/2}$ only dominates the stress fields in the proximity of the crack front (**Singularity dominated zone**).
- At a greater distance, the stress state depends on the remote stresses (participation of the other terms of the Williams series expansion).

4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

FORMULATION: LEFM



STRESSES:

$$\sigma_x = \sigma\sqrt{\frac{a}{2r}}\left[\cos\frac{\theta}{2}\left(1 - sen\frac{\theta}{2}\, sen\frac{3\theta}{2}\right)\right]$$

$$\sigma_y = \sigma\sqrt{\frac{a}{2r}}\left[\cos\frac{\theta}{2}\left(1 + sen\frac{\theta}{2}\, sen\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \sigma\sqrt{\frac{a}{2r}}\left[\cos\frac{\theta}{2}\, sen\frac{\theta}{2}\, \cos\frac{3\theta}{2}\right]$$

$\sigma_z = 0$ **(TP)** $\quad\quad \sigma_z = \upsilon\left(\sigma_x + \sigma_y\right)$ **(DP)**

DISPLACEMENTS:

$$u = \frac{\sigma}{2E}\sqrt{\frac{ar}{2}}\left(1+\nu\right)\left[\left(2\kappa - 1\right)\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right]$$

$$v = \frac{\sigma}{2E}\sqrt{\frac{ar}{2}}\left(1+\nu\right)\left[\left(2\kappa + 1\right) sen\frac{\theta}{2} - sen\frac{3\theta}{2}\right]$$

$$w = -\frac{\nu}{E}\int\left(\sigma_x + \sigma_y\right) dz$$

$\kappa = 3 - 4\nu$ **(TP)** $\quad\quad \kappa = \dfrac{3-\nu}{1+\nu}$ **(DP)**

4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

STRESS INTENSITY FACTOR:

- It completely defines the stress and strain stress in the proximity of the crack tip (singularity dominated zone).
- It depends on the remote stress (σ) and the characteristics of the deffect (a).

$$\lim_{r \to 0} \sigma_{ij}^{(I)} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{(I)}(\theta)$$

→ Units of SIF: MN · $m^{-3/2}$ = MPa · $m^{1/2}$

- The simplest, dimensionally valid, expression that can be proposed is:

$$K_I = Y\sigma\sqrt{\pi a}$$

Y: geometric factor.

- This type of expressions work well for very symmetrical geometries (infinite or semi-infinite plates).

4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

STRESS INTENSITY FACTOR:

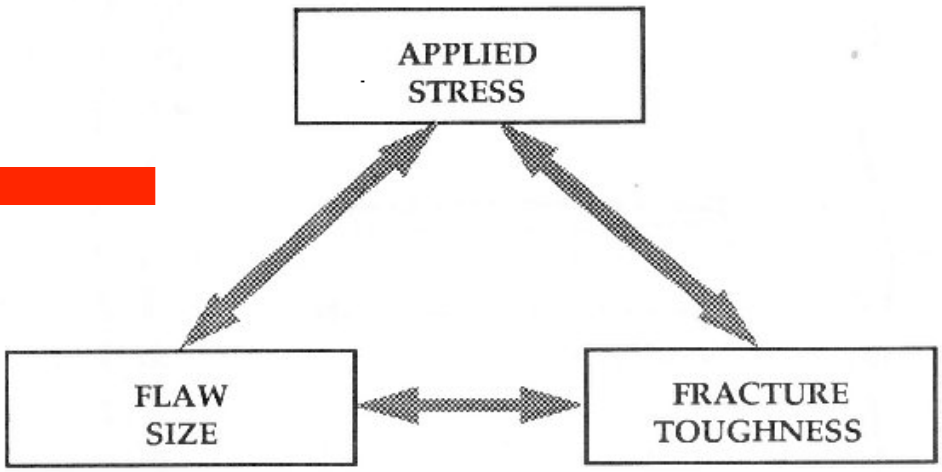
- Fracture will occur when the stress and/or strain states in the crack front (process area) reach a certain limit of the material.
- Despite not knowing well the already mentioned critical combination, and given that K_I controls the conditions in the process area, **fracture condition** will be:

$$K_I = K_{Ic}$$

K_I: Depends on the load state and the geometry of the defect.

K_{Ic}: Property of the material: **FRACTURE TOUGHNESS**.

$$K_I(\sigma, a) = K_{Ic}(material)$$

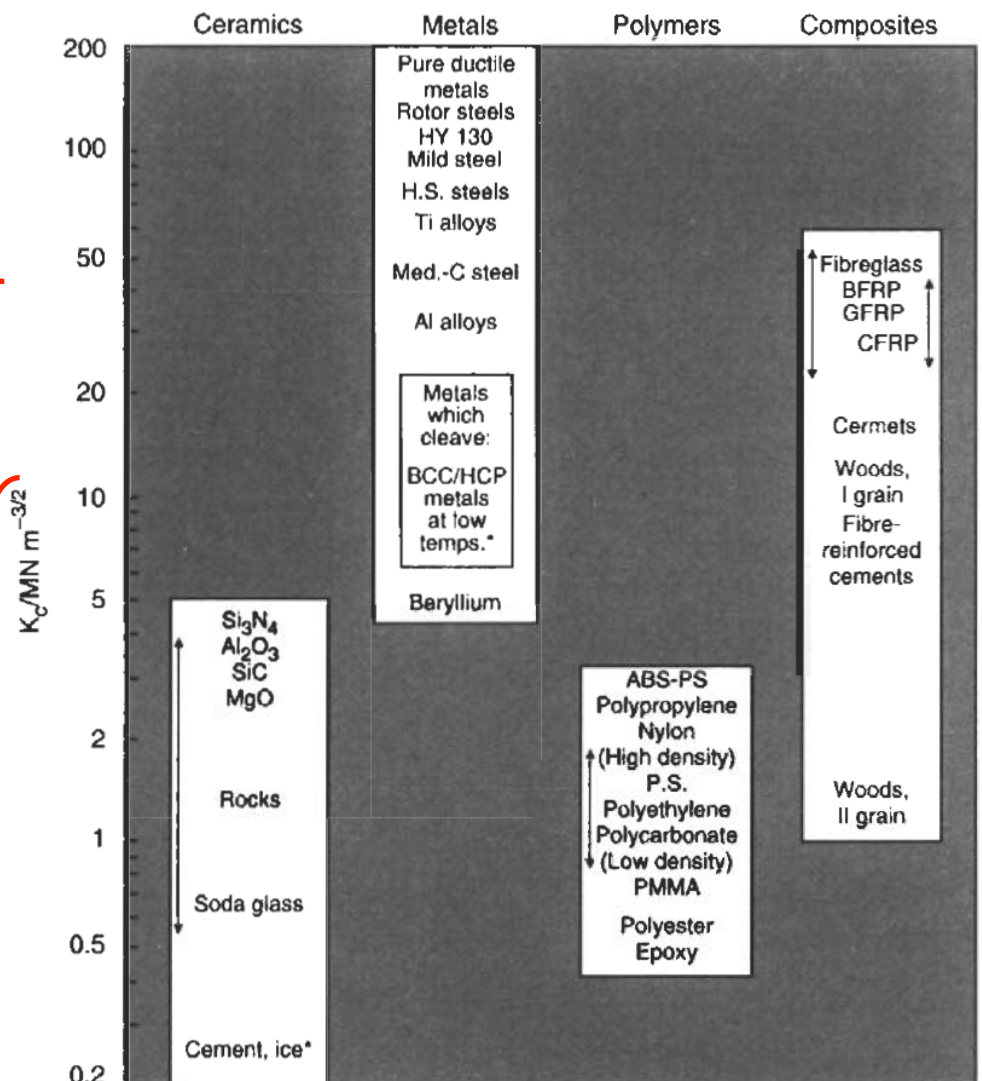


4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

Typical values of K_{Ic}:

TOUGH MATERIALS

BRITTLE MATERIALS



4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

Suggested exercise:

1) Obtain the maximum permissible crack in a semi-infinite plate (geometric factor Y = 1.12) made out of a steel with toughness K_{Ic} = 100 MPa · $m^{1/2}$, subjected to a stress state (mode I) σ = 100 MPa.

Solution:

$$K_I = Y\sigma\sqrt{\pi a} = K_{Ic} \qquad a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{100}{1.12\cdot 100}\right)^2 = 25\ mm$$

4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

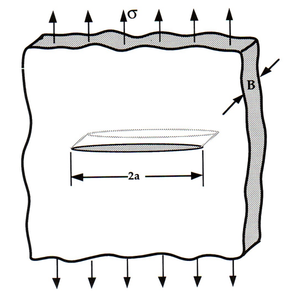
DETERMINATION OF THE STRESS INTENSITY FACTOR:

Various possibilities:

- Summaries of solutions.
- Obtaining the solution for an elastic problem with boundary conditions.
- Experimental techniques (extensometer, photoelasticity).
- Energetic methods.
- **Numerical methods (Finite Elements).**

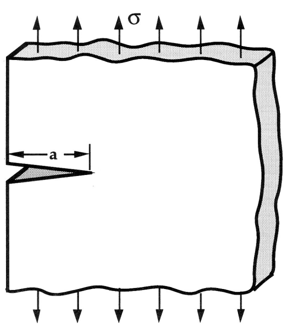
4.5. FRACTURE CRITERIA (LEFM): STRESS APPROACH

DETERMINATION OF THE STRESS INTENSITY FACTOR:

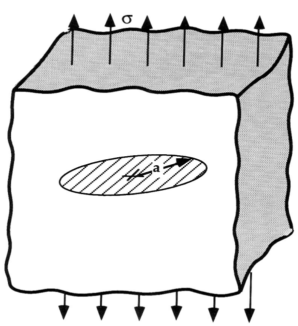


\ through-thickness crack in an infinitely wide plate subjected to a remot

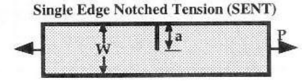
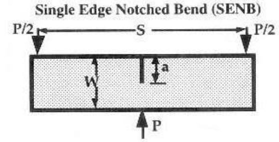
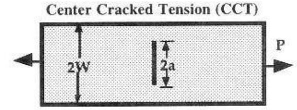
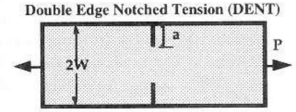
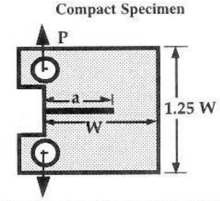
$$K_I = \sigma\sqrt{\pi a}$$



$$K_I = 1.12\sigma\sqrt{\pi a}$$



$$K_I = \frac{2}{\pi}\sigma\sqrt{\pi a}$$

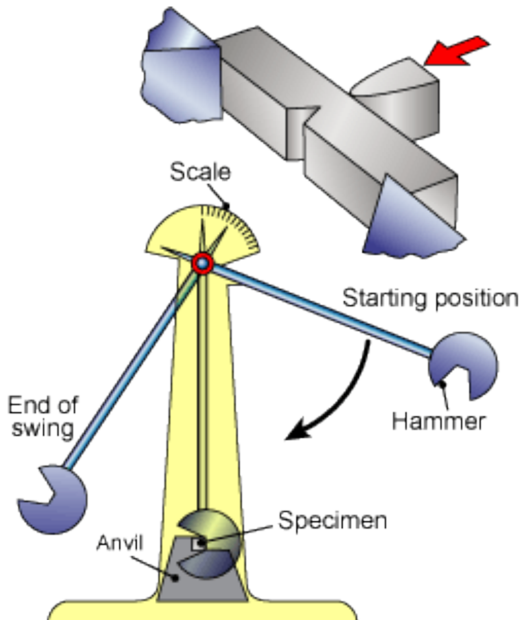
GEOMETRY	f(a/W)*
Single Edge Notched Tension (SENT) 	$\frac{\sqrt{2\tan\frac{\pi a}{2W}}}{\cos\frac{\pi a}{2W}}\left[0.752+2.02\left(\frac{a}{W}\right)+0.37\left(1-\sin\frac{\pi a}{2W}\right)^3\right]$
Single Edge Notched Bend (SENB) 	$\frac{3\frac{S}{W}\sqrt{\frac{a}{W}}}{2\left(1+2\frac{a}{W}\right)\left(1-\frac{a}{W}\right)^{3/2}}\left[1.99-\frac{a}{W}\left(1-\frac{a}{W}\right)\left\{2.15-3.93\left(\frac{a}{W}\right)+2.7\left(\frac{a}{W}\right)^2\right\}\right]$
Center Cracked Tension (CCT) 	$\sqrt{\frac{\pi a}{4W}\sec\frac{\pi a}{2W}}\left[1-0.025\left(\frac{a}{W}\right)^2+0.06\left(\frac{a}{W}\right)^4\right]$
Double Edge Notched Tension (DENT) 	$\frac{\sqrt{\frac{\pi a}{2W}}}{\sqrt{1-\frac{a}{W}}}\left[1.122-0.561\left(\frac{a}{W}\right)-0.205\left(\frac{a}{W}\right)^2+0.471\left(\frac{a}{W}\right)^3+0.190\left(\frac{a}{W}\right)^4\right]$
Compact Specimen 	$\frac{2+\frac{a}{W}}{\left(1-\frac{a}{W}\right)^{3/2}}\left[0.886+4.64\left(\frac{a}{W}\right)-13.32\left(\frac{a}{W}\right)^2+14.72\left(\frac{a}{W}\right)^3-5.60\left(\frac{a}{W}\right)^4\right]$

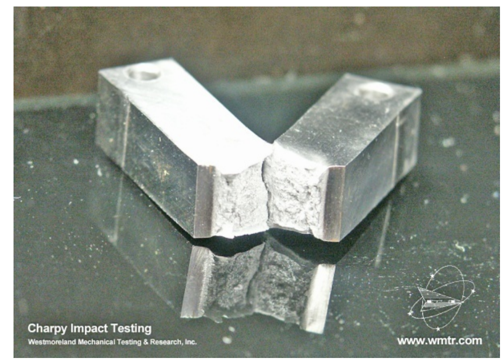
* $K_I = \frac{P}{B\sqrt{W}} f(a/W)$ where B is the specimen thickness.

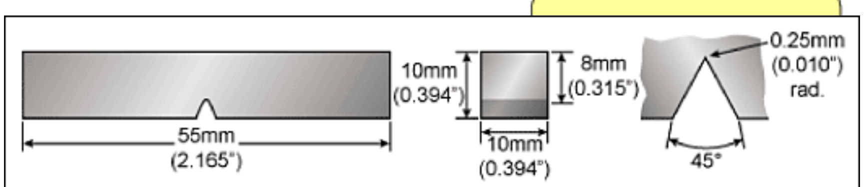
4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

- **HISTORICALLY:** charpy impact test.
- **CURRENTLY:** LEFM and EPFM own tests.

CHARPY IMPACT TEST:







4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

CHARPY IMPACT TEST:

TEST RESULTS:

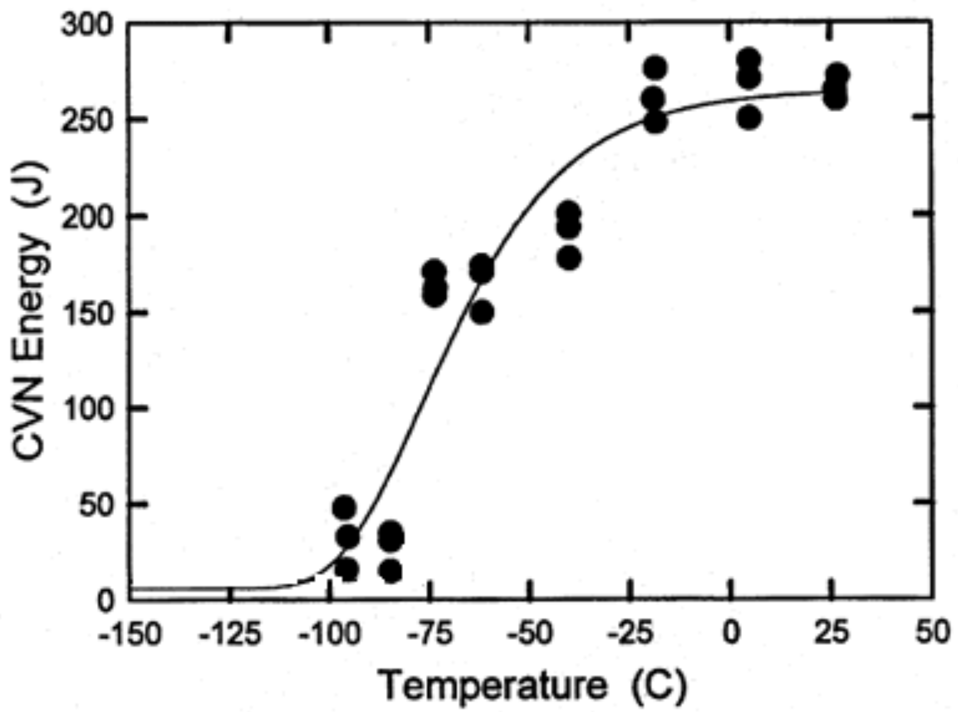
- The main result of the test is the **energy absorbed** by the specimen; **resilience** represents the energy absorbed per unit of cross section.
- Other relevant results are the fraction of ductile / brittle fracture or the lateral expansion of the specimen.
- It is regularly used to determine the influence of temperature on toughness.

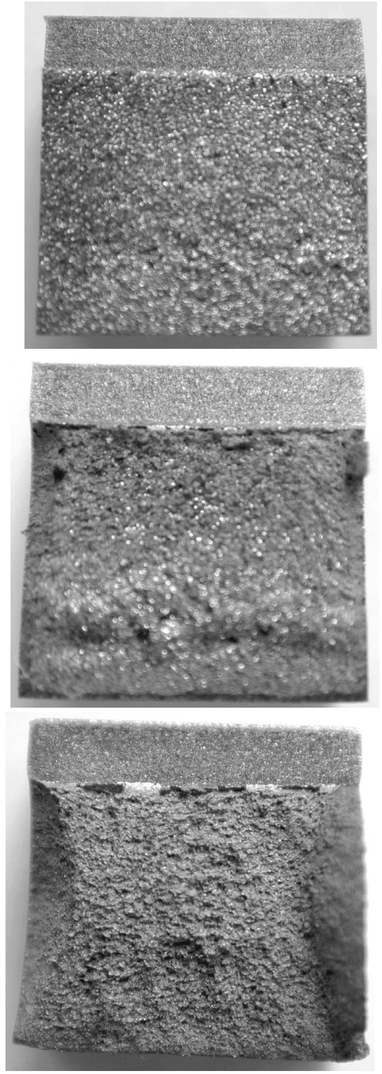
4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

CHARPY IMPACT TEST:

TEST RESULTS:

- Influence of temperature on toughness:



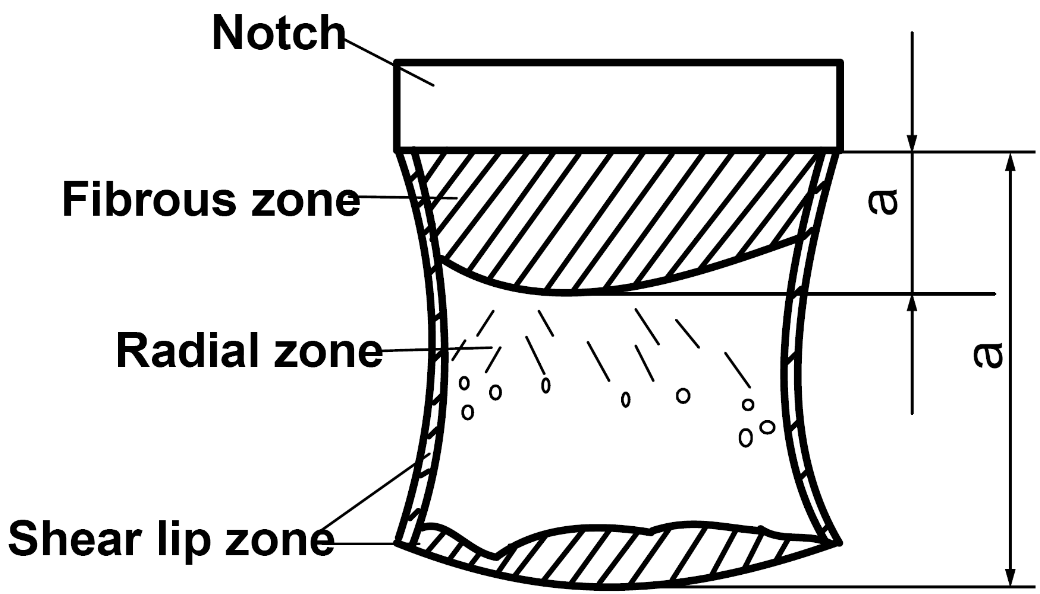


4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

CHARPY IMPACT TEST:

TEST RESULTS:

• Influence of temperature on toughness:



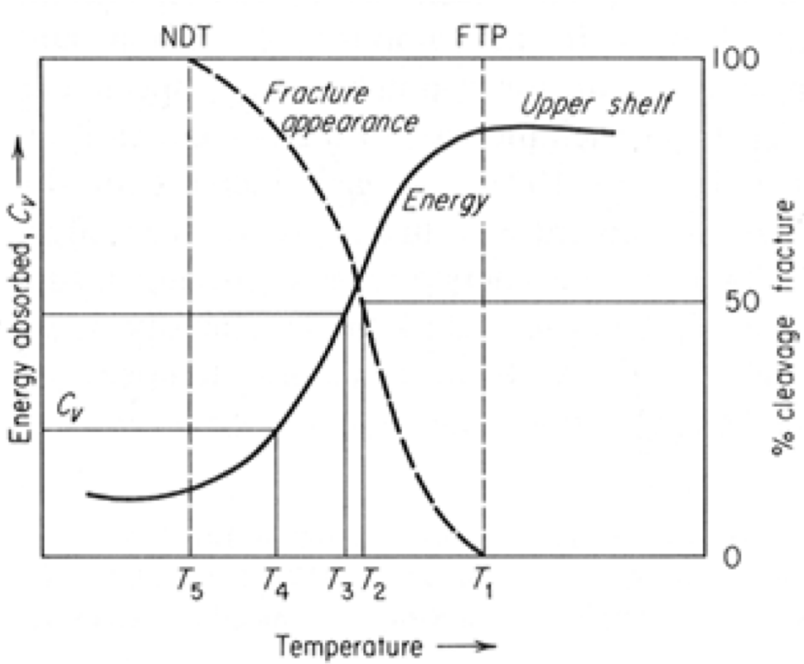
A sketch of the appearance of fractured surface of impact specimen after Charpy test

4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

CHARPY IMPACT TEST:

TEST RESULTS:

- Influence of temperature on toughness:



✓ T_1: Conservative, above T_1 fracture is 100% fibrous. *Fracture Transition Plastic* (FTP) very demanding.

✓ T_2: 50% cleavage - 50% ductile *Fracture Appearance Trans. Temp*. (FATT).

✓ T_3: Average of upper and lower shelf values. (often approx = T_2)

✓ T_4: Arbitrary value of energy absorbed, (CVN) e.g. 20 J (15 ft.lb) for low strength ship steel. *Ductility Transition Temp*.

✓ T_5: 100% cleavage fracture. *Nil Ductility Temperature* (NDT)

4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

CHARPY IMPACT TEST:

REMARKABLE CHARACTERISTICS:

- The test was developed in 1905 by French engineer Georges Charpy.
- Widely used by industry (construction, naval, nuclear, etc.) because it is simple to prepare, quick and inexpensive to perform.
- Test with a dynamic nature (impact).
- Notched, not precracked specimen.
- The major drawback is that the information obtained only can be used in comparative terms.

4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

Designation: E 399 – 90 (Reapproved 1997)

Standard Test Method for
Plane-Strain Fracture Toughness of Metallic Materials[1]

Designation: E 561 – 98

AMERICAN SOCIETY FOR TESTING AND MATERIALS
100 Barr Harbor Dr., West Conshohocken, PA 19428

Standard Practice for
***R*-Curve Determination[1]**

Designation: E 1820 – 01

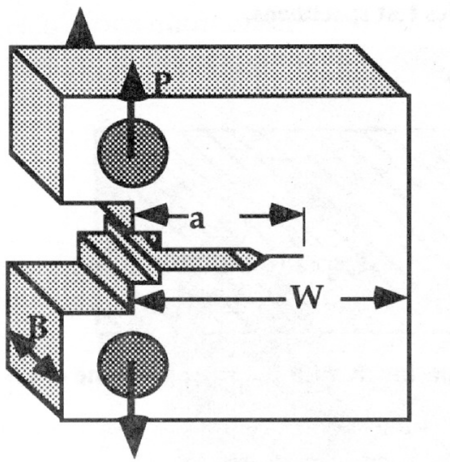
Standard Test Method for
Measurement of Fracture Toughness[1]

Designation: E 1823 – 96 (Reapproved 2002)

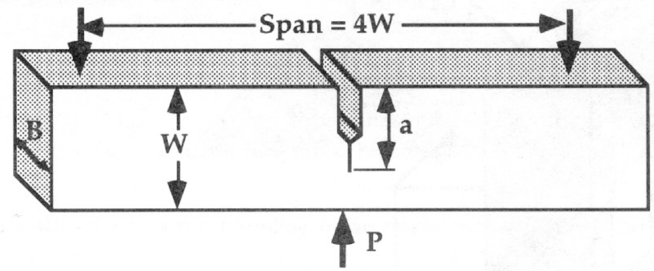
Standard Terminology
Relating to Fatigue and Fracture Testing[1]

4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

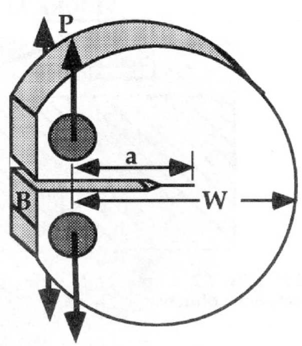
- Several experimental typologies; precracked specimens (fatigue).



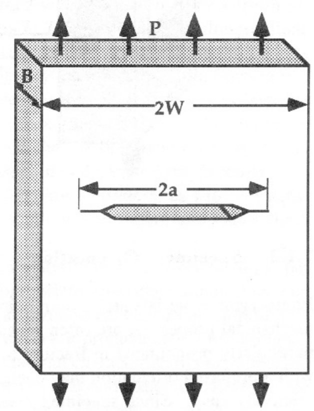
(a) Compact specimen.



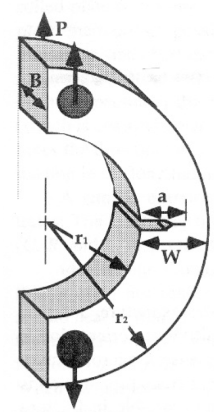
(c) Single edge notched bend (SENB) specimen.



(b) Disk shaped compact specimen.

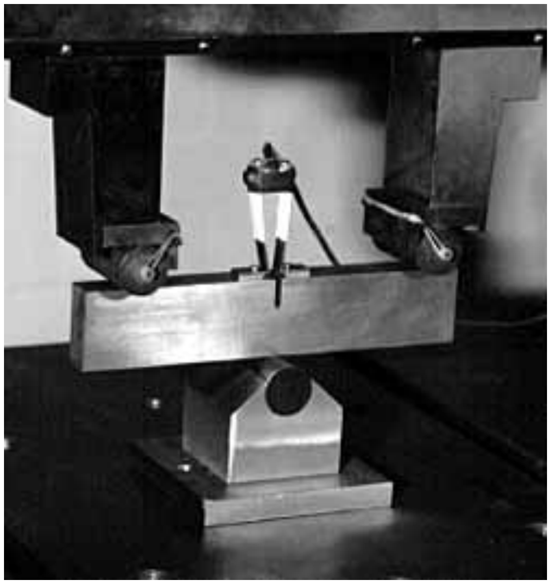


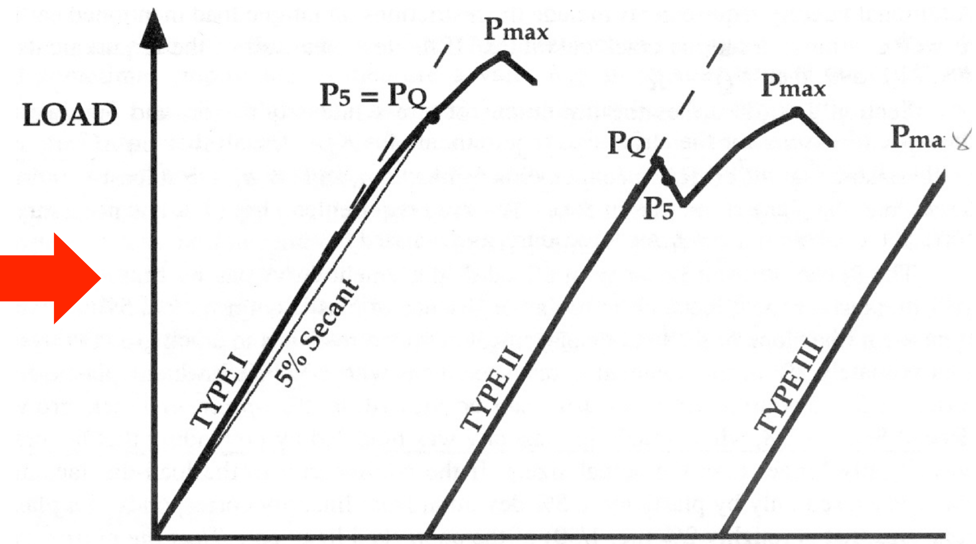
(e) Middle tension (MT) specimen.



(d) Arc shaped specimen

4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

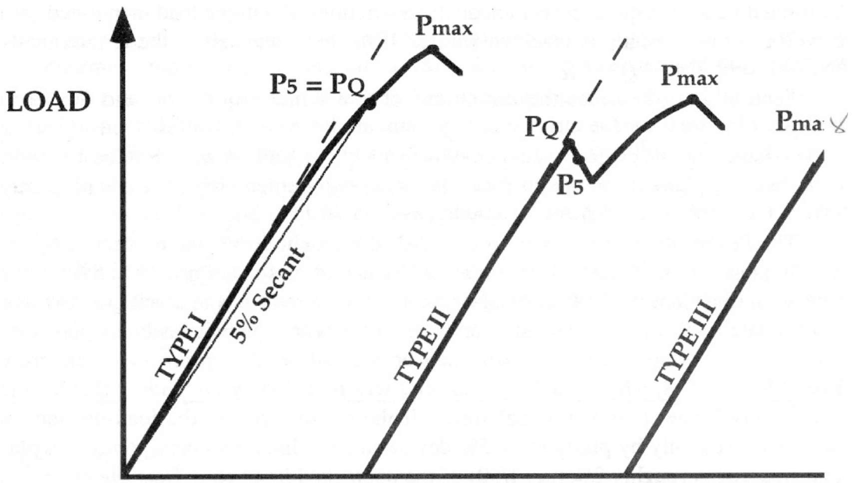




REQUIREMENTS (ASTM E 399):

- Linear elastic behavior until failure.
- Negligible size of the plastic zone.
- Brittle fracture (no stable propagation).

4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM



SEQUENCE:

1) Determining P_Q.

2) Obtaining K_Q.

$$K_Q = \frac{P_Q}{B\sqrt{W}} f\left(\frac{a}{W}\right)$$

3) Validity conditions:

$$0.45 \leq \frac{a}{W} \leq 0.55$$ ← Deep crack to ensure a high confinement (conservatism).

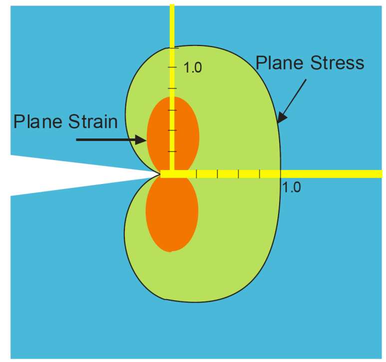
$$B, a \geq 2.5\left(\frac{K_Q}{\sigma_{YS}}\right)^2$$ ← Plane strain conditions (conservatism).

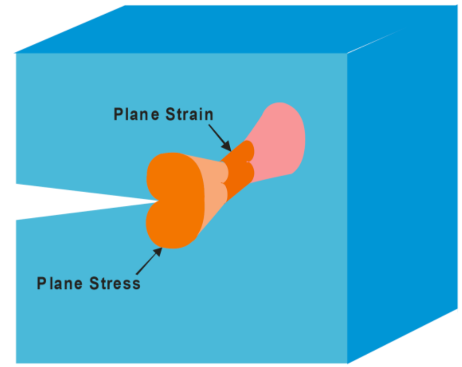
$$P_{max} \leq 1.10 P_Q$$ ← Representativity

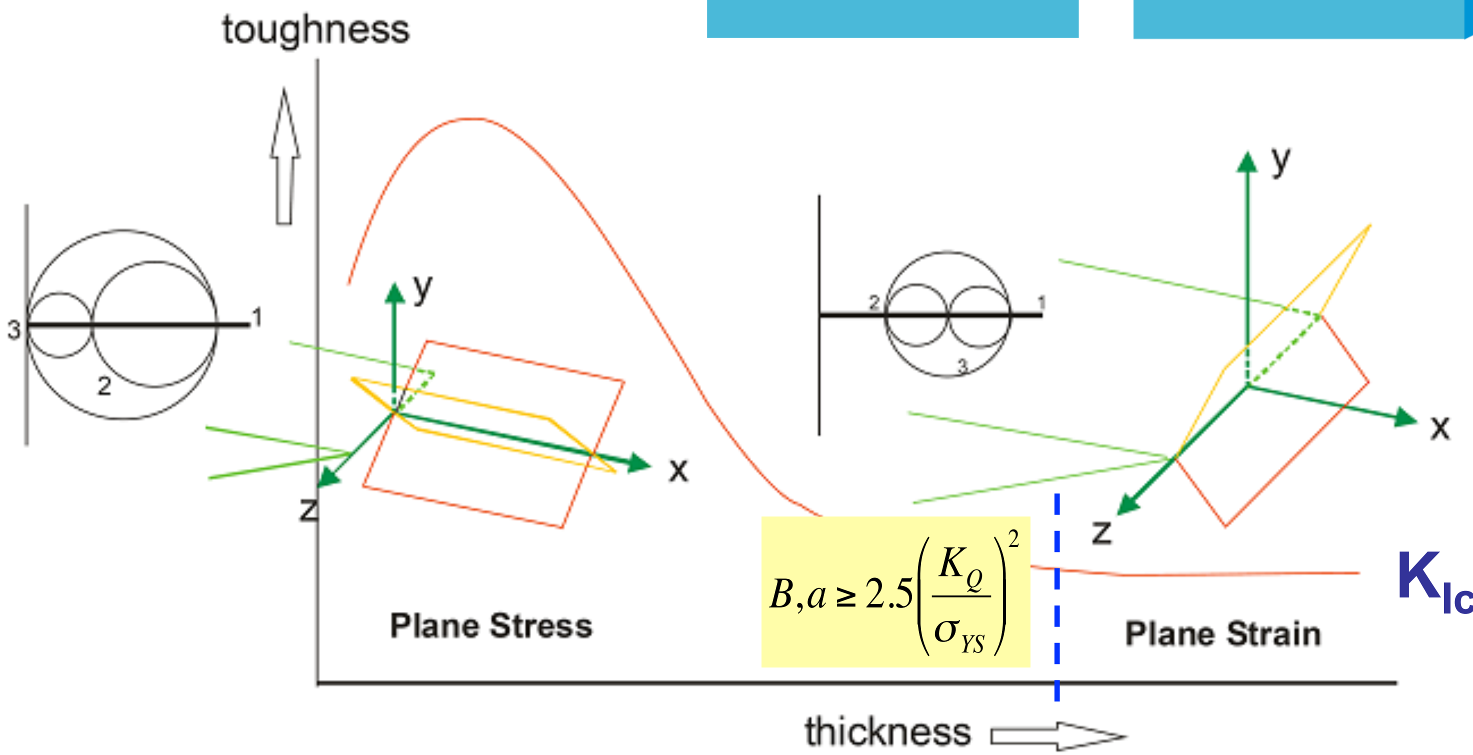
4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

TP Vs. DP:

- Triaxiality conditions change along the crack.

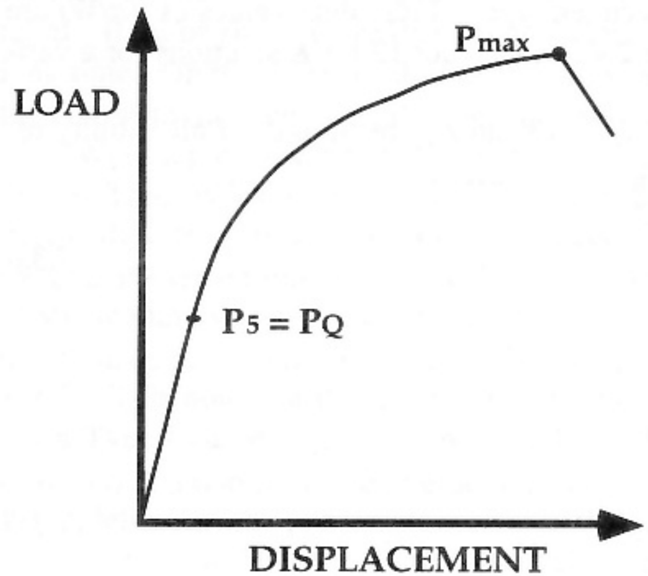






$$B, a \geq 2.5\left(\frac{K_Q}{\sigma_{YS}}\right)^2$$

4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM

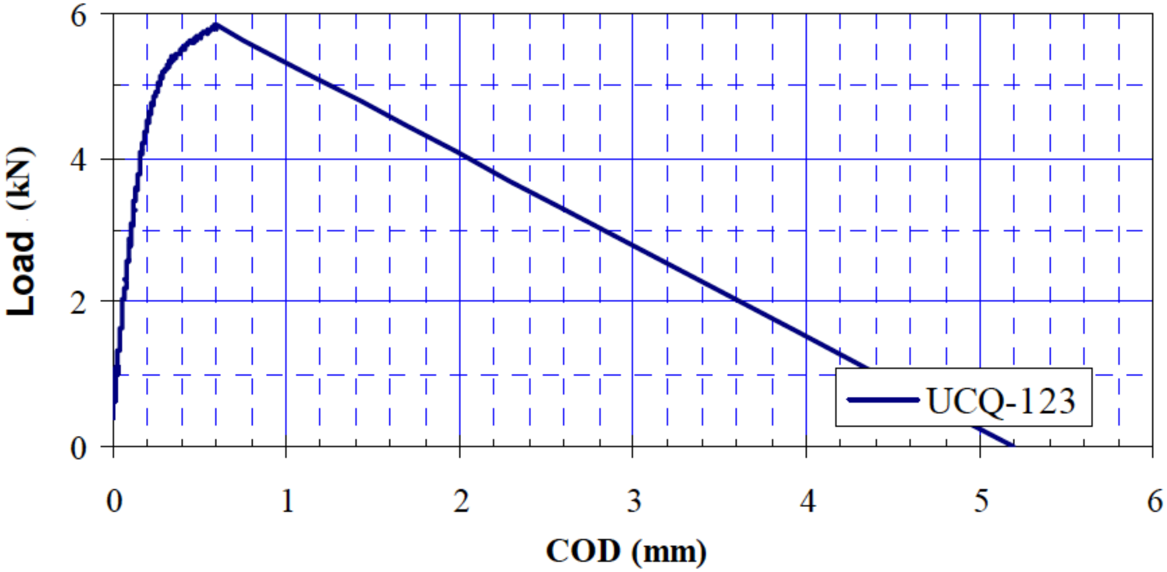


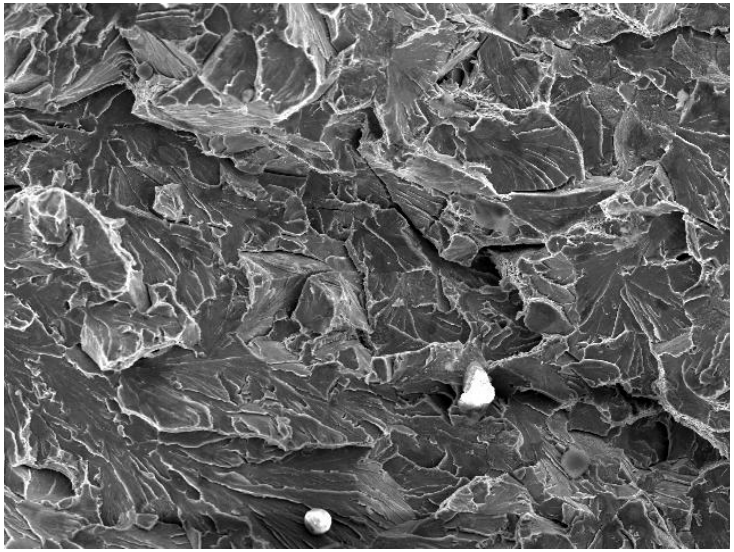
$$P_{max} \leq 1.10 P_Q$$

REPRESENTATIVITY:

- If this condition is not fulfilled, P_Q will hardly represent the fracture of this material.

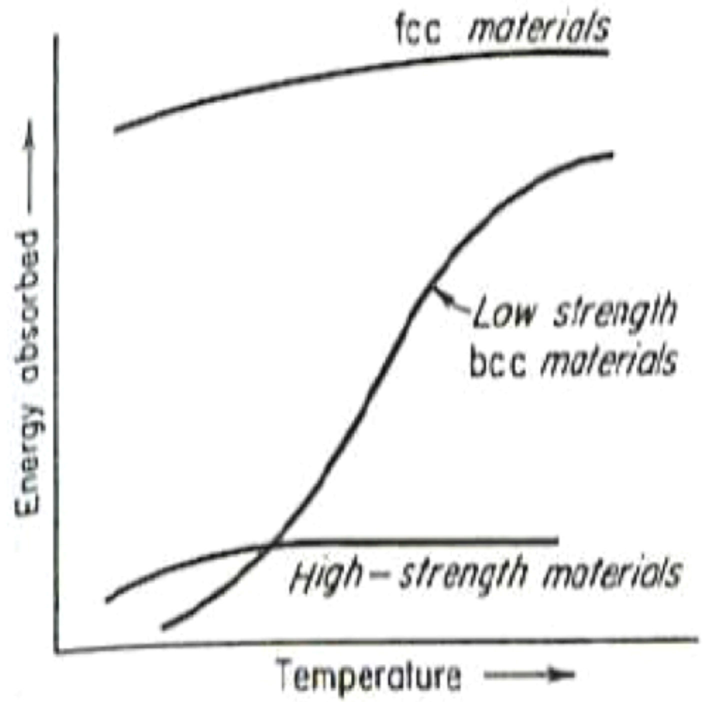
4.6. FRACTURE TOUGHNESS CHARACTERIZATION IN LEFM





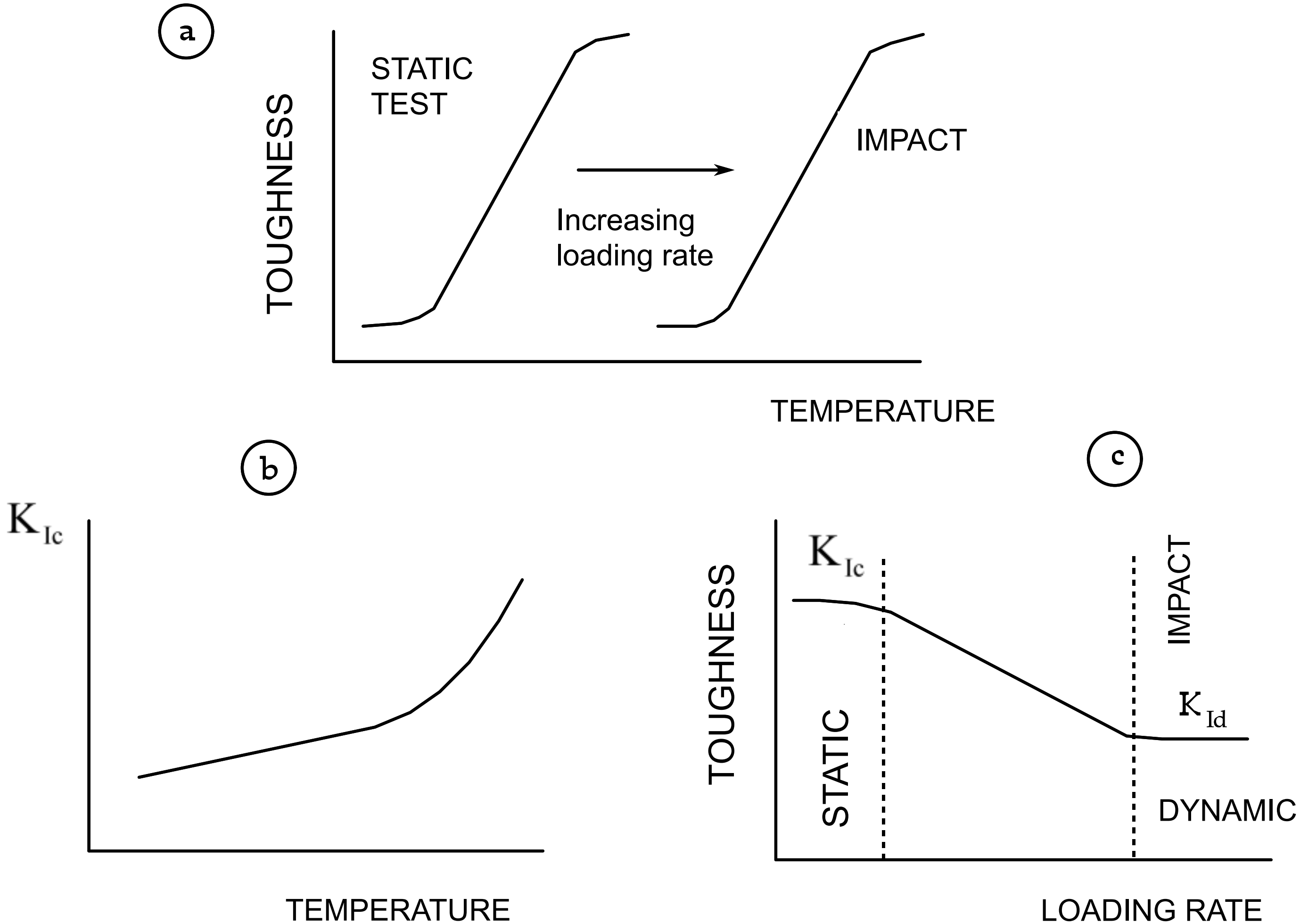
4.7. FACTORS AFFECTING TOUGHNESS

- Temperature.
- Loading rate.
- Orientation.
- Grain size.
- Neutron irradiation (steel).
- Carbon content (steel).
- Others…



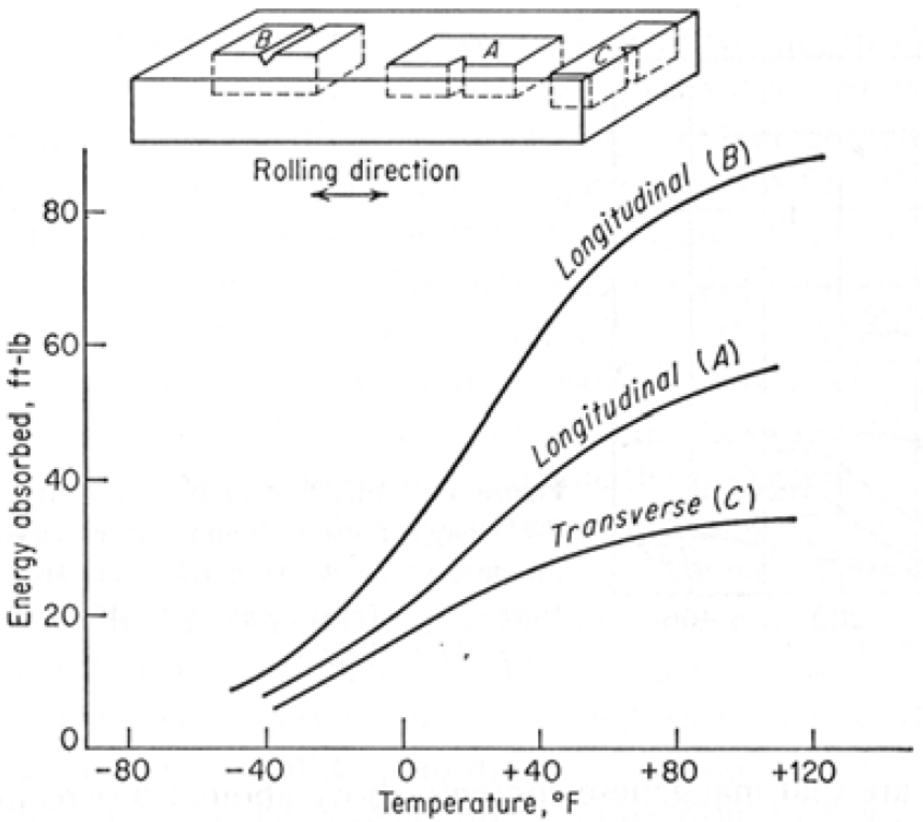
4.7. FACTORS AFFECTING TOUGHNESS

LOADING RATE:



4.7. FACTORS AFFECTING TOUGHNESS

ORIENTATION:



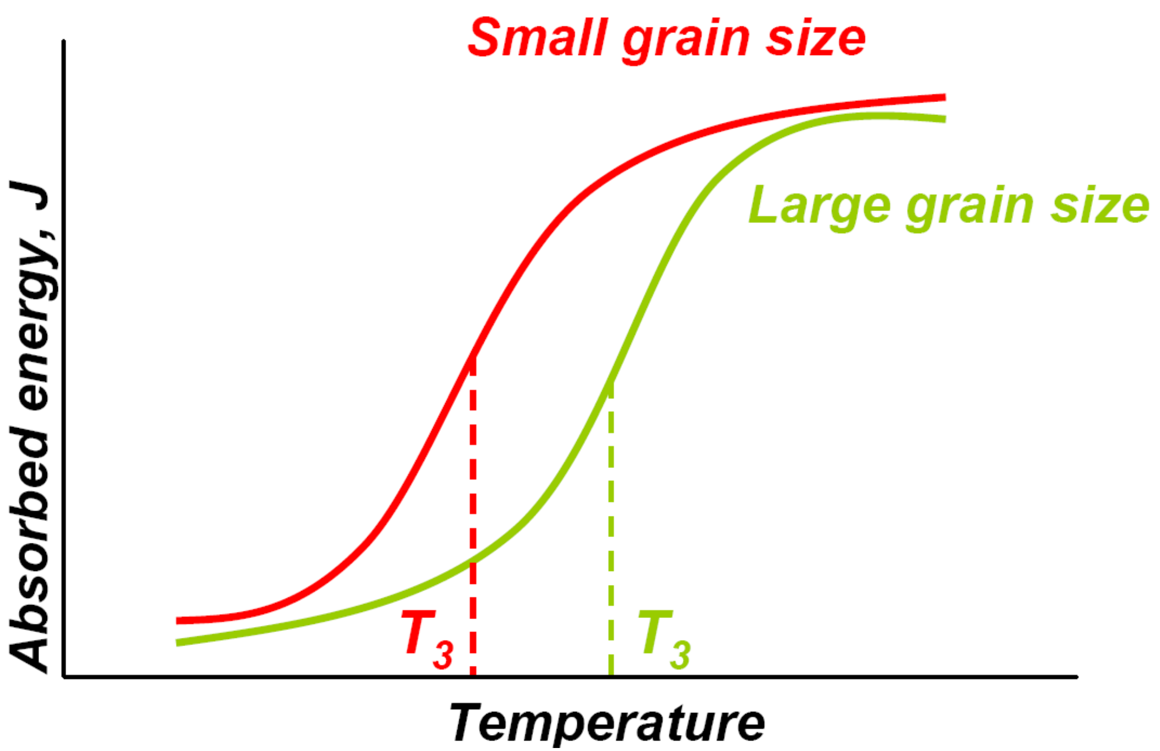
- Rolled and forged products may have varying impact behaviour due to grain orientation.
- Note that the difference is not as large at lower temperatures.

4.7. FACTORS AFFECTING TOUGHNESS

GRAIN SIZE:

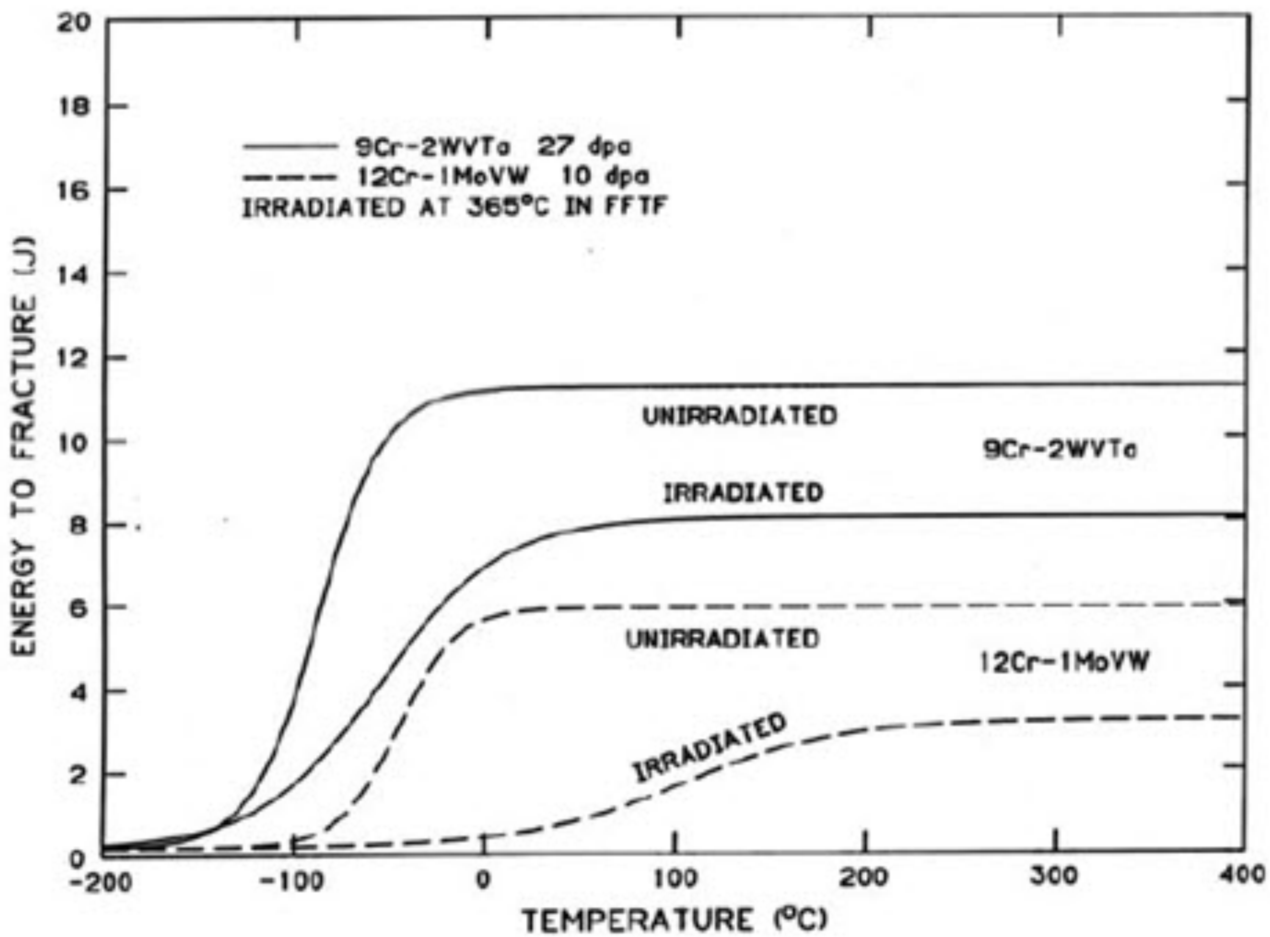
- Grain size has a strong effect on transition temperature.

Grain size ↓ **Transition temperature** ↓



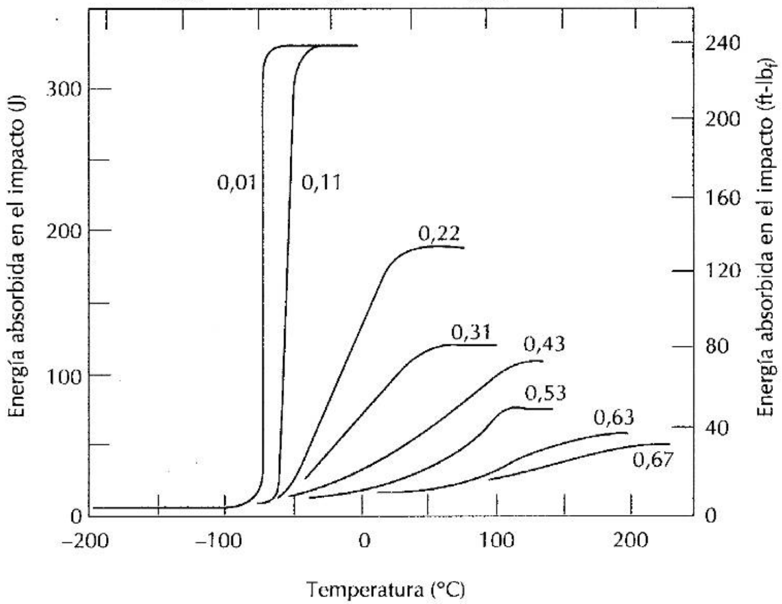
4.7. FACTORS AFFECTING TOUGHNESS

IRRADIATION:



4.7. FACTORS AFFECTING TOUGHNESS

CARBON CONTENT:



- For steels: As %C↑ ⇒ σ_y↑, σ_{TS}↑, H↑, % El↓, CVN↓ and T_T↑

- *This can be countered by adding Manganese - Mn : C should be 3:1*

- Phosphorous increases T_T, Oxygen in steel increases T_T:

  - *semi-killed (add Si) and*
  - *fully-killed (add Si +Al) to remove oxygen*

<u>Remember also:</u> as grain size ↓ toughness ………. and T_T↓. *Niobium* and *vanadium* added to keep grain size small.

4.8. PLASTICITY IN FRACTURE (EPFM)

LEFM: stress state around the defect.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{22} \end{Bmatrix} = \frac{K_I}{(2\pi r)^{1/2}} \cos(\theta/2) \begin{Bmatrix} 1-\sin(\theta/2)\sin(3\theta/2) \\ \sin(\theta/2)\cos(\theta/2) \\ 1+\sin(\theta/2)\sin(3\theta/2) \end{Bmatrix}$$

$$\sigma_{ij} = \frac{k}{\sqrt{r}} f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{m/2} g_{ij}^{(m)}(\theta) \quad \Rightarrow \quad \lim_{r \to 0} \sigma_{ij} = \frac{k}{\sqrt{r}} f_{ij}(\theta) \Rightarrow \infty$$

- Structural materials yield when the stress state exceeds a certain limit (Yielding criterion: $\sigma_{eq}(\sigma_{ij}) = \sigma_y$).
- In this case, the elastic solution is not applicable in all points of the material.

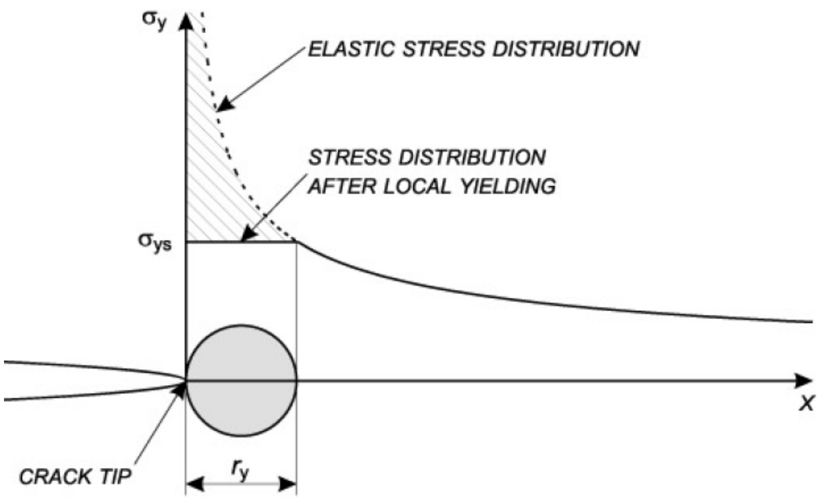
4.8. PLASTICITY IN FRACTURE (EPFM)

FIRST ESTIMATE:

- Axis X (θ = 0): $\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} = \sigma_Y \quad \Rightarrow \quad r_Y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$

LIMITATIONS:

- Describes the PZ as a circle (in principle, each direction is different).
- It doesn't take into account stress redistribution and, therefore, **it's not an equilibrium solution**.
- Considers a perfect elastic-plastic material.
- Underestimates the size of the PZ.
- It doesn't consider the possible influence of the triaxiality (plane stress vs. plane strain).



4.8. PLASTICITY IN FRACTURE (EPFM)

IRWIN APPROACH:

- It is an attempt to take stress redistribution into consideration.
- Still a simplistic solution:
  - PZ circular geometry (analysis in $\theta = 0$).
  - Perfect elastic-plastic material in Plane Stress.
- **Equivalent crack** concept: plasticity implies a lower stiffness of the com-ponent, which can be approximately described, by imposing the existence of a crack slightly longer than the actual one in the structure (a).

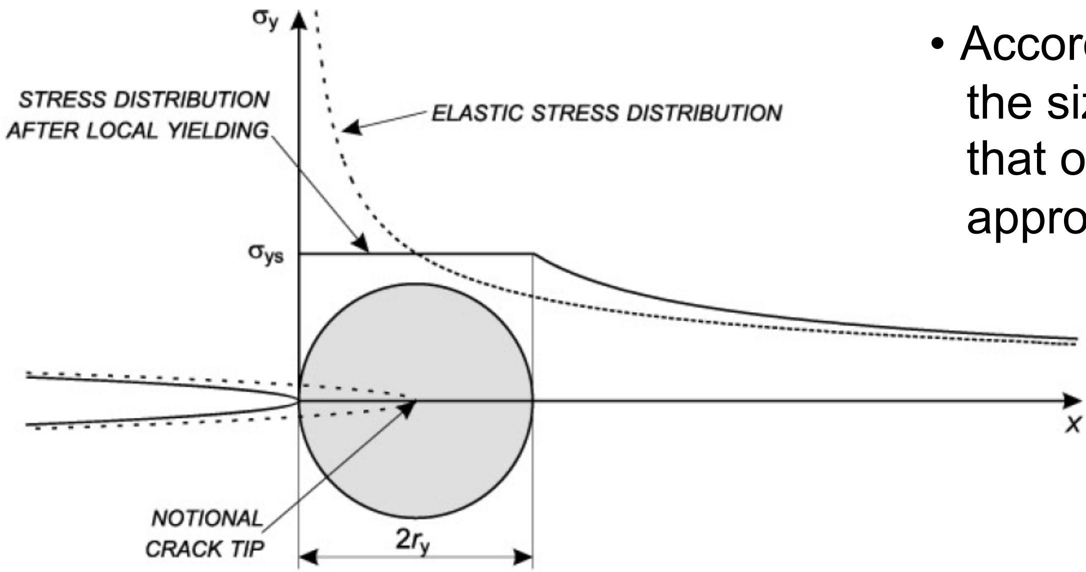
$$a_{ef} = a + \Delta a$$

To determine, by imposing local equilibrium conditions in the crack front.

4.8. PLASTICITY IN FRACTURE (EPFM)

IRWIN APPROACH: $\Delta a = r_Y$

SIZE OF THE PZ: $r_P \approx 2\, r_Y$



- According to Irwin model, the size of the PZ is twice that obtained in the first approximation.

$$r_P = 2 r_Y = \frac{1}{\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

- The main effect of plasticity is that stresses relax in the PZ because they increase outside of it.
- SIF is expressed as: $$K_I = \sigma\sqrt{\pi\left(a+\Delta a\right)} = \sigma\sqrt{\pi\left(a+r_Y\right)}$$
- The procedure applies only if $r_Y << a$.

4.8. PLASTICITY IN FRACTURE (EPFM)

APPLICATION OF THE METHOD:

1) Get the SIF for the actual crack (a).

2) Calculate r_Y: $$r_Y = \frac{1}{n\pi}\left(\frac{K_I}{\sigma_Y}\right)^2 \quad \begin{cases} n = 2 \; Plane\ stress \\ n = 6 \; Plane\ strain \end{cases}$$

3) Get the effective crack: $$a_{ef} = a + \Delta a$$

4) Recalculate the SIF: $$K_I = \sigma\sqrt{\pi\left(a + r_Y\right)}$$

5) Iterate (because K_I depends on r_Y and r_Y depends on K_I):

- The SIF calculated this way represents the stress state outside the PZ as well as its size; it seems reasonable, then, that the SIF is a valid crack parameter even in SSY (Small Scale Yielding).

4.8. PLASTICITY IN FRACTURE (EPFM)

APPLICATION OF THE METHOD:

- In some cases it is not necessary to iterate:

$$K_I = Y\sigma\sqrt{\pi a} \quad \Rightarrow \quad K_{I,eff} = Y\sigma\left\{\pi\left[a + \frac{1}{n\pi}\left(\frac{K_{I,eff}}{\sigma_Y}\right)^2\right]\right\}^{1/2} \quad \Rightarrow$$

$$K_{I,eff} = \frac{Y\sigma\sqrt{\pi a}}{\sqrt{1 - \frac{1}{n}\left(Y\frac{\sigma}{\sigma_Y}\right)^2}}$$

FRACTURE CONDITION:

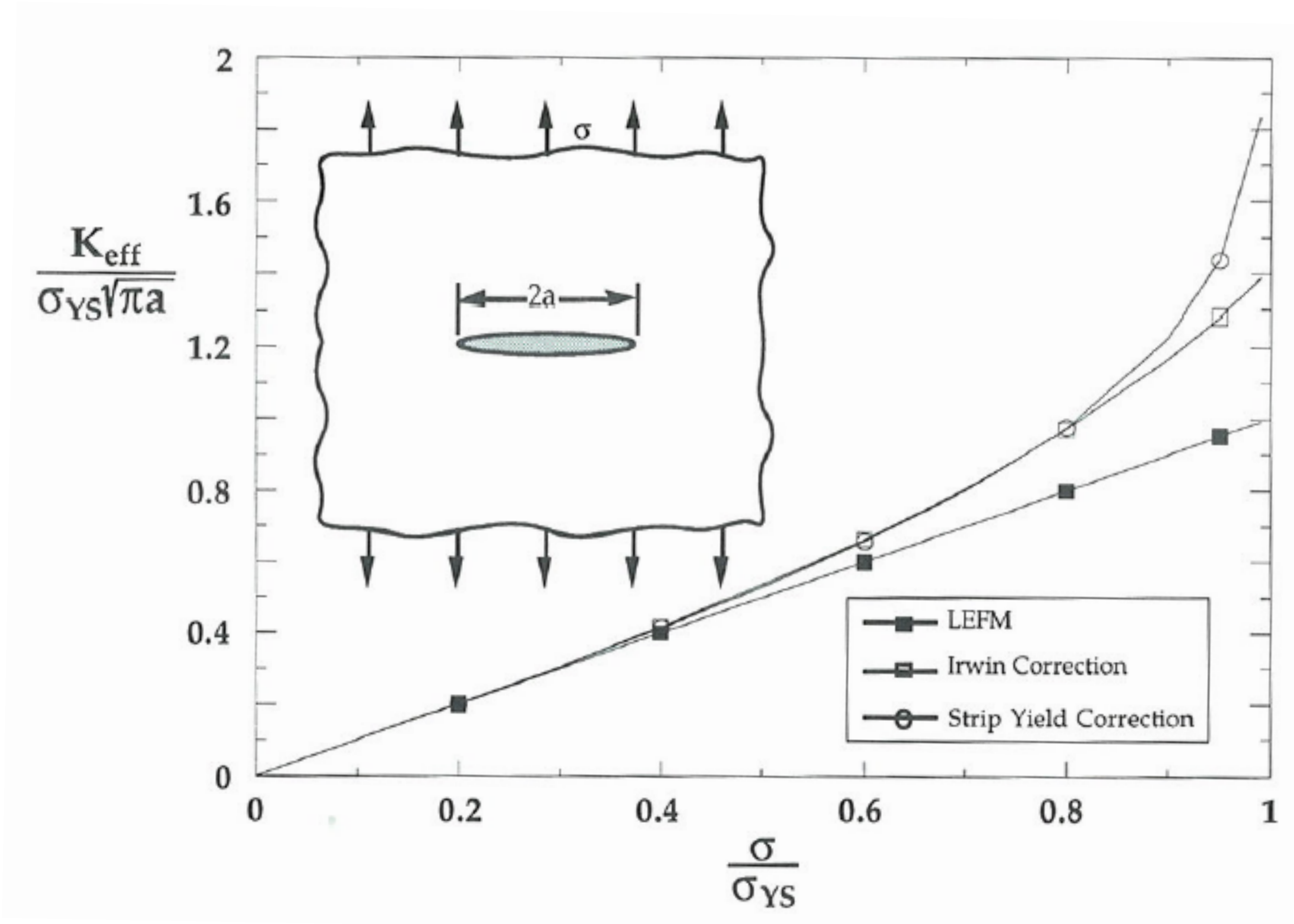
$$K_{I,eff} = K_{Ic}$$

A BETTER APPROACH: STRIP YIELD MODEL:

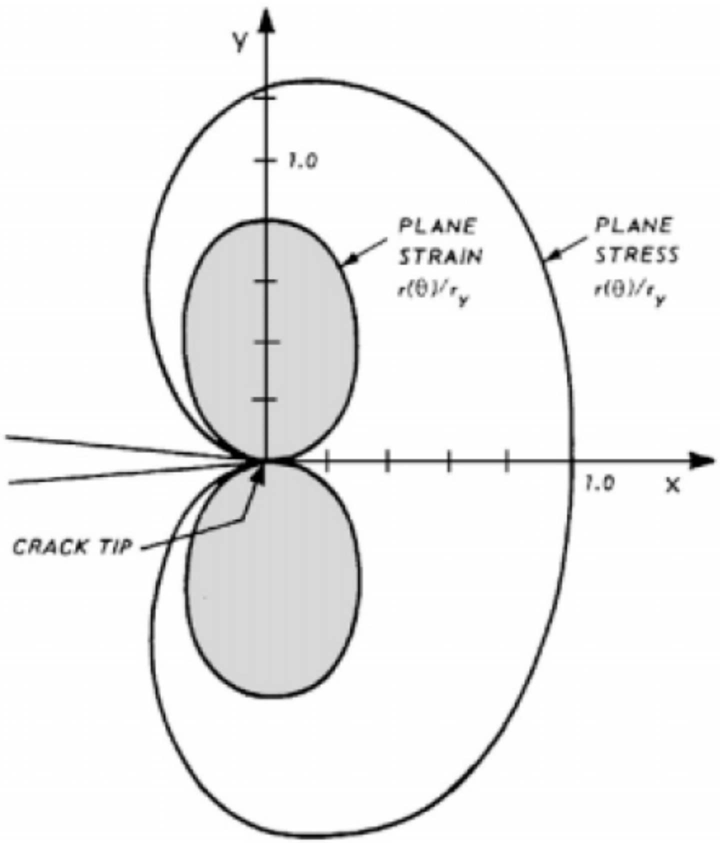
- Dugdale, Barenblatt, Burdekin and Stone:

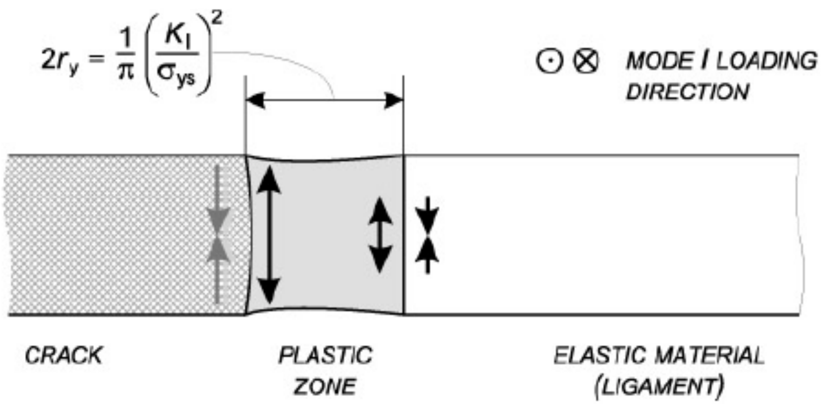
$$K_{I,eff} = \sigma_y\sqrt{\pi a}\left[\frac{8}{\pi^2}\ln\sec\left(\frac{\pi\sigma}{2\sigma_y}\right)\right]^{1/2}$$

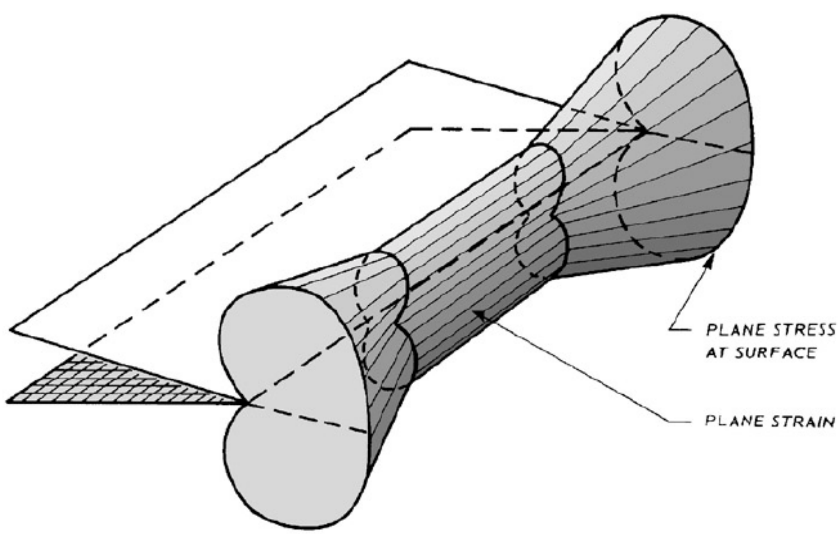
4.8. PLASTICITY IN FRACTURE (EPFM)



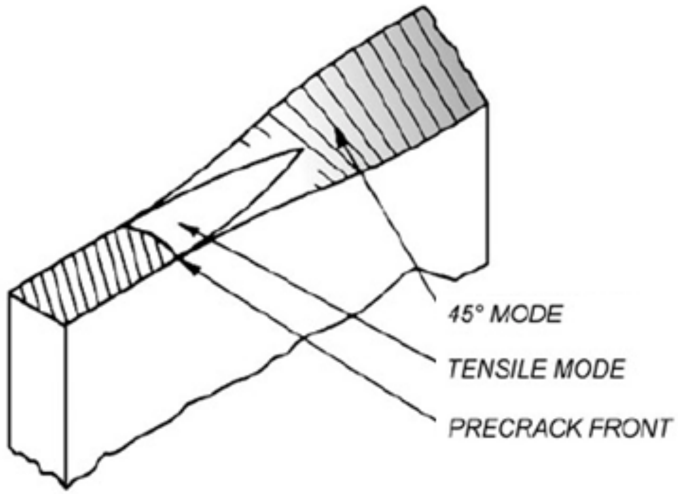
4.8. PLASTICITY IN FRACTURE (EPFM)



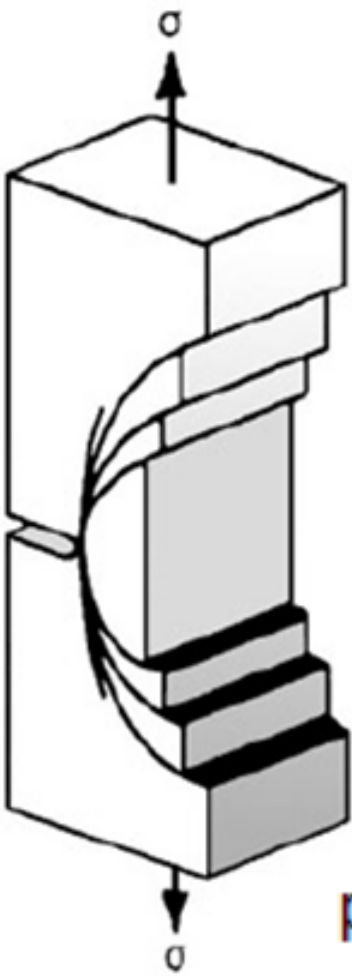




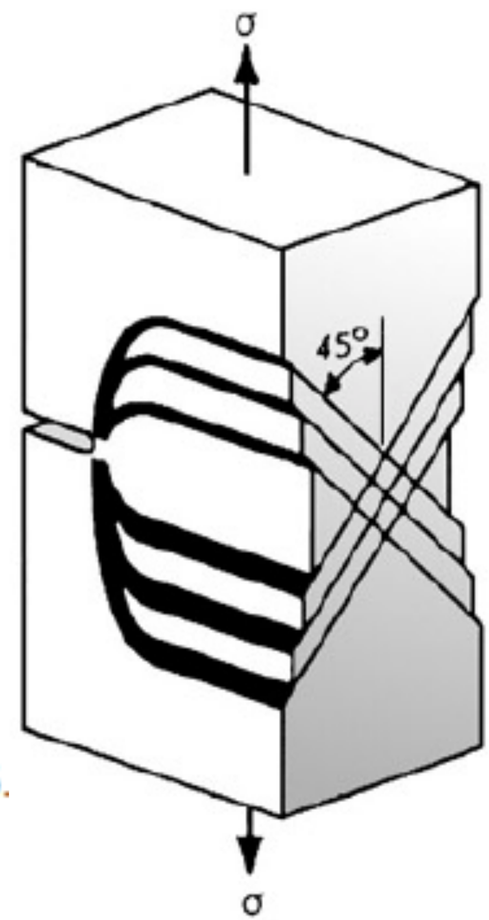
4.8. PLASTICITY IN FRACTURE (EPFM)



General appearance of monotonic overload fracture from a precrack

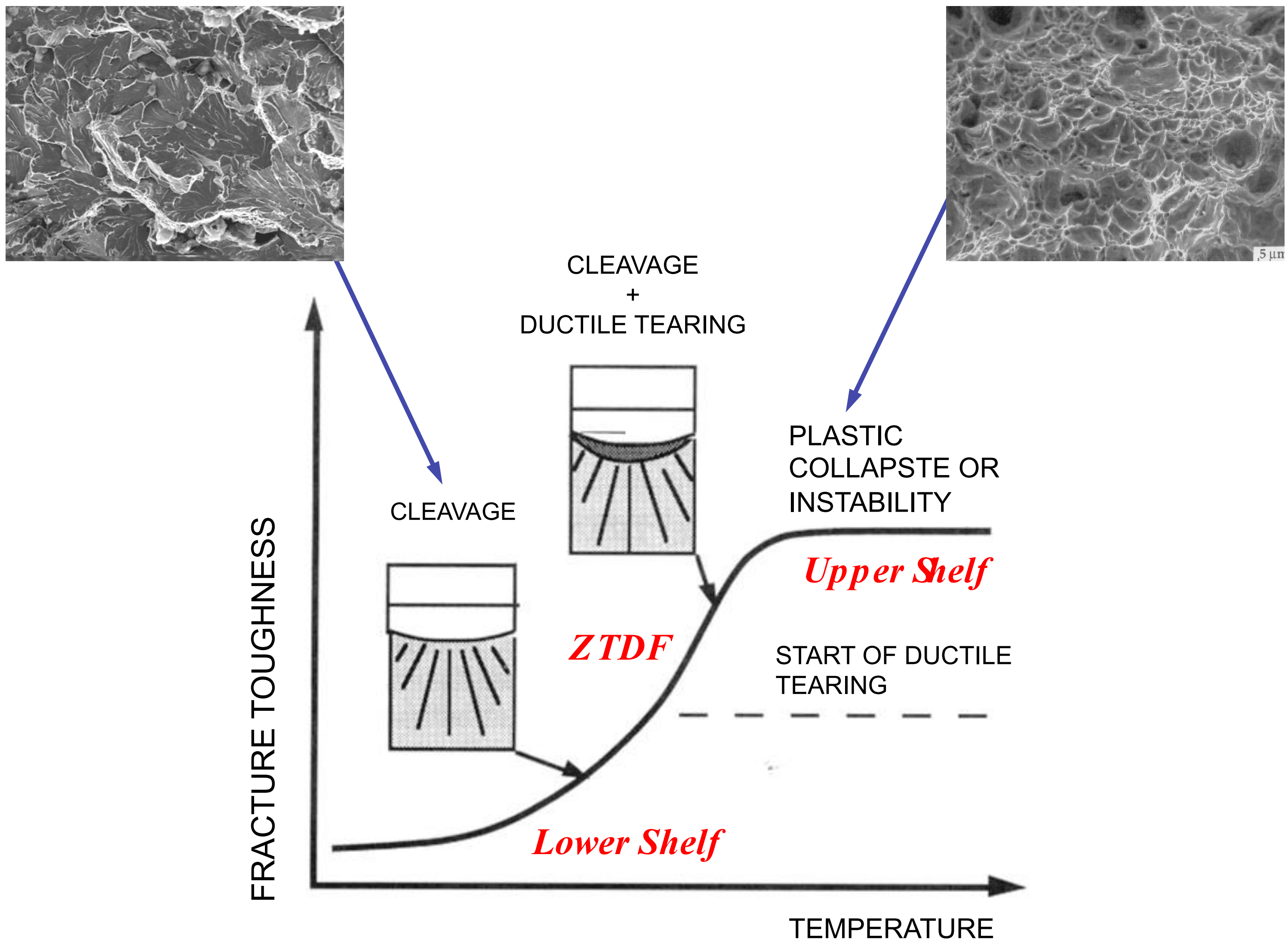


plane strain



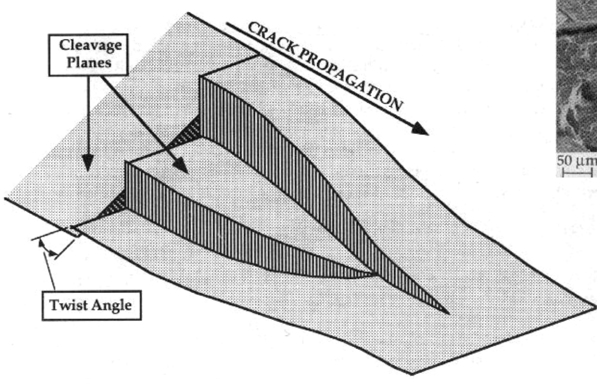
plane stress.

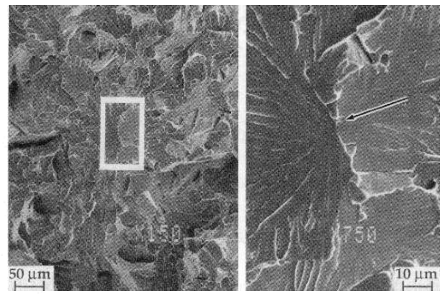
4.9. FRACTURE MICROMECHANISMS

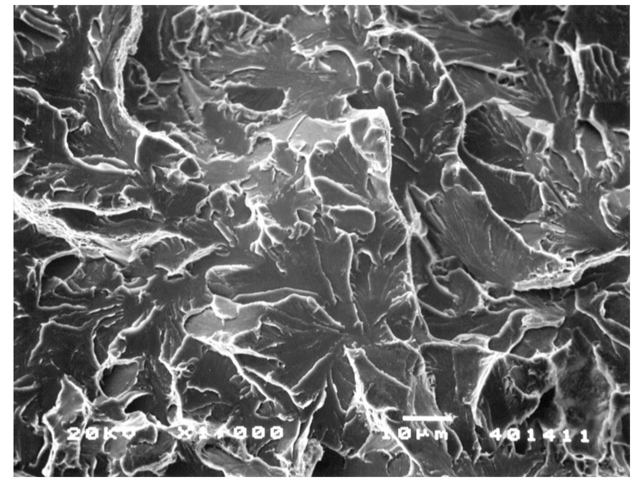


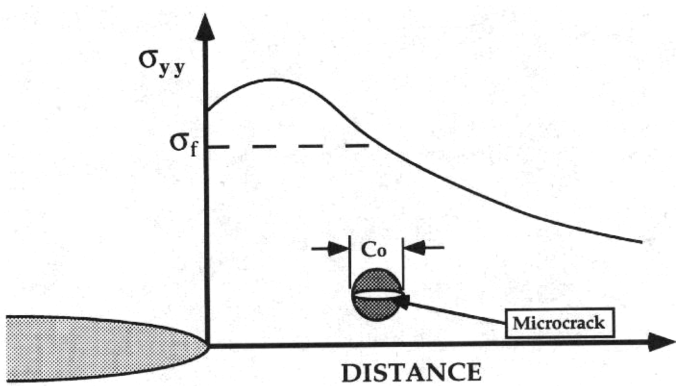
4.9. FRACTURE MICROMECHANISMS

BRITTLE FRACTURE: CLEAVAGE:







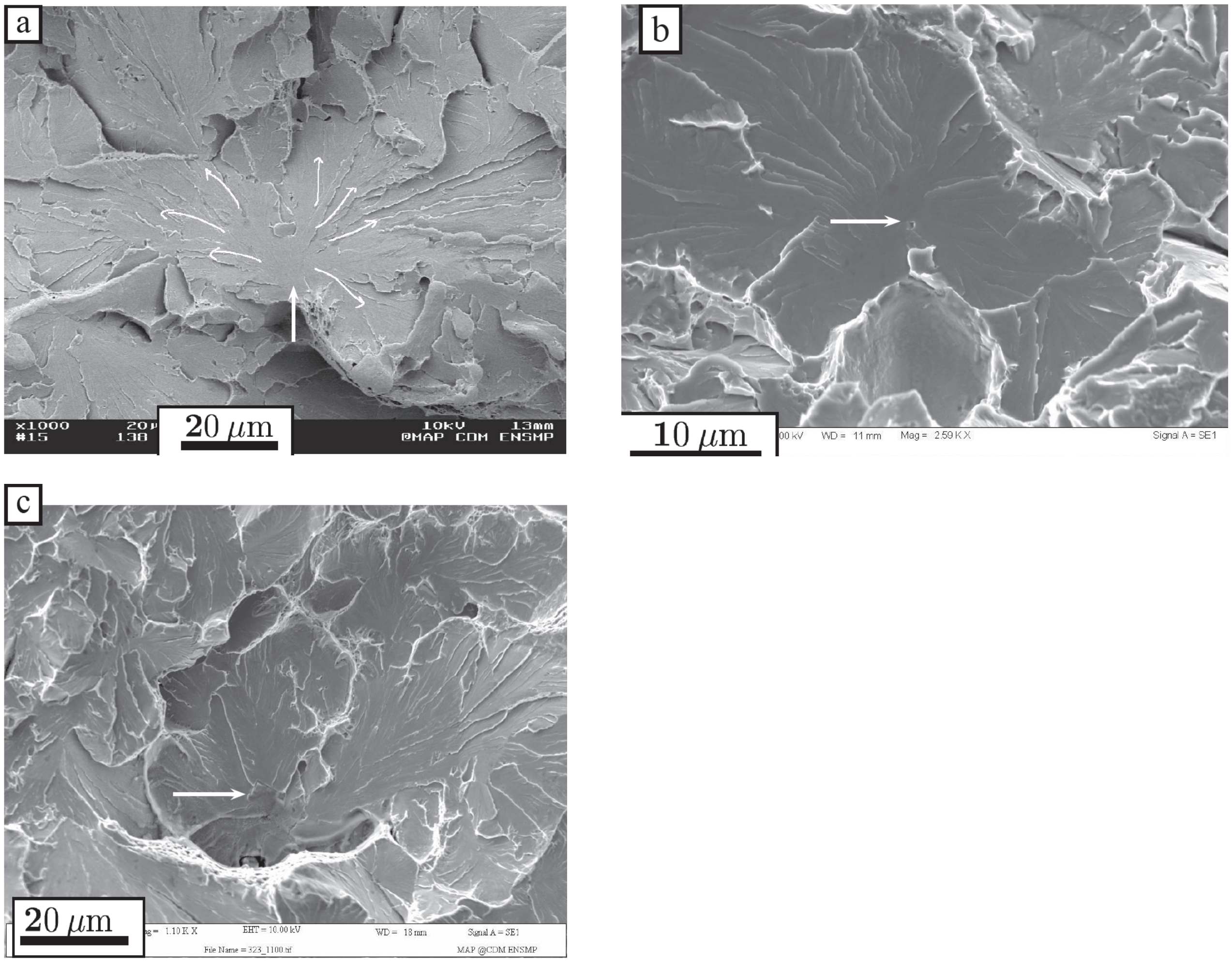


It occurs on brittle metallic materials:

- Favored by low temperatures and high loading rates.
- Favored in materials with high σ_Y.

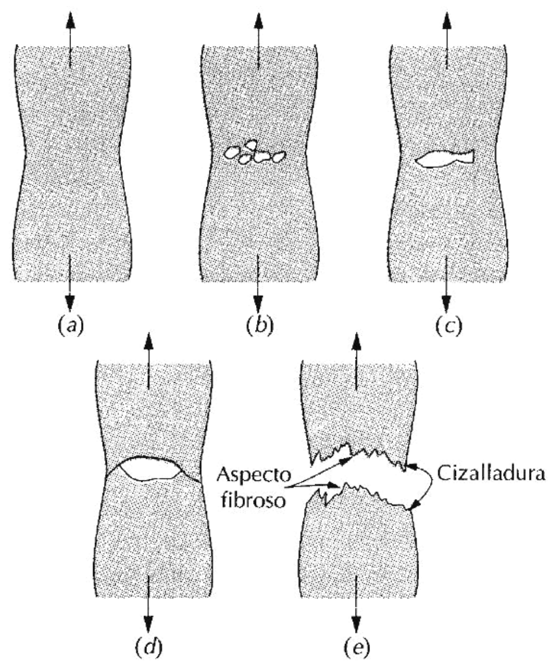
4.9. FRACTURE MICROMECHANISMS

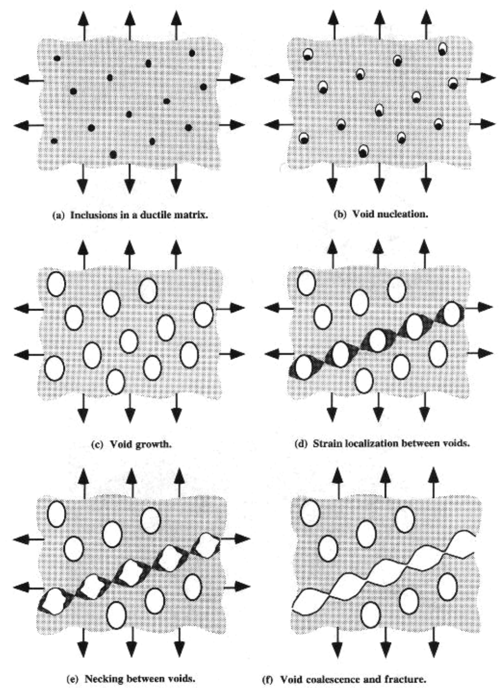
BRITTLE FRACTURE: CLEAVAGE:

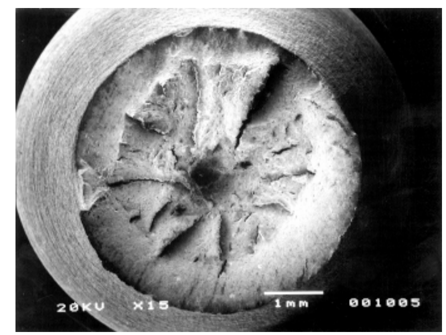


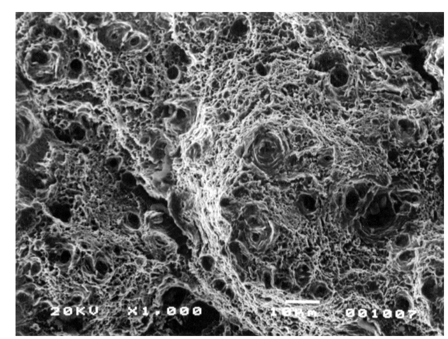
4.9. FRACTURE MICROMECHANISMS

<u>DUCTILE FRACTURE</u>: FORMATION, GROWTH AND COALESCENCE OF MICROVOIDS







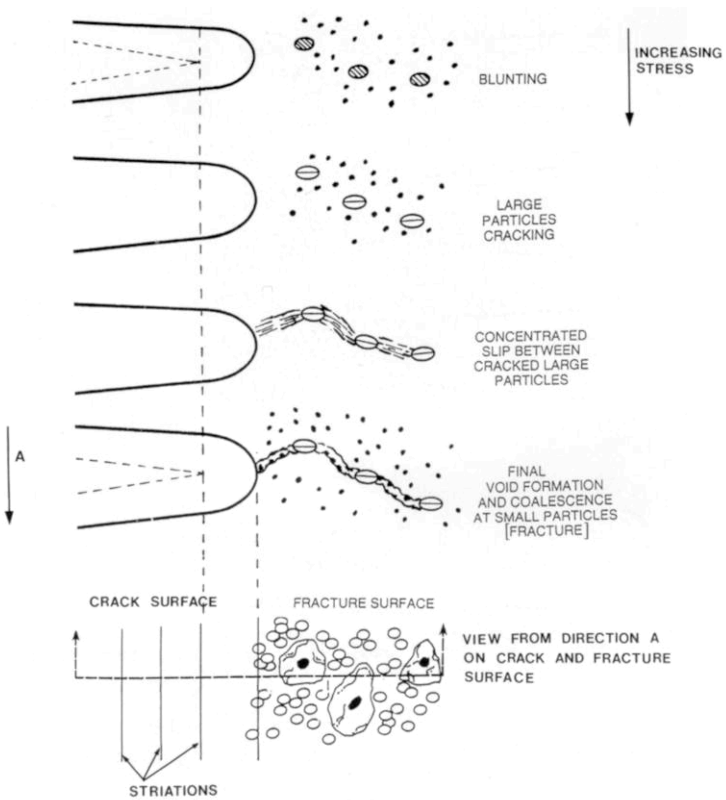


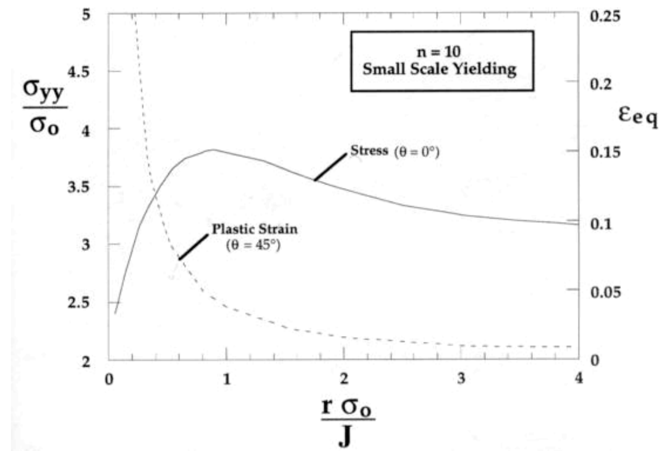
Metallic materials with plastic behavior:

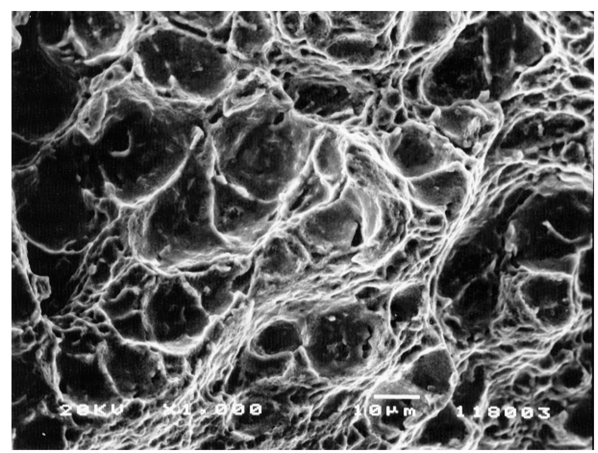
- Favored by T↑, σ_Y ↓, $d\sigma / dt$ ↓

4.9. FRACTURE MICROMECHANISMS

DUCTILE FRACTURE: FORMATION, GROWTH AND COALESCENCE OF MICROVOIDS

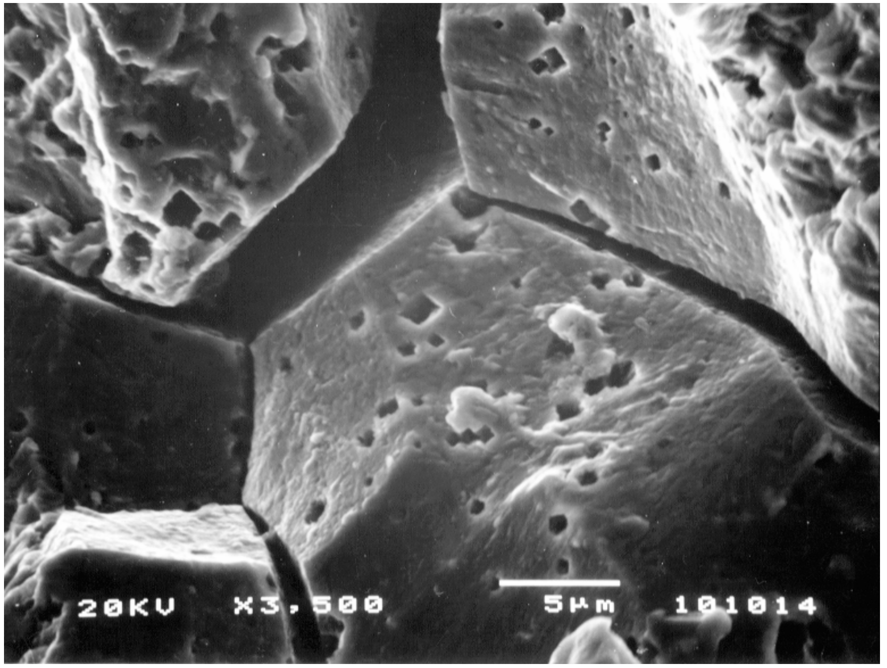






4.9. FRACTURE MICROMECHANISMS

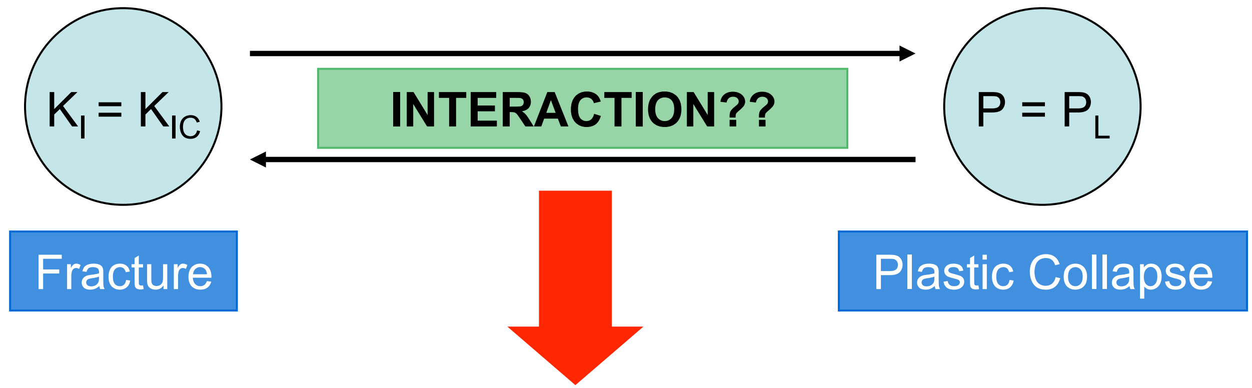
INTERGRANULAR FRACTURES:



© Because of the environment or grain boundary segregations.

4.10. FAILURE ASSESSMENT DIAGRAMS (FAD)

SEPARATED ASSESSMENT

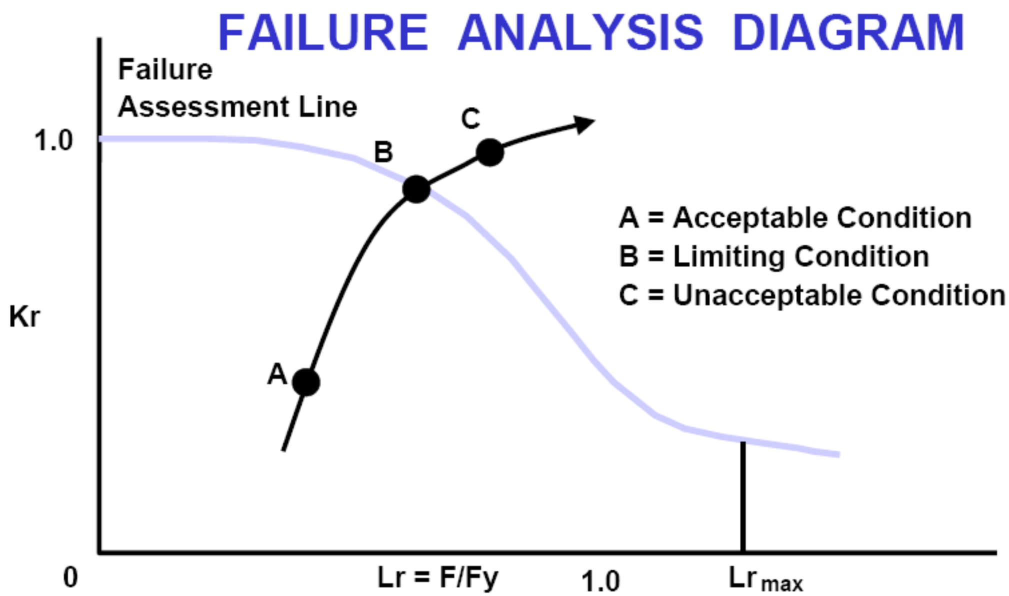


- It is necessary a relationship between $K_r = K_I/K_{IC}$ y $L_r = P/P_L$ that separates admissible and non admissible situations.
- Equating $K_{I,eff}$ of the Strip Yield model to fracture toughness K_{IC}: $K_{I,eff} = K_{Ic}$

$$\frac{K_{IC}}{K_I} = \frac{\sigma_y}{\sigma}\left[\frac{8}{\pi^2}\ln\sec\left(\frac{\pi\sigma}{2\sigma_y}\right)\right]^{1/2} \Rightarrow K_r = L_r\left[\frac{8}{\pi^2}\ln\sec\left(\frac{\pi}{2}L_r\right)\right]^{-1/2}$$

4.10. FAILURE ASSESSMENT DIAGRAMS (FAD)

- FAD can simultaneously evaluate fracture and plastic collapse. The coordinates of the assessment point indicates the type of failure (fracture vs. collapse).



$$Fracture: \frac{K_I}{K_{IC}} = 1$$

$$Plastic-collapse: \frac{P}{P_L} = 1$$

$$(without-yield-plateau)$$

$$K_r = \frac{K_I}{K_c}$$

$$L_r = \frac{P}{P_L}$$

4.10. FAILURE ASSESSMENT DIAGRAMS (FAD)

<u>INTERGRANULAR FRACTURES</u>: