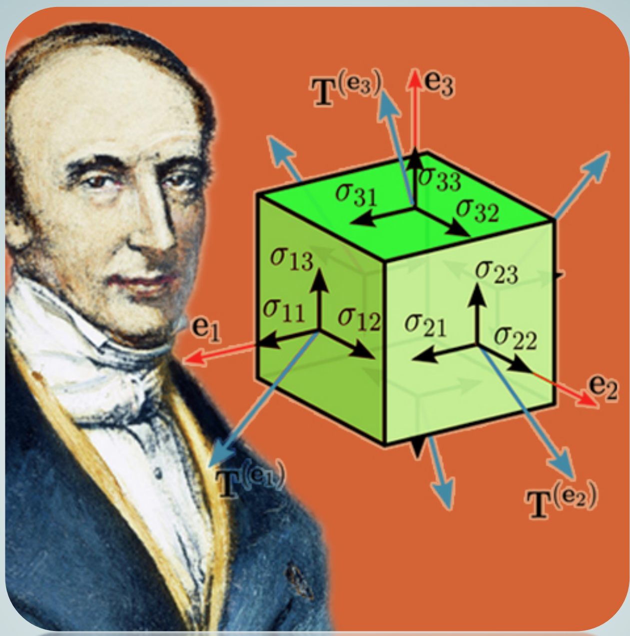
Mechanical Properties of Materials, Processing and Design

Exercises Lesson 1. Elastic behavior



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MECHANICAL PROPERTIES OF MATERIALS **2019-2020**

1: Elastic behaviour

1. Determine the relation between the bulk modulus, K, of a material, its Young's modulus, E, and its Poisson's ratio, ν.

2. The elastic properties of an isotropic and homogeneous, linear elastic, continuous medium, can be described using only two elastic constants. Although there are many possibilities, it is usual to use the modulus of elasticity, E, and Poisson's ratio, ν, for that purpose. According to this, the rest of elastic constants can be expressed in terms of the previous ones, E and ν, by using expressions that are independent of the stress state considered.

In this case, we want to obtain the expression G (E,ν), G being the shear modulus of the material. For this purpose, consider an infinitesimal element of the continuous medium, with dimensions L x L (the third dimension can be considered as unit value), subjected to a pure shear state, as shown in Figure 1. In this figure, the deformation of this element is also shown, which corresponds to a distortion γ.

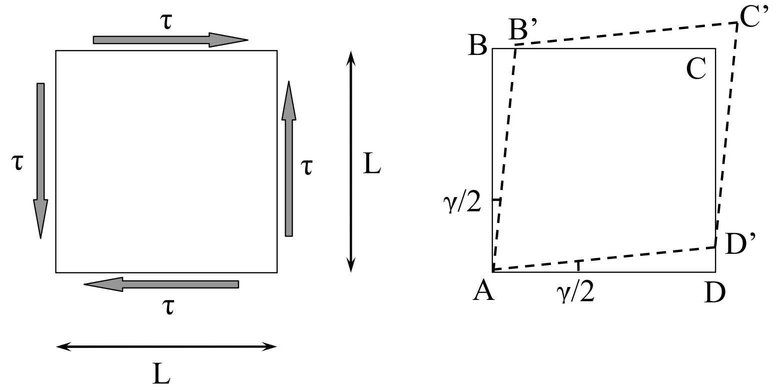

Figure 1

To obtain the requested relationship, you must follow this procedure:

a) Establish the constitutive relation between the stress τ and the distortion γ.
b) Express the longitudinal strain, ε, experienced by the diagonal AC (Figure 1) s a function of the distortion, γ (that is, apply compatibility conditions). For this purpose, small strain conditions must be considered.
c) Considering, for convenience another element 1234, as shown in Figure 2, obtain the stress state in the edge 14. For this purpose it is recommended to study the equilibrium of the triangular prism BCD (that is, apply a cut BD to the original prism ABCD and impose equilibrium conditions).
d) Repeat the previous procedure to obtain, in this case, the stress state at the edge 12. What kind of stress state is that of the element 1234?

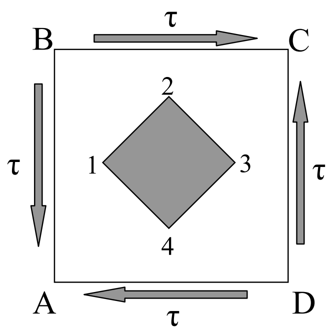

Figure 2

e) Given the stress state of the element 1234, apply the generalized Hooke's law to calculate the strain, ε, experienced by the line MN in Figure 3, depending on the modulus of elasticity, E, and Poisson's ratio, ν, of the material.

f) Obtain the expression, G(E,ν), by using the strains obtained in b) and e) and considering the relation in a).

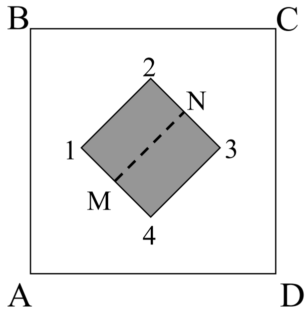

Figure 3

3. Obtain the ratio between the elastic limits of two linear elastic materials based on their elastic constants, so that they simultaneously reach those limits when they work together as fibers of the same composite material (isodeformation conditions).

4. Determine the equivalent Young's modulus of a compound material reinforced with unidirectional fibers as a function of the percentage of fibers in the volume and on the elastic constants of the matrix and the fibers, both isotropic. Can the compound material be isotropic?

5. A reinforced concrete bar, with a cross section of $20 \times 20\, cm^2$ with 4 steel wires $\Phi 20$ on the corners, is subjected to a tension force of $20\, Tm$. Determine the stress and strain states in the concrete and the steel supposing that they are both working in a linear elastic regime and that the adherence between them is perfect.

Data: $E_s = 200\, GPa$; $E_c = 20\, GPa$; $\sigma_{Ys} = 400\, MPa$; $\sigma_{ct} = 2\, MPa$

6. Determine the theoretical modulus of elasticity of the common salt (NaCl), knowing that its interatomic distance is 2.5 Å. Use a Born exponent, $n = 8$. Compare the result obtained with the value determined in a real test of this crystal, $50\, GN/m^2$, justifying the possible differences.

7. In the attached figure, the geometry (dimensions in mm) of a hollow thin-wall cylinder, with its ends closed by two plates (supposed to be non-deformable) is shown. Four high-strength bolts keep the two plates together. The cylinder contains a pressurized fluid. The closure of the assembly is based on prestressing the system by (simultaneously) tightening the bolts a quarter of a turn beyond the contact between the nuts and the plates.

a) Determine the stress in each bolt immediately after prestressing the bolts.

Assuming that, after prestressing, the internal pressure of the fluid rises up to a value “p”, answer the following questions:

b) Establish the equilibrium equation/s of the problem.
c) Establish the compatibility equation/s of the problem.
d) Determine the condition for the fluid to escape the cylinder.
e) From the previous results, obtain the internal pressure of the fluid that causes the escape.
f) Answer again question a) assuming that the cylinder is perfectly non-deformable.
g) Answer again question e) supposing that the cylinder is perfectly non-deformable.

Relevant data:

- The four bolts that adjust the plates have a thread with a pitch of 15 fillets/inch.
- Mechanical properties for all the materials: $E = 2\cdot10^5$ MPa, $\nu=0.25$.
- Cross section of the bolts: $A_p = 50\ mm^2$.

Note:

- All the questions must be solved symbolically (that is, without substituting the numerical values); in addition, sections a) and e) must be solved numerically.