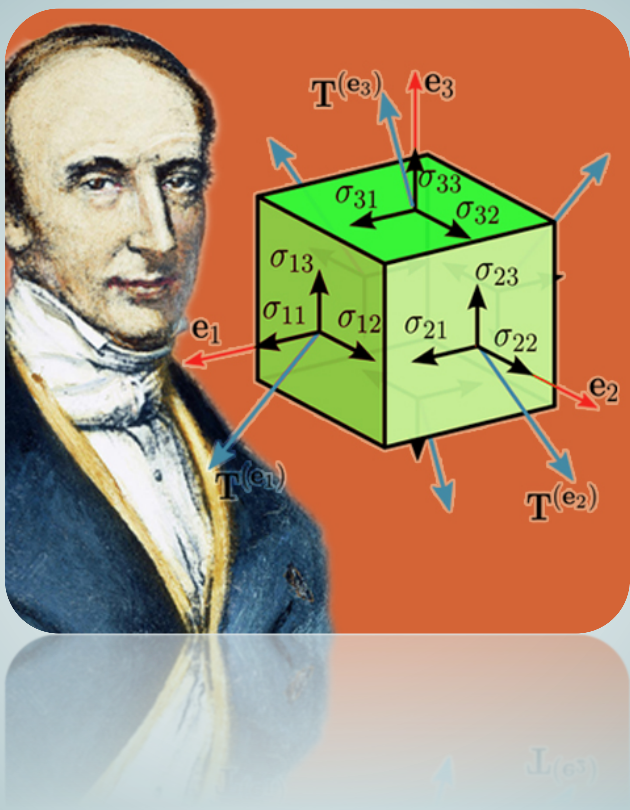
Mechanical Properties of Materials, Processing and Design

Exercises Lesson 4. Fracture mechanics



Diego Ferreño Blanco
Borja Arroyo Martínez
José Antonio Casado del Prado

Department of Terrain and Materials Science and Engineering

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DEPARTAMENTO DE CIENCIA E INGENIERÍA DEL TERRENO Y LOS MATERIALES

ETS de Ingenieros de Caminos, Canales y Puertos

MECHANICAL PROPERTIES OF MATERIALS ***2019-2020***

4: Fracture Mechanics

1. A large steel plate will be subjected to a mode I stress state (tensile mode). The design stress is $\sigma = 0.5 \cdot \sigma_u$, where σ_u represents the material's tensile strength; the sensitivity of the device used to detect defects is $2\ mm$. In order to reduce the weight of the structure it is proposed to change the heat treatment of the steel to raise its ultimate stress from $1500\ MPa$ to $2000\ MPa$.

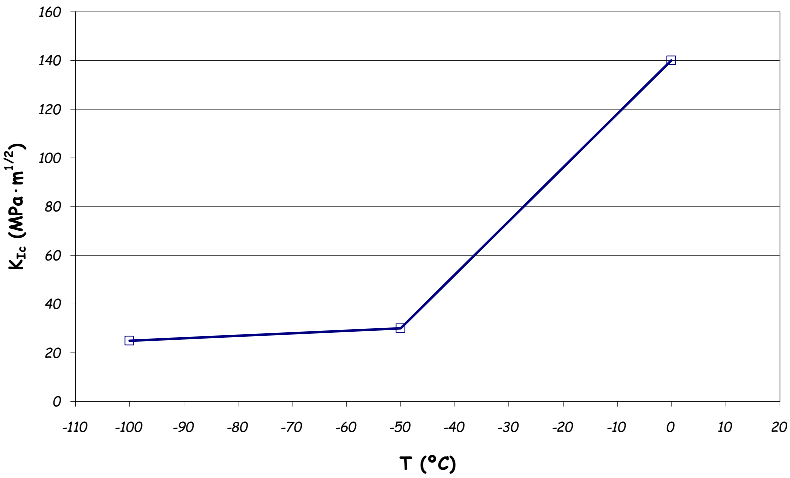
Simultaneously, as a result of this heat treatment, a reduction in the toughness, K_{Ic}, will take place, from $70\ MPa \cdot m^{1/2}$ to $35\ MPa \cdot m^{1/2}$. Assuming plane strain conditions and knowing that the inspection has given a negative result, decide if this proposal is recommended.

Once the appropriate treatment has been done and during an inspection previous to the installation of the plate, a through-thickness crack with a semi-length $2.4\ mm$ is observed. Furthermore, it is found that the real work stress exceeds by $10\ \%$ that considered in the design stage. Determine whether the installation of the plate is compatible with the safety of the structure or not.

Data: $K_I = \sigma \sqrt{\pi a}$; $\sigma_Y = 0.8 \cdot \sigma_u$

2. For the construction of a $6\ m$-diameter spherical tank to store pressurized nitrogen, a plate with thickness $e = 35\ mm$ is going to be used. The plate is made of a steel with a yield stress of $490\ MPa$ and in which some cracks ($6\ mm$ deep) were detected.

The maximum working pressure is $9.8\ MPa$, corresponding to the highest storage temperature predicted ($0\ ^oC$). Given the possibility that that temperature decreases, a set of tests to determine the influence of temperature on the fracture toughness are performed, obtaining as a result the following graph:



a) Determine the minimum safety factor guaranteed by a classical calculation.
b) Determine the storage temperature range that maintains a fracture safety factor equal or higher than the one obtained in a).

It can be assumed that nitrogen behaves like a perfect gas and, for calculation purposes, surface cracks can be considered elliptically shaped, with a stress intensity factor given by:

$$K_I = 2.42 \cdot \sigma \cdot \sqrt{a\left(1.19 + \frac{a}{e}\right)}$$

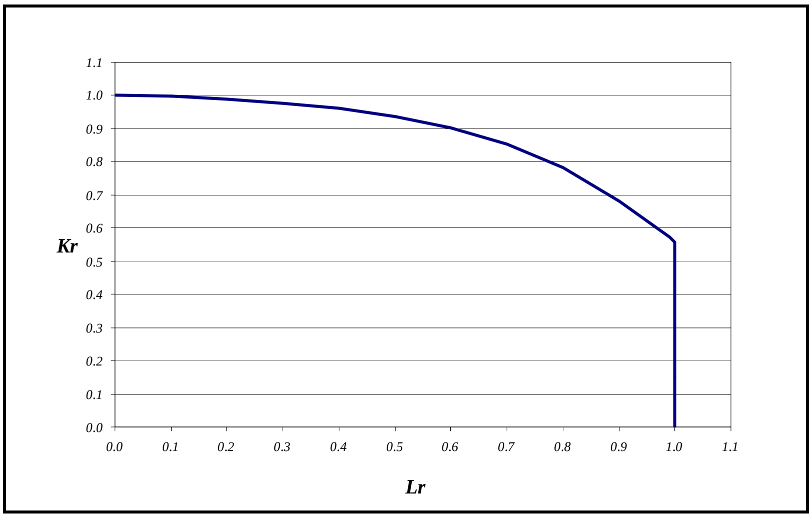
being 'σ' the applied stress, 'a' the crack depth and 'e' the thickness of the plate.

3. Obtain the critical value a crack in a wire ($K_I = 1.2\sigma\sqrt{\pi a}$) working under an applied stress of $\sigma = \sigma_Y / 1.2$, applying the following methods:

a) With the stress intensity factor procedure, considering the Irwin corrections for the local plasticity (plane strain).
b) Using the failure assessment diagram (FAD) attached.

Compare the results obtained.

$$K_r = \frac{K_I}{K_{Ic}}; \; L_r = \frac{\sigma}{\sigma_Y}; \; \sigma_Y = 600\ MPa; \; K_{Ic} = 120\ MPa \cdot m^{1/2}$$

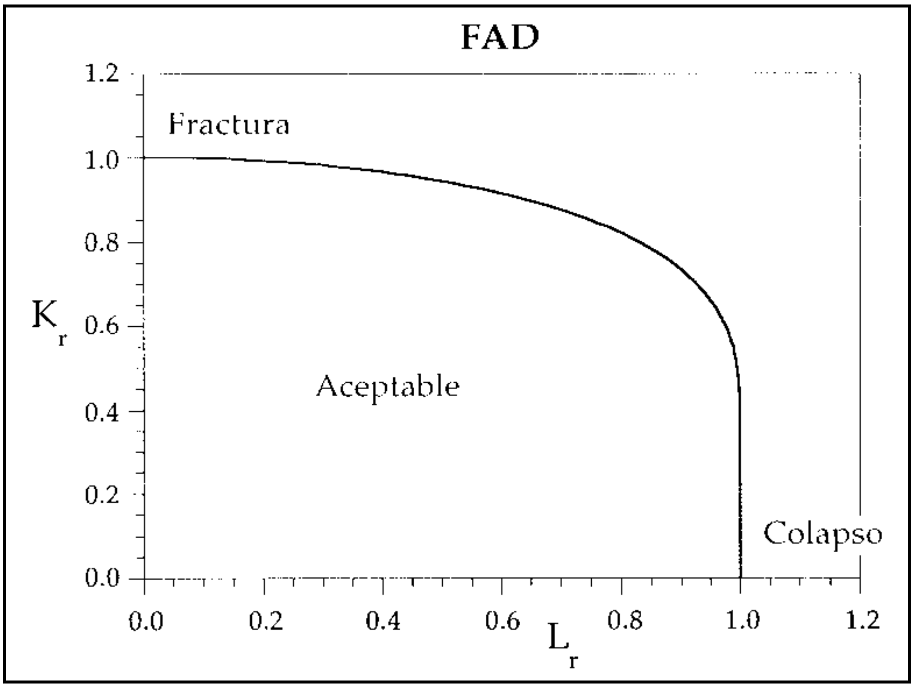


4. A large metallic plate with cracks of length $2a = 20\ mm$ is subjected to working stresses of 50, 150, 250 y $350\ MPa$. Moreover, the material properties over time change as shown in the following table:

$t\,(años)$	0	5	10	15	20
$\sigma_Y\ (MPa)$	500	510	540	565	585
$K_{Ic}\ (MPa{\cdot}m^{1/2})$	150	135	120	100	85

a) Represent in the failure assessment diagram the evolution of the safety conditions in years 0, 10 and 20.
b) Which is the most critical condition?
c) Determine the period of time during which a safety factor higher than 1.2 is guaranteed.

$$K_r = \frac{K_I}{K_{Ic}} \qquad L_r = \frac{\sigma}{\sigma_Y}$$



5. The Structural Integrity of vessels and pressure vessels of American design is regulated by the ASME (*American Society of Mechanical Engineers*) code. This document provides expressions to obtain the stress intensity factor, SIF, assuming that the vessel has a very conservative postulated crack, elliptically-shaped, with a depth (crack length), a, equal to one quarter of the thickness of the wall, t, ($a = t/4$) and length $1.5 \cdot t$.

For the simplest case considered in the code the expression for the SIF, assuming an axial defect (crack plane contains the axis of the cylinder), is the following (p being the pressure and R_i the internal radius of the vessel):

$$K_I\left(MPa \cdot m^{1/2}\right) = 0.450 \cdot \sqrt{t(m)} \cdot p(MPa) \cdot \frac{R_i}{t}$$

The component to be studied is a cylindrical vessel of thickness $20\,cm$ and an internal radius $4.5\,m$, that is subjected to two working cycles that involve different pressure and temperature conditions as shown in the following table. Both temperatures belong to the ductile to brittle transition region (DBTR) of the steel that constitutes the vessel

CICLO	$T(^\circ C)$	$p(MPa)$
1	-10	6
2	-50	18

Recently, there has been developed an approach that describes the behavior of a broad family of steels in the DBTR, known as the Master Curve, which reproduces the statistical dispersion of toughness in the DBTR, see the attached figure, including the failure possibility as a parameter. Toughness depends exclusively on the so-called Reference Temperature that, for the steel considered in this problem, is equal to $T_0 = -100^\circ C$. The toughness, $K_C\left(MPa \cdot m^{1/2}\right)$, as a function of the temperature, $T(^\circ C)$, for a failure probability of 5% and 1%, respectively, corresponds to the following expressions (with an upper bound also included)

$$K_{C,0.05} = 25.2 + 36.6 \cdot e^{0.019 \cdot (T - T_0)}$$
$$K_{C,0.01} = 23.5 + 24.4 \cdot e^{0.019 (T - T_0)}$$
$$K_C \leq 200\ MPa \cdot m^{1/2}$$

From all this information, analyze the fracture behavior of the vessel in each of the working cycles.