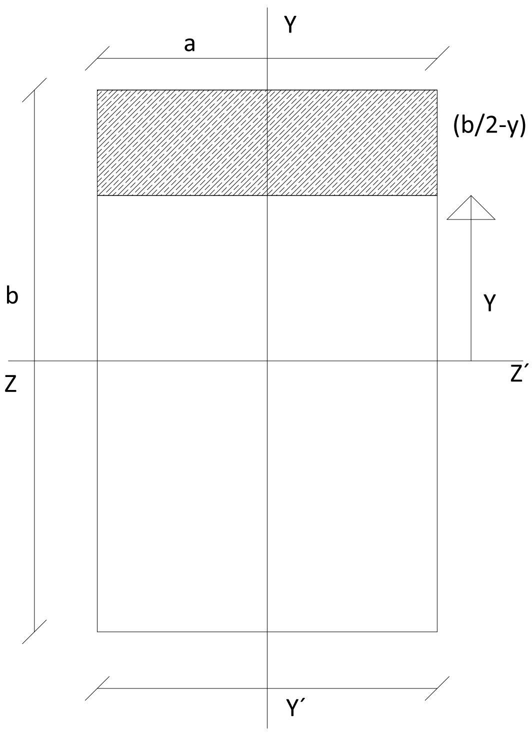
TENSIONES CORTANTES

1º) SECCIÓN RECTANGULAR;



$$Z_y = \frac{QMe}{Ia}$$

Momento Estático:

$$Me = a\left(\frac{b}{2}-y\right)\bullet\left[y+\frac{b/2-y}{2}\right] \rightarrow Me = \frac{a\left(b^2-4y^2\right)}{8}$$

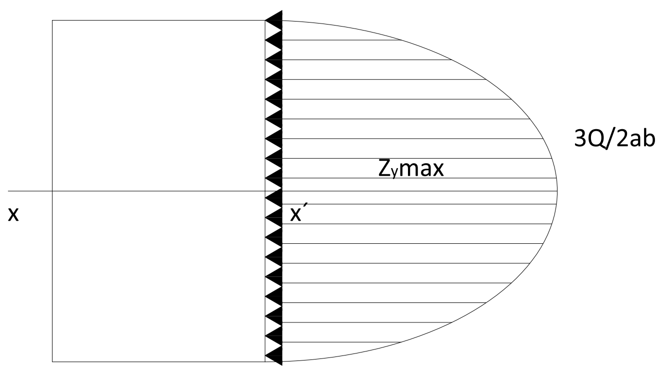
Momento de inercia:

$$I\frac{1}{12}ab^3$$

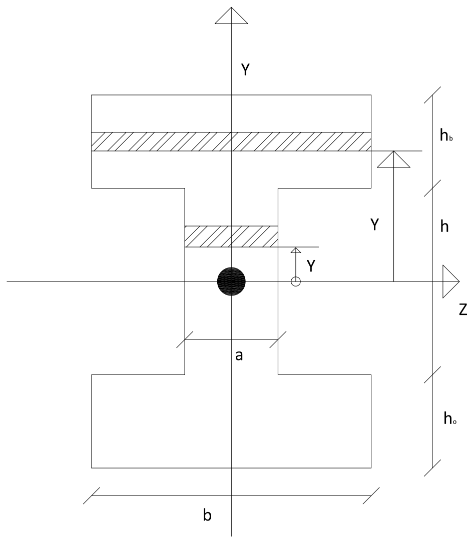
Luego:

$$Z_Y = \frac{Q\frac{a(b^2-4y^2)}{8}}{\frac{1}{12}ab^3a}$$

$$\boxed{Z_Y = \frac{3Q(b^2-4y^2)}{2ab^3}}$$



2º) SECCIONES EN DOBLE T



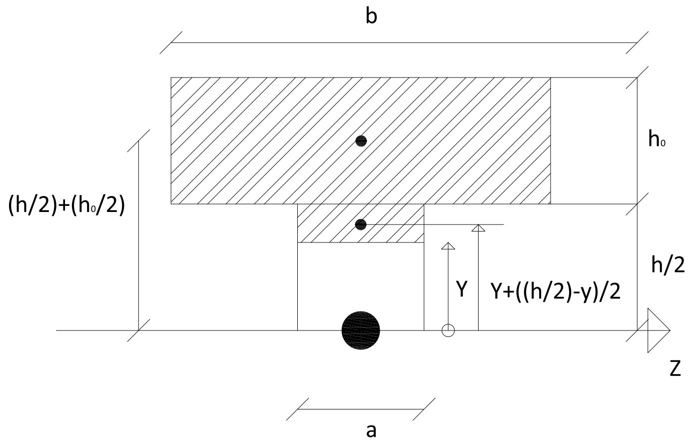
Momento de Inercia;

$$I = 2\int_0^{h/2} a\,dy\,y^2 + 2\int_{h/2}^{h/2+h} b\,dy\,y^2 = 2\,a\left[y^3\big/3\right]_0^{h/2} + 2\,b\left[\frac{y^3}{3}\right]_{h/2}^{h/2+h_0} = \frac{2a}{3}\frac{h^3}{8} + \frac{2b}{3}\left[\left(\frac{h}{2}+h_0\right)^3 - \left(\frac{h}{2}\right)^3\right]$$

$$I = \frac{2ah^3}{24} + \frac{2b}{3}\left[h_0^3 + 3\frac{h^2h_0}{4} + \frac{3hh_0^2}{2}\right]$$

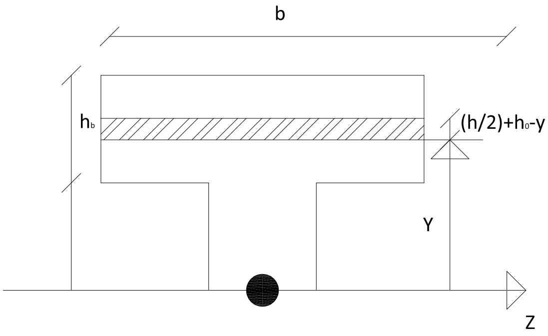
Momento estático;

Dos leyes: $0 \le y \le h\big/2$ y $h\big/2 \le y \le h\big/2 + h_0$



$$Me = bh_0\frac{(h+h_0)}{2} + a\left(\frac{h}{2}-y\right)\left[y + \frac{\frac{h}{2}-y}{2}\right] \rightarrow Me = \frac{bh_0}{2}\left(h_0 + h\right) + a\left(\frac{h}{2}-y\right)\left(\frac{h+2y}{4}\right)$$

Momento estático para $\frac{h}{2} \leq y \leq \frac{h}{2} + h_0$

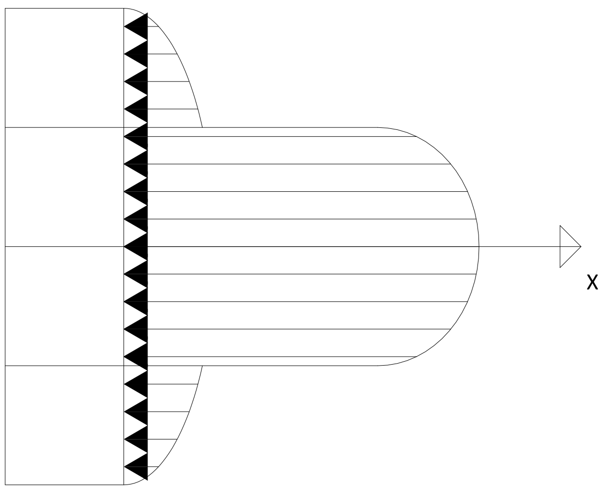


$$Me = b\left(\frac{h}{2} + h_0 - y\right)\left[y + \frac{\frac{h}{2} + h_0 - y}{2}\right] \rightarrow Me = \frac{b}{2}\left(\frac{h^2}{4} + h_0^2 + h_0 h y^2\right)$$

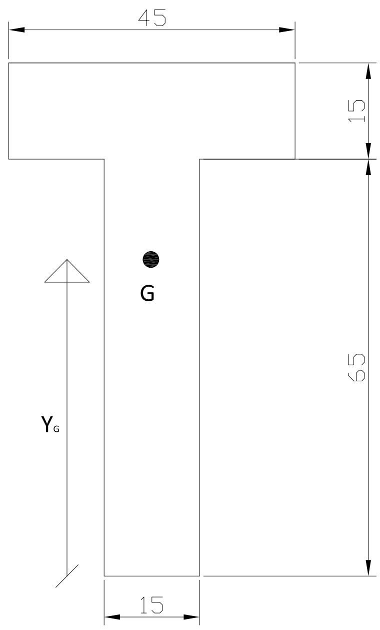
Por tanto el cortante tiene dos leyes;

$$0 \leq y \leq \frac{h}{2} \rightarrow \mathrm{Z} = \frac{Q\left[bh_0(h+h_0) + a\left(h^2/4 - y^2\right)\right]}{2aI}$$

$$\frac{h}{2} \leq y \leq \frac{h}{2} + h_0 \rightarrow \mathrm{Z} = \frac{Q(h_0 + h_0 h + h^2/4 - y^2)}{2I}$$



3º) La sección adjunta está sometida a un My=15 Mpxm en el C.d.G. y a un Q=20 Mp.
Hallar las máximas tensiones.



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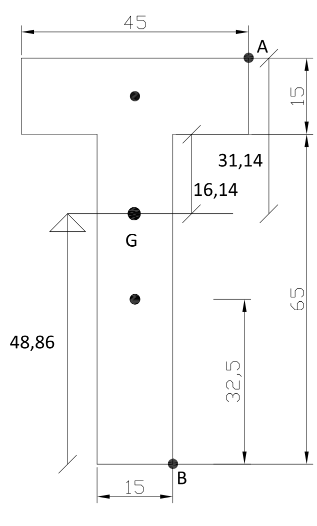
1º - Cálculo C.d.G.

$$Y_g = \frac{\sum Me_i}{\sum A_i} = \frac{45 \bullet 15(65+\frac{15}{2}) + 65 \bullet 15 \bullet \frac{65}{2}}{45 \bullet 15 + 65 \bullet 15} = \frac{80625}{1650} = 48,86cms.$$

2º - Cálculo del Momento de Inercia Ig.

Aplicamos Steiner;



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$$I_G = \frac{1}{12} 45 \bullet 15^3 + 45 \bullet 15 \left(16,14 + \frac{15}{2}\right)^2 + \frac{1}{12} \bullet 15 \bullet 65^3 + 65 \bullet 15 \left(48,86 - \frac{65}{2}\right)^2$$

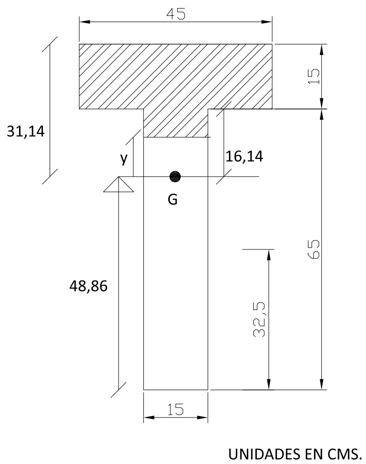
$$I_G = 994.119,34 cm^4$$

Tensiones normales;

- Compresión; $\sigma_A = \frac{My_1}{I} = \frac{15.10^5 \bullet 31,14}{994.119,34} = 47,10 \, {}^{kp}\!/_{cm^2}$
- Compresión; $\sigma_B = \frac{My_2}{I} = \frac{15.10^5 \bullet 48,86}{994.119,34} = 73,9 \, {}^{kp}\!/_{cm^2}$

Tensiones cortantes;

$$Z_y = \frac{QMe}{IQ_y}$$



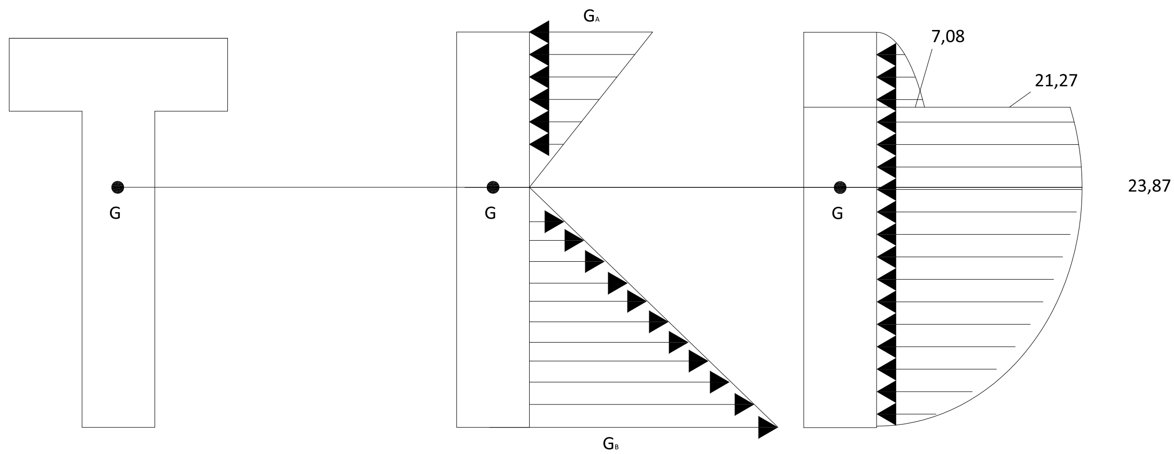
Para $0 \le y \le 16,14$

$$Me = 45 \bullet 15(31,14 - \frac{15}{2}) + (16,14 - y) \bullet 15(y + \frac{16,14 - y}{2})$$

Entonces $Z_y = \dfrac{20.10^3 \bullet 15 \bullet (1193,44 - \frac{y^2}{2})}{994.119,34 \bullet 15} \approx 23,87 - 0,01y^2$

Del mismo modo $16,13 \le y \le 31,14$; $Z_y = 9,69 - 0,01y^2$

Y para $0 \ge y \ge -48,87$; $Z_y = 23,87 - 0,01y^2$



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