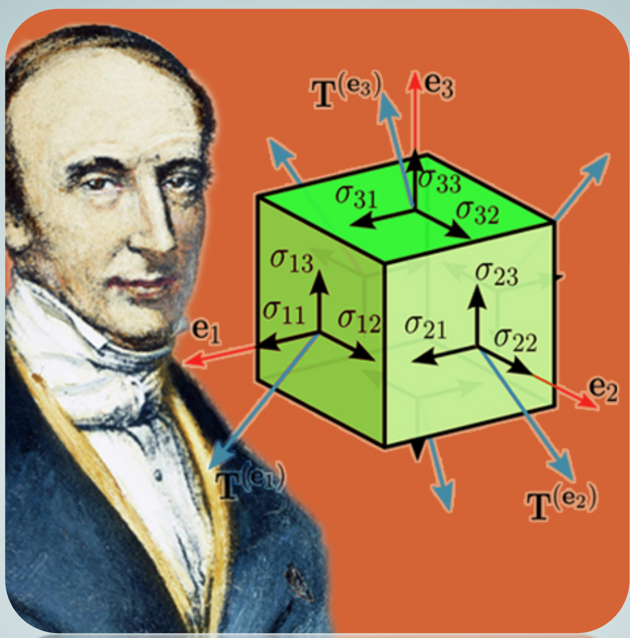
Mechanical Properties of Materials, Processing and Design

Exercises Lesson 5. Fatigue of materials



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DEPARTAMENTO DE CIENCIA E INGENIERÍA DEL TERRENO Y LOS MATERIALES

ETS de Ingenieros de Caminos, Canales y Puertos

UNIVERSIDAD DE CANTABRIA

MECHANICAL PROPERTIES OF MATERIALS ***2019-2020***

5: Fatigue

1. With the purpose of manufacturing a component using an A533 steel with a yield stress of $450\,MPa$ and a fracture toughness of $120\ MPa{\cdot}m^{1/2}$, answer the following questions:

a) Obtain the minimum thickness required to guarantee plane strain conditions.
b) Determine the critical size of an edge crack in a semi-infinite plate subjected to a stress of $0.75{\cdot}\sigma_Y$ in mode I.
c) Calculate the critical crack size of an edge crack in a plate of finite width (W=100 mm).
d) Determine for that case the crack length that leads to a general yielding. Decide whether this will occur or not.
e) A very wide piece of this material (comparable to an infinite plate) is going to be used as part of a high responsibility structural element in a railway bridge, subjected to fatigue stresses between 200 and -100 MPa due to passing trains. A centered crack of length 10 mm (2a) is detected in the plate; determine the number of circulations that it will resist until failure and the failure mode knowing that its fatigue behavior is defined by the following Paris law:

Infinite plate: $K_I = \sigma\sqrt{\pi a}$

Semi-infinite plate: $K_I = 1.12{\cdot}\sigma\sqrt{\pi a}$

Finite plate: $K_I = \dfrac{P}{B\sqrt{W}}\cdot\dfrac{\sqrt{2{\cdot}tg\left(\dfrac{\pi a}{2W}\right)}}{\cos\left(\dfrac{\pi a}{2W}\right)}\cdot\left\{0.752+2.02\dfrac{a}{W}+0.37\left[1-sen\left(\dfrac{\pi a}{2W}\right)\right]^3\right\}$

Paris law: $\dfrac{da}{dN}(m/ciclo)=6.7{\cdot}10^{-10}\left(\Delta K_I\right)^{2.5};\ \ \Delta K_I\left(MPa{\cdot}m^{1/2}\right)$

Propagation threshold: $\Delta K_{th} = 10\ MPa{\cdot}m^{1/2}$

2. An aluminum alloy, normally used in aircraft components, presents a fatigue behavior, analyzed with Basquin's law with the parameters $b=-0.073$ and $\sigma'_f = 340\ MPa$:

a) Determine the number of cycles that the component will resist until final failure if the it is subjected to a stress amplitude $\sigma_a = 75\ MPa$ with $R=-1$.
b) A second structural component, made with the same alloy, continues in service after $3.5{\cdot}10^8$ cycles in the conditions described in a). Determine the maximum stress amplitude admissible to ensure, at least, 10000 cycles of remaining life.
c) It is known that after those $3.5{\cdot}10^8$ cycles, a crack of length 1 mm has been detected. Explain qualitatively how this new information modifies the resolution of b).

d) Assuming that the Stress Intensity Factor is given by $K_I = 1.12\sigma\sqrt{\pi a}$ and that the fatigue crack propagation law is expressed as follows:

$$\frac{da}{dN}(m/ciclo) = 4\cdot 10^{-12}(\Delta K)^{3.0}; \quad \Delta K\left(MPa{\cdot}m^{1/2}\right)$$

estimate the number of cycles used for initiation and propagation of the crack, being the stress level $\sigma_a = 75\ MPa$ with $R = -1$.

$\Delta K_{th} = 2.0\ MPa{\cdot}m^{1/2}$

$K_{Ic} = 40\ MPa{\cdot}m^{1/2}$

Basquin: $\frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f\left(2N_f\right)^b$

Palmgren-Miner: $\sum_i \frac{n_i}{N_{fi}} = 1$

3. The design of structural components to work in regions close to the core of nuclear power plants (NPP) must take into consideration the very specific environmental conditions in which those components have to work during the operating period. The effect of the neutron irradiation caused by the combustible fission produces damage on the crystalline structure of the steels nearby that, from a macroscopic perspective, result in a process of increasing embrittlement with the exposure time.

The surveillance programs of the NPP, take into account this situation and provide the characteristics that the monitoring of the properties of interest of the material must fulfill. Briefly:

- From the beginning of the operation some capsules containing specimens of different nature (toughness, impact, tensile test, …) of the material being monitored are introduced into the vessel, in strategic locations.
- During the refueling outage periods of the plant, the specimens are removed from the capsules to be tested.
- Such tests provide reliable information about the evolution of the properties of the material.

According to the considerations above, study the evolution over time of the safety condition in of one of the beams that supports the jet pumps of the recirculation system. The material properties gathered from the Surveillance Program for the last 3 refueling outages are collected in the following chart:

Outage date	$s_Y(MPa)$	$K_{Ic}\left(MPa{\cdot}m^{1/2}\right)$
June 7, 1996	572	192
June 7, 2000	651	169
June 7, 2004	729	151

Considering the information above:

a) Fit the yield stress and fracture toughness by means of a linear relation over time assuming that the first date given above can be considered as the origin of time and that the time is expressed in months.

b) Estimate the value of the yield stress an fracture toughness that will be (presumably) obtained in the next refueling outage, scheduled for June 2008.

The working conditions of the studied beams imply oscillating solicitations, which, in practice, result in stress states ranging between 300 and 500 MPa. The following information is also available:

- The number of cycles per year is estimated to be 2000.
- The geometrical factor for the calculation of K_I is 1.17.
- During the 1996 outage an ultrasound analysis to detect and measure cracks was performed, with a negative result (no cracks were observed), being the sensitivity of the detection system of 2 mm.
- Paris law is as follows:

$$\frac{da}{dN} = 10^{-11} \cdot (\Delta K_I)^{3.0}$$

c) Calculate the slopes of the lines that in a failure assessment diagram describe the state of the material for the three conditions of damage included

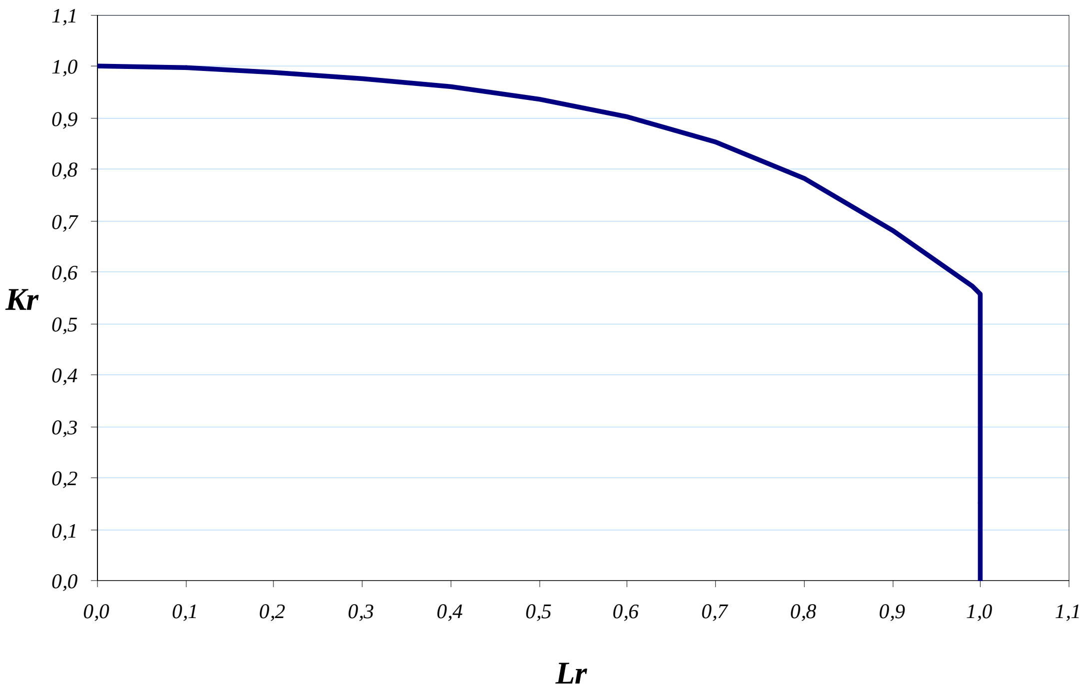
d) in the chart, assuming the worst-case scenario. Add the calculation corresponding to the next refueling outage forecast.

e) Represent on the FAD given, using the results from c), the working conditions of the component exactly after each refueling outage, as well as the ones corresponding to the June 2008 one.

f) Is the component able to resist, with guarantees, until the next refueling outage?

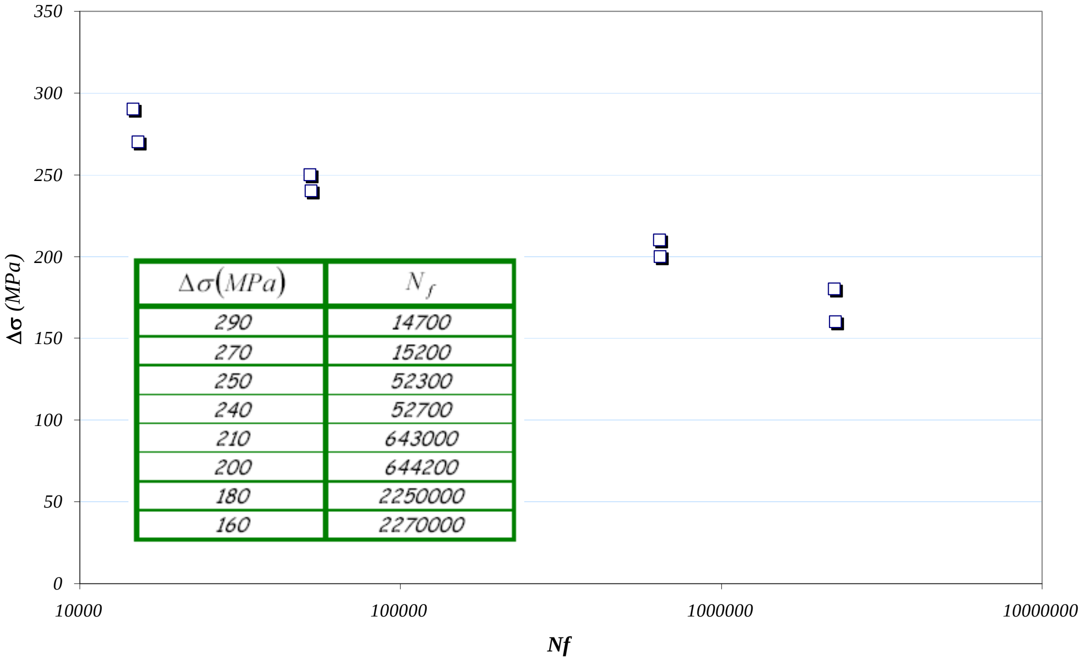
g) Extrapolating the graphical results from d), locate on the FAD the position corresponding to the component failure. Indicate the expected failure mode.

h) Calculate the safety factor values for the four conditions considered.



4. The quality department of a company that manufactures automobiles, aims to study the behavior of a large metal plate that is a part of the frame of a motor. In this kind of components, there has been systematically reported the existence of fatigue problems from edge cracks. As this is a very carefully fabricated element that has passed very strict inspections, the presence of initial cracks can be dismissed.

In order to find out the material properties, a series of tests on small dimension specimens, consisting on fatiguing to failure each of the 8 available specimens, were performed. In all cases, the mean stress value was $\sigma_m = 100\ MPa$ and different stress intervals $\Delta\sigma$ were applied, recording the number of cycles to failure. The results obtained are shown in the table below and represented in the figure.



$\Delta\sigma(MPa)$	N_f
290	14700
270	15200
250	52300
240	52700
210	643000
200	644200
180	2250000
160	2270000

Once the material has been characterized, it is decided to study the problematic behavior in operating conditions. It is known that, on average, the component is subjected every day to three sequences of 10000 cycles each one, with $\Delta\sigma$ = 100, 125 and 150 MPa, respectively, being $\sigma_m = 50\ MPa$. These sequence repeats indefinitely.

Taking these conditions into account:

a) From the experimental results, obtain Basquin's model.
b) Using Miner's method, find out the number of sequences that can be applied on the component until failure; estimate the lifetime.
c) Assuming that only one block is applied ($\Delta\sigma = 150\ MPa$ and $\sigma_m = 50\ MPa$) and that the material responds according to the Paris law shown below, obtain the total number of cycles to failure and the fraction of these that are consumed in the processes of initiation and propagation, respectively.

Basquin's model: $\Delta\sigma \cdot \left(N_f\right)^a = C_1$, being $\Delta\sigma \equiv \Delta\sigma_{\sigma_m = 0}$

Soderberg's law: $\Delta\sigma_{\sigma_m \neq 0} = \Delta\sigma_{\sigma_m = 0} \cdot \left(1 - \dfrac{\sigma_m}{\sigma_Y}\right)$; $\sigma_Y = 500\ MPa$

Paris' law: $\dfrac{da}{dN} = C\left(\Delta K_I\right)^m; C = 1.30 \cdot 10^{-12}; m = 2.6$

Toughness: $K_{Ic} = 120\ MPa \cdot m^{1/2}$

Stress intensity factor: $K_I = 1.12 \cdot \sigma\sqrt{\pi a}$

Propagation threshold: $\Delta K_{th} = 9\ MPa \cdot m^{1/2}$