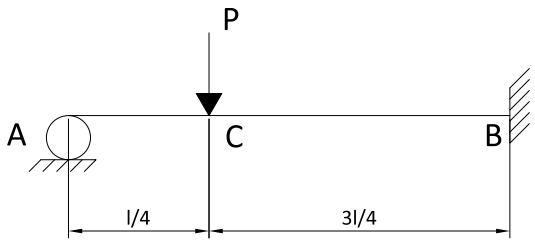
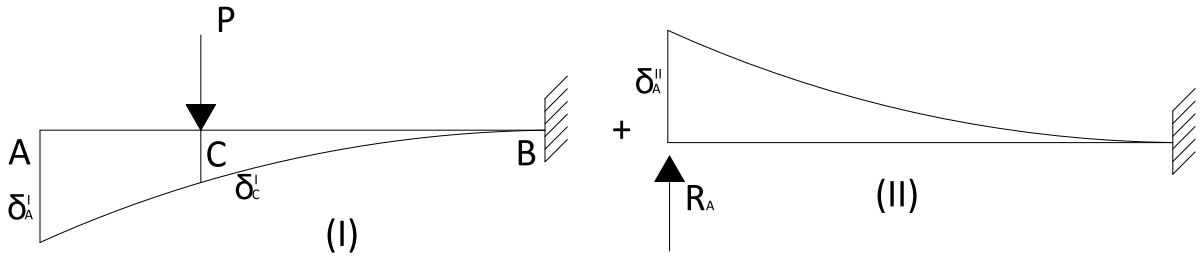
VIGAS HIPERESTÁTICAS

EJERCICIO 1º:



Cálculo de reacciones: Descomponemos en dos estados



$$\delta_A^I = \delta_C^I + \vartheta_C^I \frac{l}{4} \rightarrow \vartheta_C^I = \frac{P(\frac{3l}{4})^2}{3EI}$$

$$\delta_A^{II} = \frac{R_A l^3}{3EI}$$

$$\delta_C^I = \frac{P(\frac{3l}{4})^3}{3EI}$$

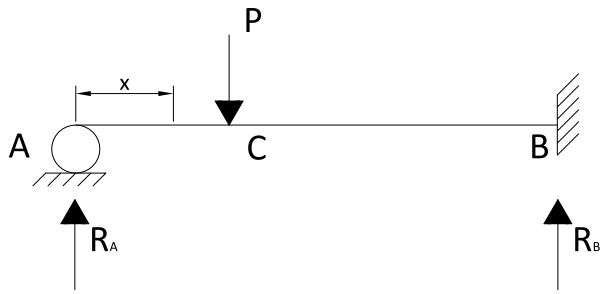
Igualamos los desplazamientos;

$$\frac{P(\frac{3l}{4})^3}{3EI} + \frac{P(\frac{3l}{4})^2}{3EI} \bullet \frac{l}{4} = R_A \frac{l^3}{\cdot EI} \rightarrow R_A = \frac{81P}{128}$$

$$R_B = \frac{47P}{128}$$

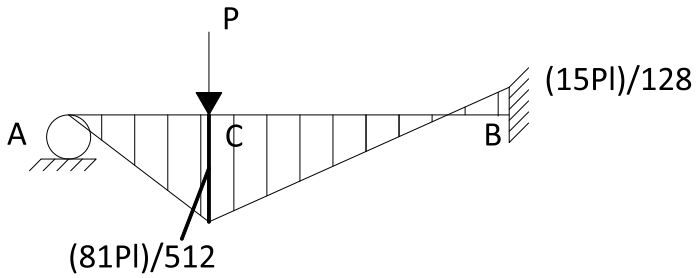
Flectores;

Entre A y C;

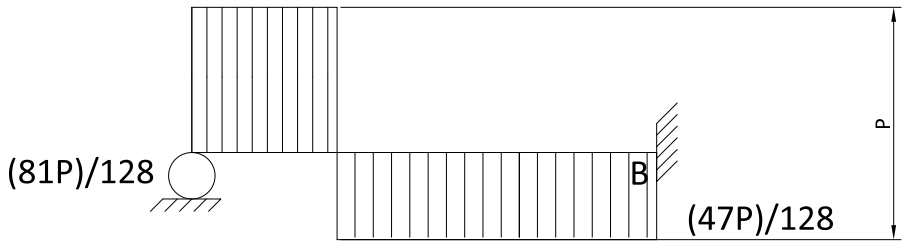


$$M(x) = -\frac{81P_x}{128} \rightarrow Mc = -\frac{81Pl}{512}$$

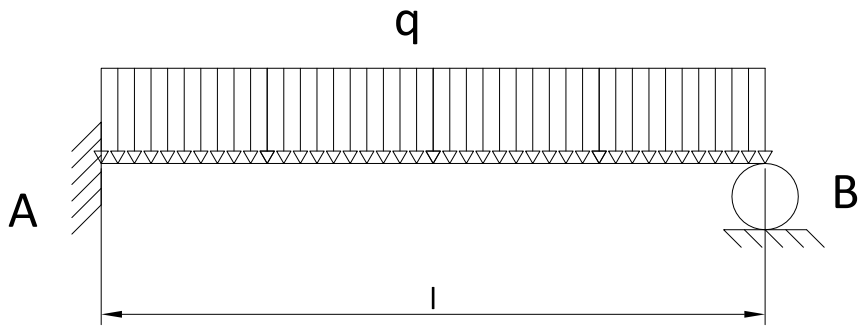
$$M_A - R_A l + P\frac{3l}{4} \rightarrow M_A = 15\frac{Pl}{128}$$



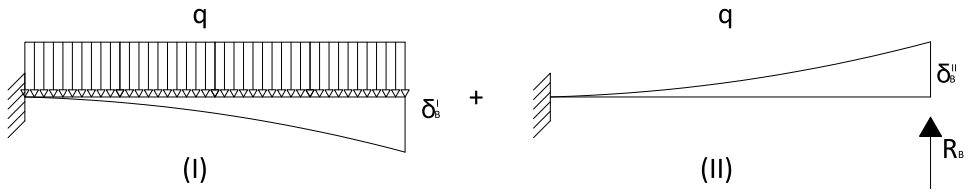
CORTANTES;



EJERCICIO 2º:



Cálculo de reacciones;

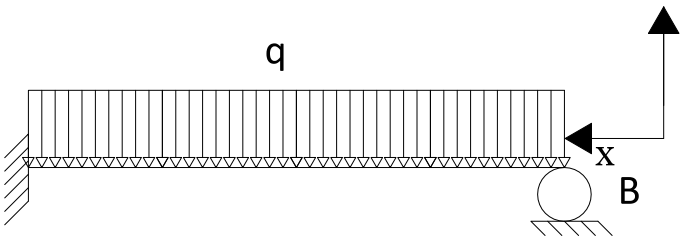


$$\delta_B^I = \frac{ql^4}{8EI} \qquad \delta_B^{II} = R_B \frac{l^3}{3EI}$$

Igualando los desplazamientos;

$$\frac{ql^4}{8EI} = R_B \frac{l^3}{3EI} \rightarrow R_B = \frac{3ql}{8} \rightarrow R_A = \frac{5ql}{8}$$

FLECTORES;



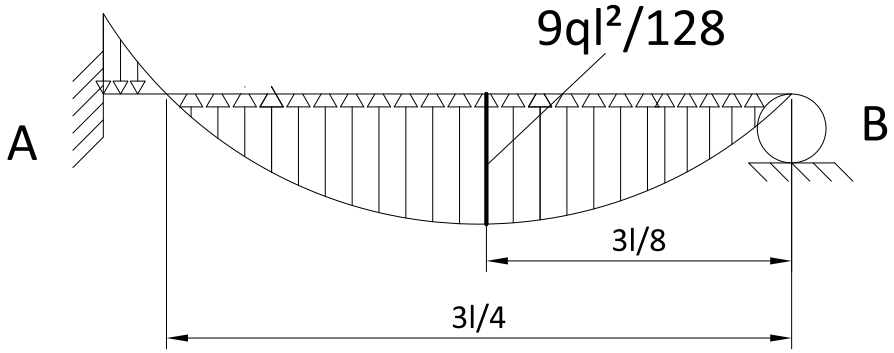
Entre B y A;

$$M(x) = \frac{3qlx}{8} - \frac{qx^2}{2}$$

$$M_A = -\frac{ql^2}{8}$$

Abcisa de momento nulo;

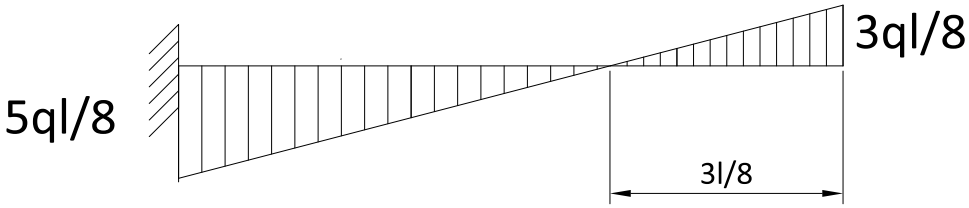
$$M(x) = 0 \rightarrow \frac{3qlx}{8} - \frac{qx^2}{2} = 0 \rightarrow x = 0 \,//\, x = \frac{3l}{4}$$



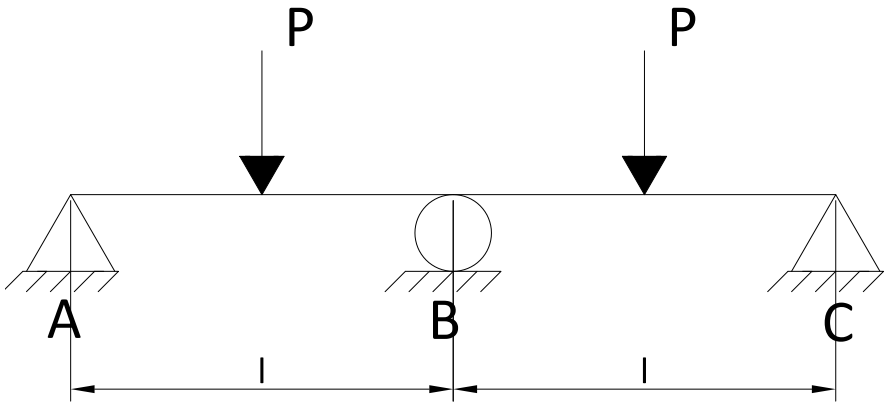
Momento máximo;

$$M'(x) = 0 \rightarrow \frac{3ql}{8} - qx = 0$$

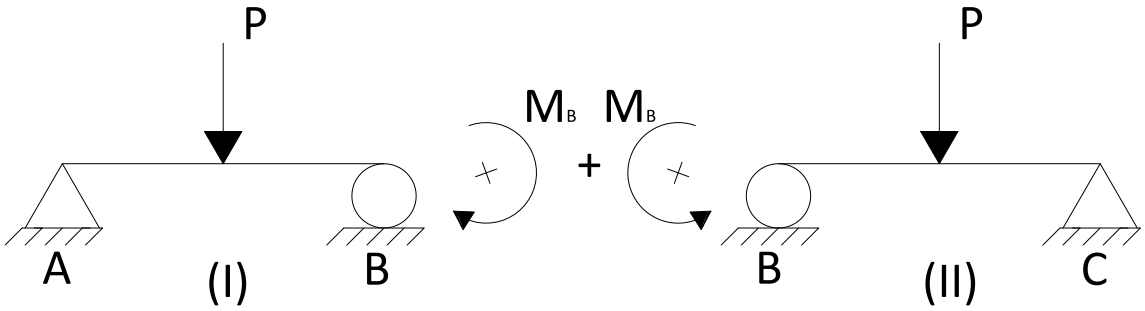
$$M_{\max} = \frac{3ql}{8} \bullet \frac{3l}{8} - \frac{q}{2}\left(\frac{3l}{8}\right)^2 = \frac{9ql^2}{128}$$



EJERCICIO 3º:



Descomponemos en dos estados isostáticos;



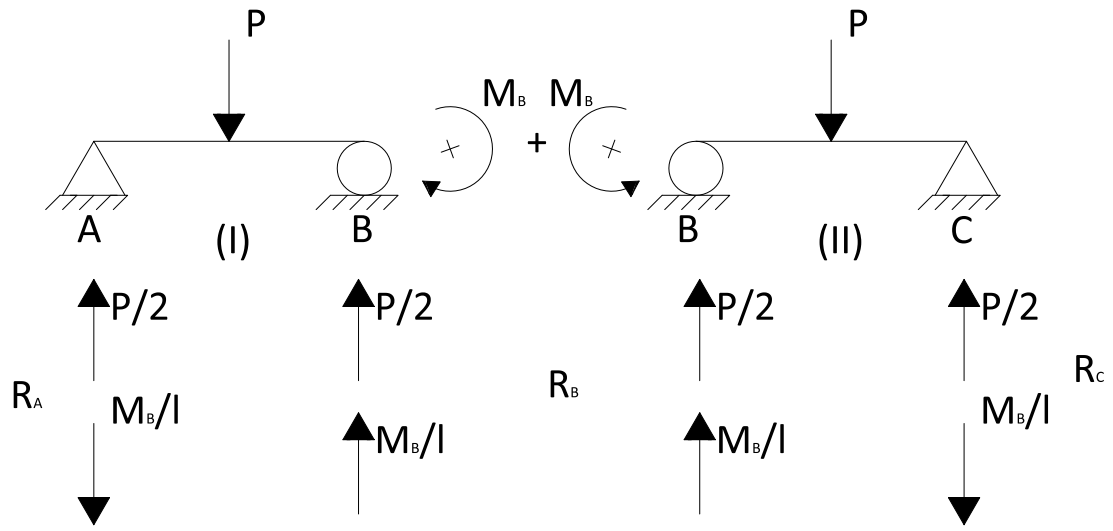
Igualamos el giro en B.

Sabemos que; $\vartheta_B = \frac{Pl^2}{16EI}; \vartheta_B = \frac{M_B l}{3EI}$

Por tanto;

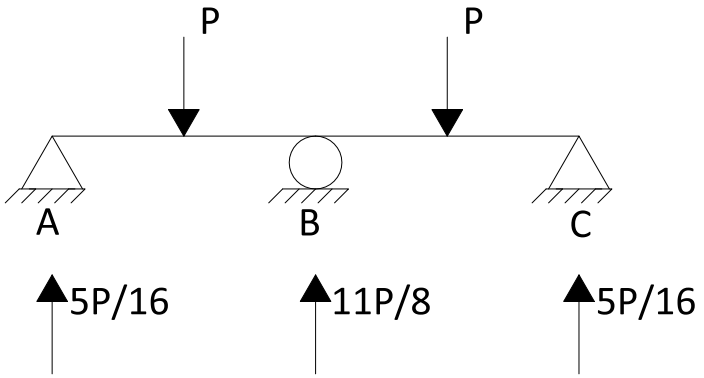
$$\vartheta_B^I = \frac{Pl^2}{16EI} - \frac{M_B l}{3EI} \qquad \vartheta_B^I = \vartheta_B^{II} \rightarrow \frac{Pl^2}{16EI} - \frac{M_B l}{3EI} = \frac{M_B l}{3EI} - \frac{Pl^2}{16EI} \rightarrow \frac{2pl^2}{16} = \frac{2M_B l}{3} \rightarrow M_B = \frac{3Pl}{16}$$
$$\vartheta_B^{II} = \frac{M_B l}{3EI} - \frac{Pl^2}{16EI}$$

Cálculo de reacciones;

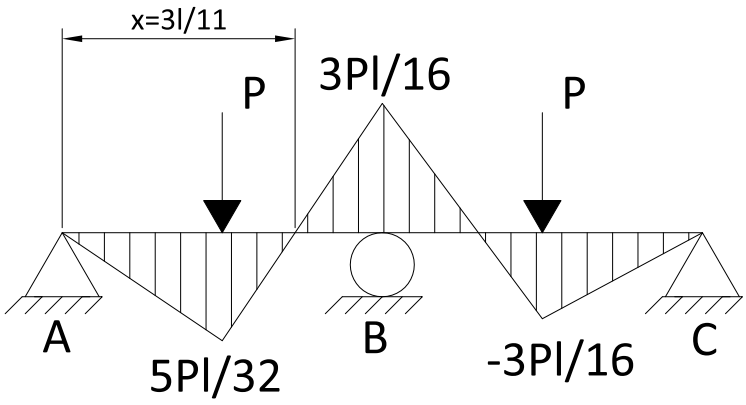


$$R_A = R_C = \frac{P}{2} - \frac{M_B}{l} \rightarrow R_A = R_C = \frac{5P}{16}$$

$$R_B = \frac{P}{2} + \frac{M_B}{l} + \frac{M_B}{l} + \frac{P}{2} \rightarrow R_B = \frac{11P}{8}$$



Por tanto la ley de flectores;

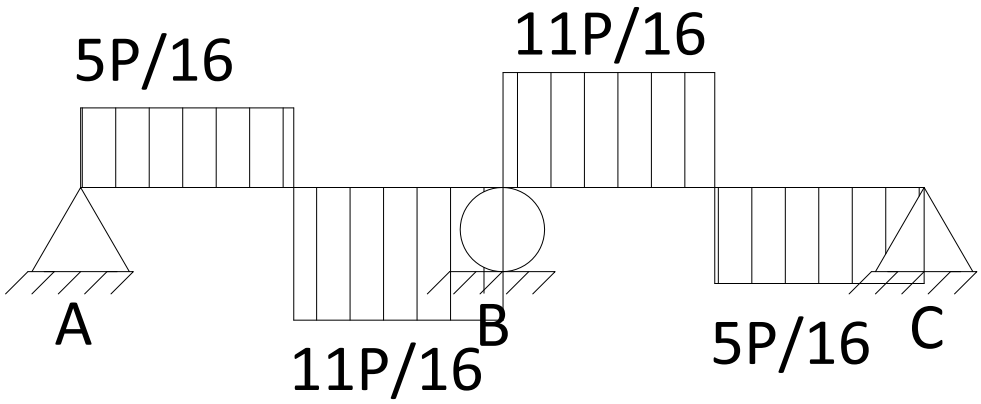


Abcisa de momento nulo;

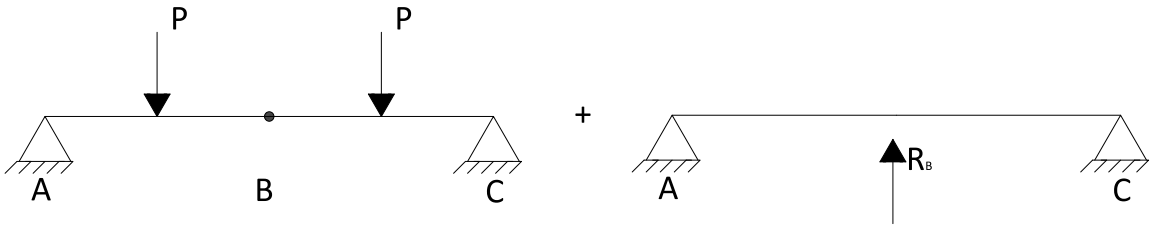
$$M_E^P(x) = -\frac{5P}{16}x + P\left(x - \frac{l}{2}\right)$$

$$M(x) = 0 \rightarrow \frac{5Px}{16} = P(x - \frac{l}{2}) \rightarrow x = \frac{8}{11}l$$

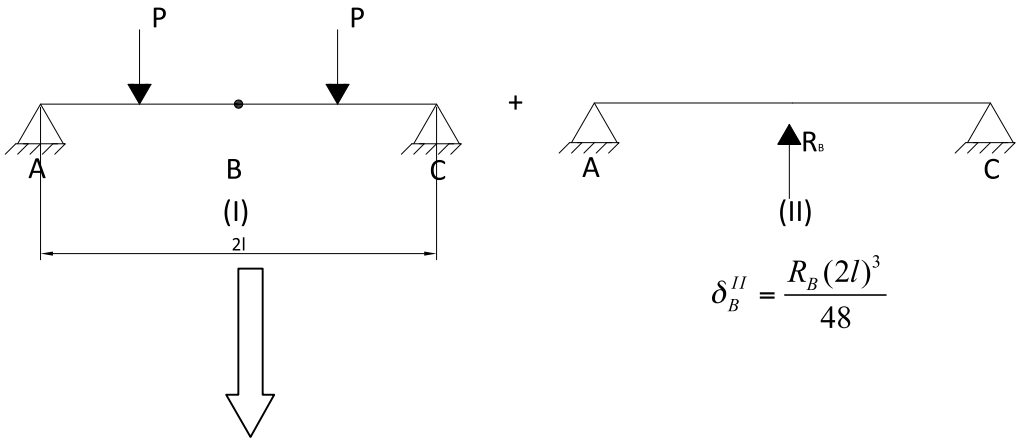
Ley de cortantes;



Otro enfoque; Descomponer en dos estados estáticos



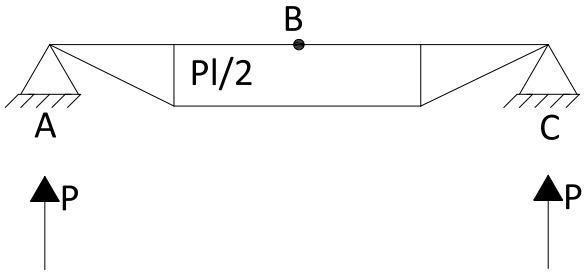
Imponer la condición; Flecha en B=0



$$\delta_B^{II} = \frac{R_B(2l)^3}{48}$$

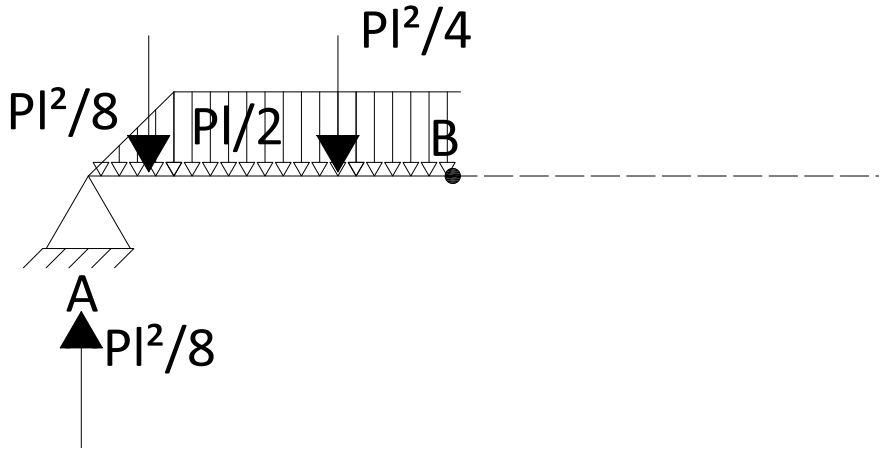
Cálculo de la flecha en B;

Los flectores:



Aplicar el teorema de Mohr;

Carga ficticia;



El momento en B;

$$M_B = -\frac{3Pl^2}{8}l + \frac{Pl^2}{8}(l/2 + \frac{1}{3}l/2) + \frac{Pl^2}{4}\frac{l}{4}$$

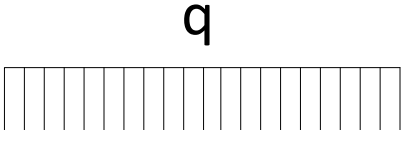
Operando; $M_B = \frac{11Pl^3}{48}$

Luego; $\delta_B^I = \frac{11Pl^3}{48EI}$

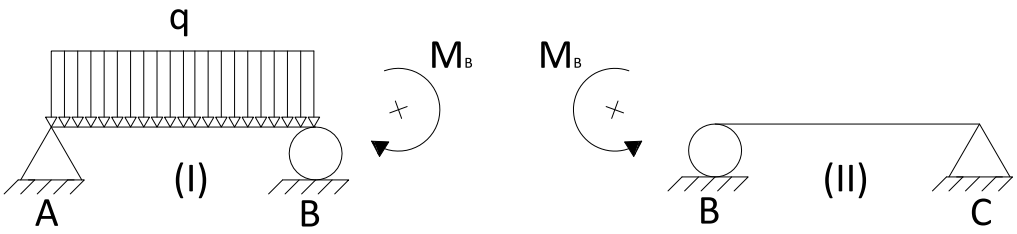
Como $\delta_B^{II} = \frac{8l^3 R_B}{48EI} \rightarrow Igualando \rightarrow \frac{11Pl^3}{48EI} = \frac{8l^3 R_B}{48EI} \rightarrow R_B = \frac{11}{8}P$

El resto es igual.

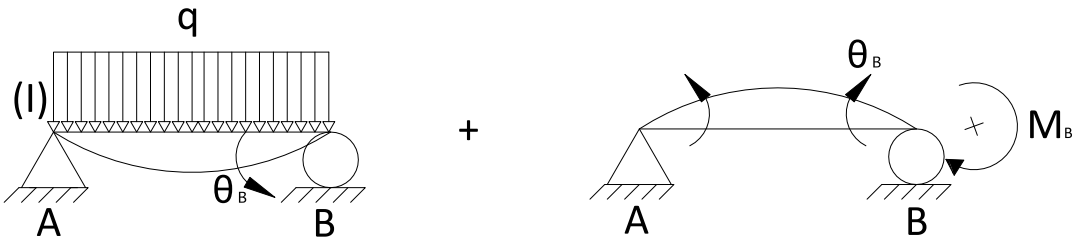
EJERCICIO 4º:



Descomponemos en dos estados;



Igualamos el giro en B;



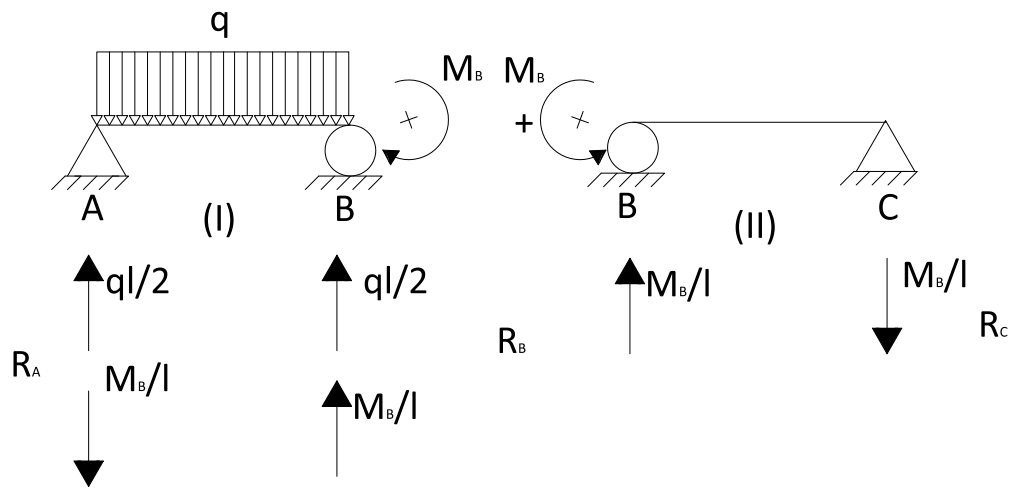
$$\vartheta_B = \frac{ql^3}{24EI} \qquad \vartheta_B = -\frac{M_B l}{3EI}$$

Luego $\vartheta_B^I = \frac{Pl^3}{24EI} - \frac{M_B l}{3EI}$; $\vartheta_B^{II} = \frac{M_B l}{3EI}$

Igualando;

$$\frac{Pl^3}{24EI} - \frac{M_B l}{3EI} = \frac{M_B l}{3EI} \rightarrow M_B = \frac{ql^3}{16}$$

Cálculo de reacciones;



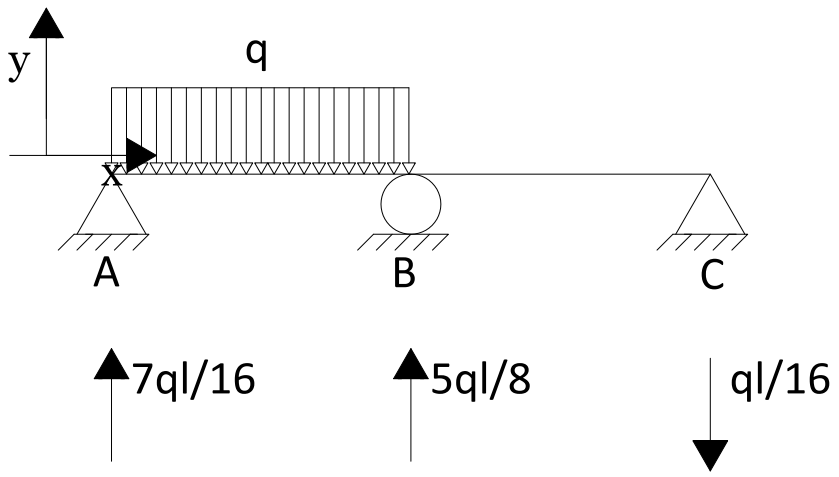
Operando;

$$R_A = \frac{7ql}{16}$$

$$R_B = \frac{5ql}{8}$$

$$R_C = -\frac{ql}{16}$$

Ley de flectores;



$$M_A^B(x) = -\frac{7ql}{16}x + \frac{qx^2}{2} \rightarrow M_A = 0; M_{B(x=l)} = \frac{ql^2}{16}$$

Momento nulo;

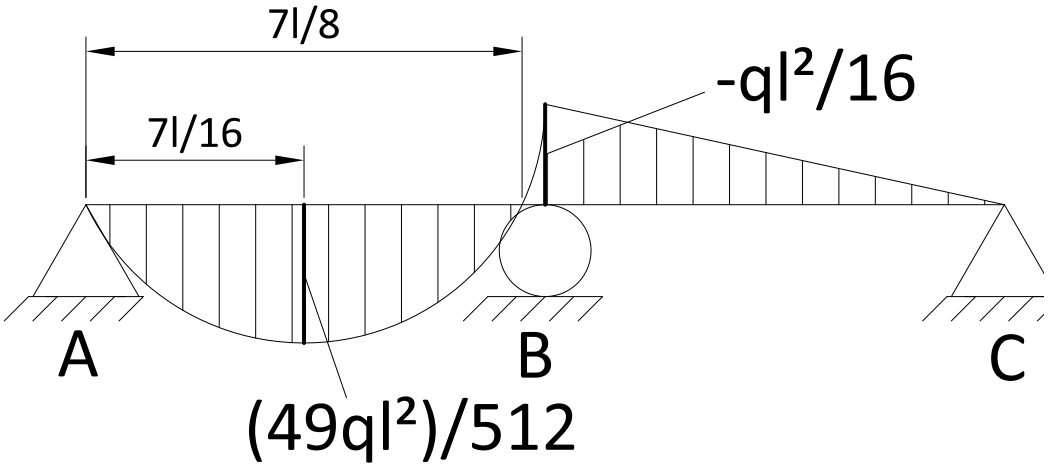
$$M(x) = 0 \rightarrow \frac{-7ql}{16} + \frac{qx^2}{2} = 0 \rightarrow x = 0; x = \frac{7l}{8}$$

Momento máximo: M´(x)=0

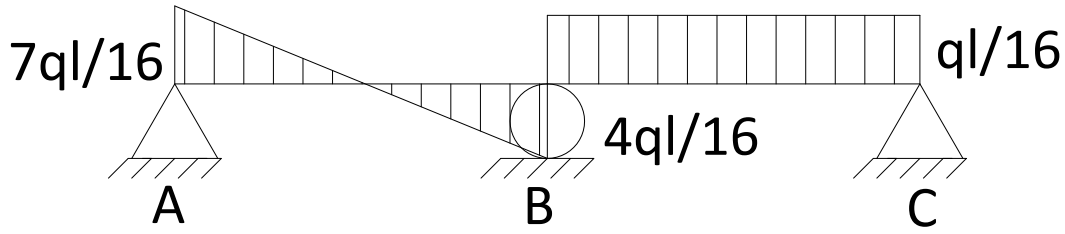
$$\frac{-7ql}{16} + qx = 0 \rightarrow x = \frac{7}{16}l$$

$$M_{\max} = \frac{-7ql}{16}(\frac{7l}{16}) + \frac{q}{2}(\frac{7}{16})^2 l^2 = \frac{49ql^2}{16^2.2}$$

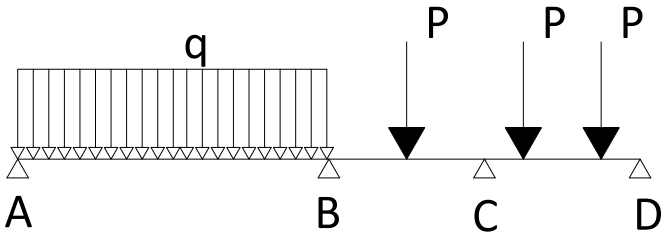
Flectores;



Cortantes;

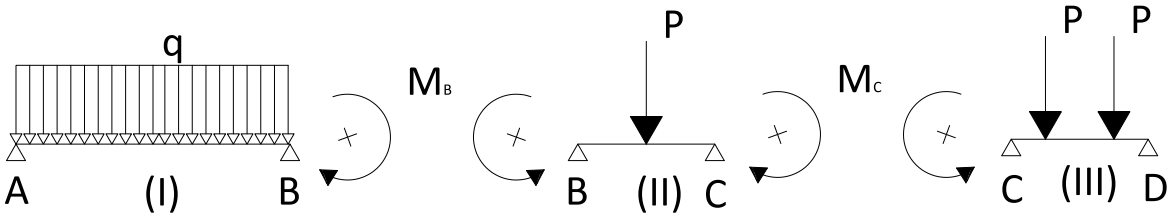


EJERCICIO 5º:



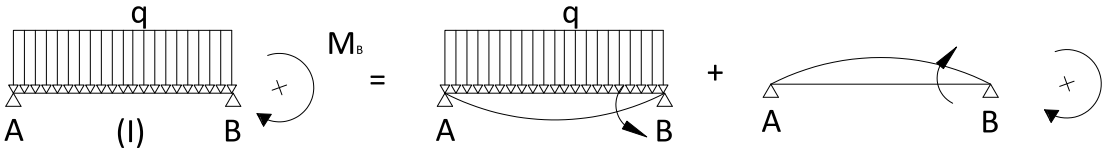
Hiperestática 2º grado:

Descomponemos en 3 estados;



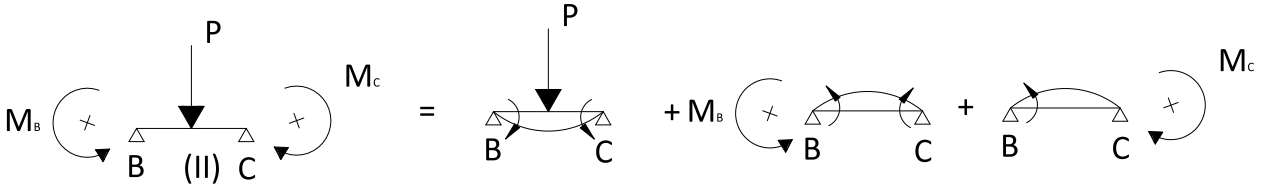
Igualamos giros;

I;



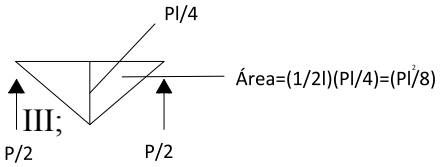
$$\vartheta_B^{'} = \frac{ql^3}{24EI} \qquad \vartheta_B^{''} = -\frac{M_B l}{3EI} \qquad \vartheta_B^{I} = \frac{ql^3}{24EI} - \frac{M_B l}{3EI}$$

II;



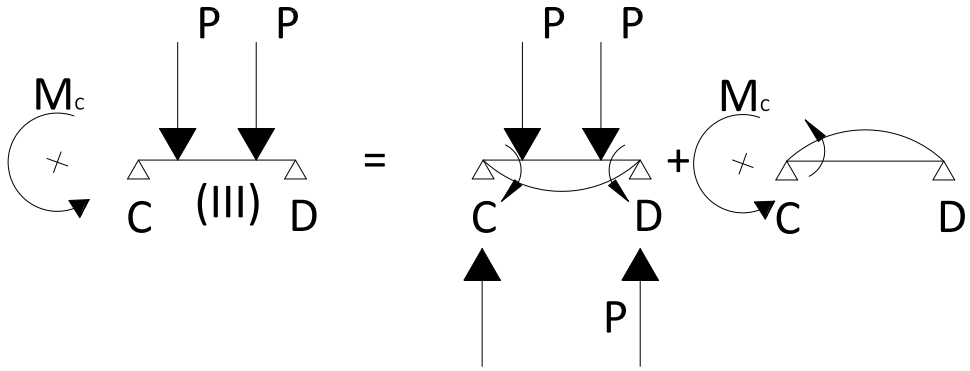
$$\vartheta_B^{'} = -\frac{Pl^3}{16EI} \qquad \vartheta_B^{''} = \frac{M_B l}{3EI} \qquad \vartheta_B^{'''} = \frac{M_C l}{6EI}$$

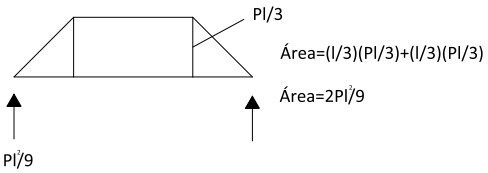
$$\vartheta_C^{'} = \frac{Pl^2}{16EI} \qquad \vartheta_C^{''} = -\frac{M_B l}{6EI} \qquad \vartheta_C^{'''} = -\frac{M_C l}{3EI}$$



$$\vartheta_B^{II} = -\frac{Pl^2}{16} + \frac{M_B l}{3} + \frac{M_C l}{6}$$

$$\vartheta_C^{II} = \frac{Pl^2}{16} - \frac{M_B l}{6} - \frac{M_C l}{3}$$





$$\vartheta_C' = -\frac{Pl^2}{9EI} \qquad \vartheta_C' = -\frac{M_C l}{3EI}$$

$$\vartheta_C^{III} = -\frac{MPl^2}{9} + \frac{M_C l}{3}$$

Igualamos giros en B y C;

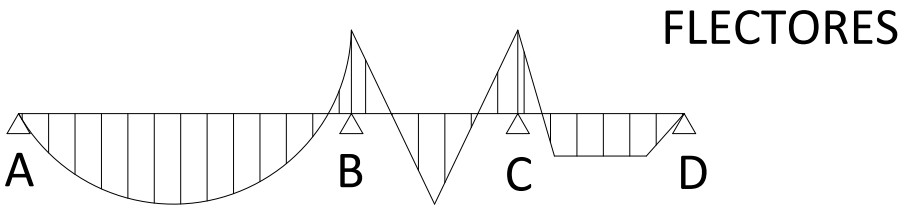
$$\frac{q^2 l^3}{24} - \frac{M_B l}{3} = \frac{-Pl^2}{16} + \frac{M_B l}{3} + \frac{M_C l}{6}$$
$$\frac{Pl^2}{16} - \frac{M_B l}{6} - \frac{M_C l}{3} = -\frac{Pl^2}{9} + \frac{M_C l}{3}$$

$$\frac{ql^2}{24} + \frac{Pl}{16} = \frac{M_B}{3} + \frac{M_C}{6} \qquad \frac{ql^2}{4} + \frac{3Pl}{8} = 4M_B + M_C$$
$$\frac{Pl}{9} + \frac{Pl^2}{16} = \frac{M_B}{6} + \frac{M_C}{3} \qquad \frac{Pl}{3} + \frac{3Pl}{8} = 4M_B + 16M_C$$

$$\frac{ql^2}{4} + \frac{3Pl}{8} - \frac{4PL}{3} - \frac{3PL}{2} = -15M_C$$
$$\frac{ql^2}{4} + Pl\left(\frac{3}{8} - \frac{4}{3} - \frac{3}{2}\right) = -15M_C$$

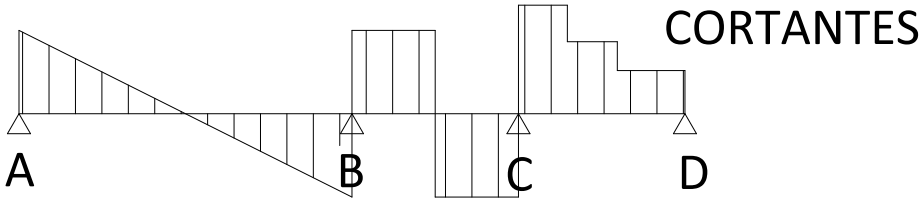
$$\frac{ql^2}{4} - \frac{57}{24}Pl = -15M_c \rightarrow M_C = \frac{57}{15.24}Pl - \frac{ql^2}{60}$$

$$\frac{ql^2}{4} + \frac{3}{8}Pl = 16M_B + M_C$$
$$\frac{Pl}{3} + \frac{3}{8}Pl = M_B + 4M_C$$
$$ql^2 + \frac{3}{2}Pl - \frac{Pl}{3} - \frac{3Pl}{8} = 15M_B$$
$$ql^2 + Pl\left(\frac{3}{2} - \frac{1}{3} - \frac{3}{8}\right) = 15M_B$$
$$ql^2 + \frac{19Pl}{24} = 15M_B$$
$$M_B = \frac{ql^2}{15} + \frac{19Pl}{15.24}$$

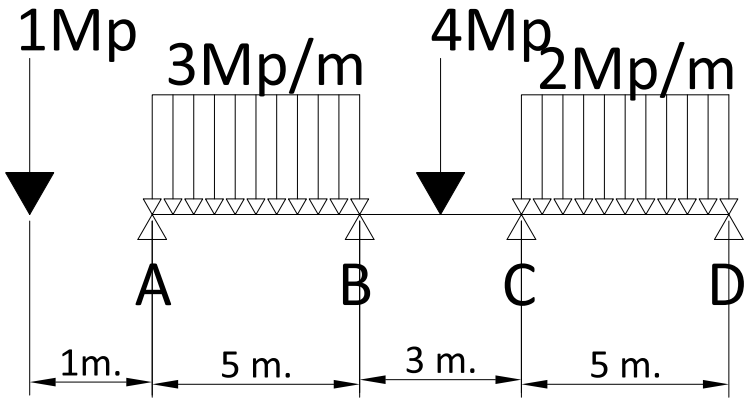


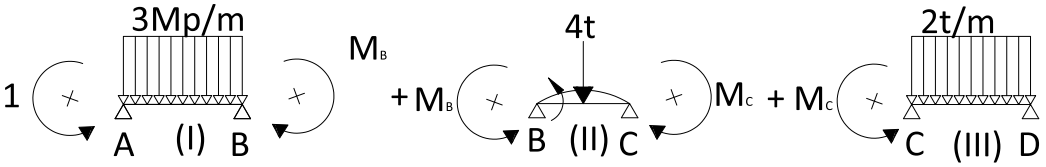
Aplicar a los datos; L=4 m., P=2 Mp., q= 1 kp.

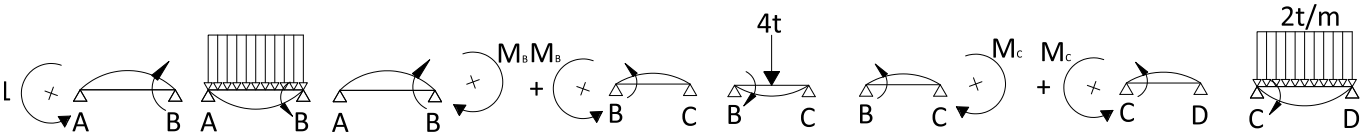
Calcular loas máximas y nudos



EJERCICIO 6º:



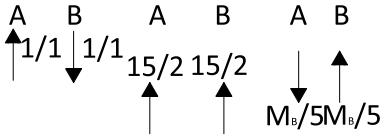




$G_B = \frac{-Ml}{6EI}$ $G_B = +\frac{-ql^3}{24EI}$ $G_B = -\frac{M_B l}{3EI}$ $G_B = \frac{M_B l}{3EI}$ $G_B = -\frac{Pl^2}{16EI} G_B = \frac{M_C l}{6EI}$ $G_C = \frac{M_C l}{3EI}$ $G_C = -\frac{9l^3}{24EI}$

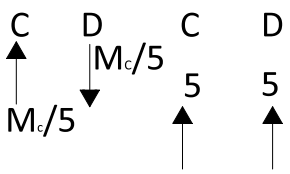
$G_B = \frac{-1.5}{6}$ $G_B = \frac{3.5^3}{24}$ $G_B = -\frac{M_B.5}{3}$ $G_B = -\frac{M_B.3}{3}$ $G_B = -\frac{4.3^2}{16} G_C = \frac{M_C.3}{6}$ $G_C = \frac{M_C.5}{3}$ $G_C = -\frac{2.5^3}{24}$

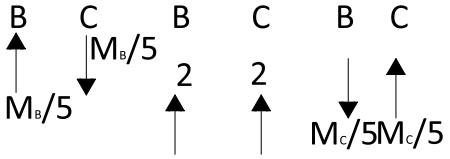
$$G_B = \frac{3.5^3}{24} - \frac{5}{6} - \frac{M_B.5}{3} \qquad G_B = M_B + \frac{M_C}{2} - \frac{4.9}{16} \qquad G_C = \frac{5}{3} M_C - \frac{2.5^3}{24}$$



$$G_C = -\frac{M_B l}{6EI} + \frac{Pl^2}{16EI} - \frac{M_C l}{3EI}$$

$$G_C = -\frac{M_B 3}{6EI} + \frac{4.3^2}{16} - \frac{M_C 3}{3EI}$$





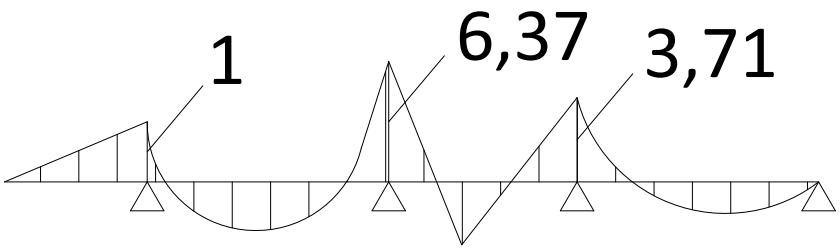
$$\frac{5^3}{8}-\frac{5}{6}-\frac{5M_B}{3}=M_B+M_C-\frac{4.9}{16} \qquad M_B+\frac{5}{3}M_B+\frac{M_C}{2}=\frac{125}{8}-\frac{5}{6}+\frac{9}{4}$$
$$\frac{9}{4}-M_C-\frac{M_B}{2}=\frac{5}{3}M_C-\frac{5^3}{12} \rightarrow \qquad -\frac{M_B}{2}-M_C-\frac{5}{3}M_C=-\frac{125}{12}+\frac{9}{4}$$

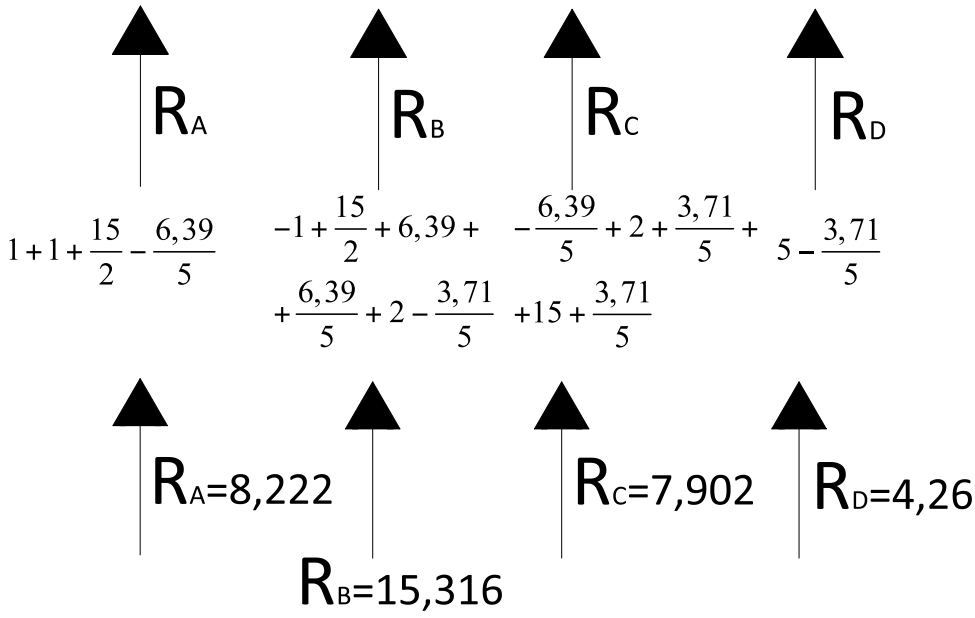
$$\frac{8}{3}M_B+\frac{M_C}{2}=\frac{375-20+54}{24} \qquad \frac{8}{3}M_B+\frac{M_C}{2}=\frac{409}{24} \rightarrow \qquad 16M_B+3M_C=\frac{409}{4}$$
$$\frac{M_B}{2}+\frac{8M_C}{3}=\frac{125+27}{12} \rightarrow \qquad \frac{M_B}{2}+\frac{8M_C}{3}=\frac{152}{12} \qquad 3M_B+16M_C=\frac{152}{12}$$

$$16M_B+3M_C=\frac{409}{4} \qquad 16.3M_B+9M_C=\frac{409.3}{4}$$
$$3M_B+16M_C=\frac{152}{12} \rightarrow \qquad 16.3M_B+16M_C=\frac{16.152}{2}$$

$$(16^2-9)M_C=8{,}15^2-\frac{409.3}{4}$$
$$245M_C=1216-306{,}75$$
$$M_C=\frac{909{,}25}{245}=3{,}71MpxM$$

$$M_B=\frac{1}{16}\left(\frac{409}{4}-11{,}13\right)$$
$$M_B=6{,}39Mpxm$$



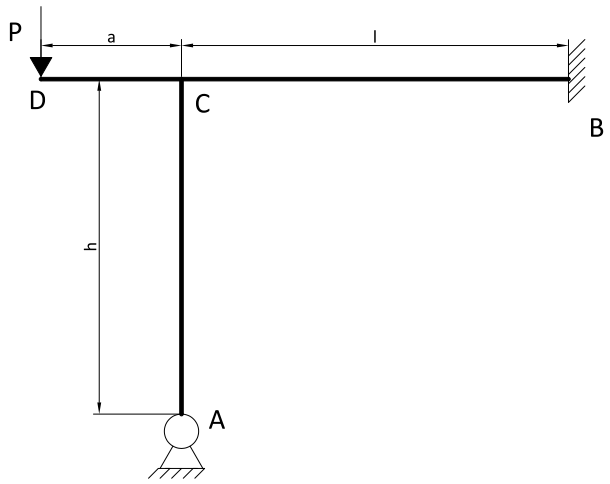


COMPROBACIÓN $\sum R_A+R_B+R_C+R_D=30Mp$

PÓRTICOS

PROBLEMA 1º;

Hallar las reacciones y las leyes Mf, Q y N.

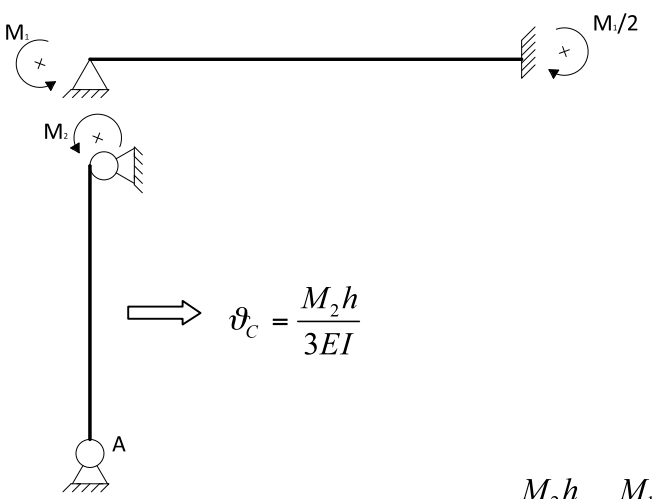


Hay 2 coacciones hiperestáticas que podemos calcular imponiendo 2 condiciones;

1º) El giro en C será igual en el soporte y en el dintel.

2º) Equilibrio del nudo C.

1º)

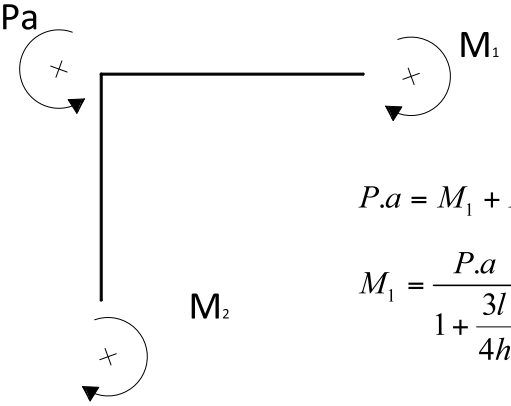


$$\Longrightarrow \vartheta_C = \frac{M_1 l}{3EI} - \frac{M_1}{2}\frac{l}{6EI}$$

$$\Longrightarrow \vartheta_C = \frac{M_2 h}{3EI}$$

Luego; $\dfrac{M_2 h}{3EI} = \dfrac{M_1 l}{4}$

2º)

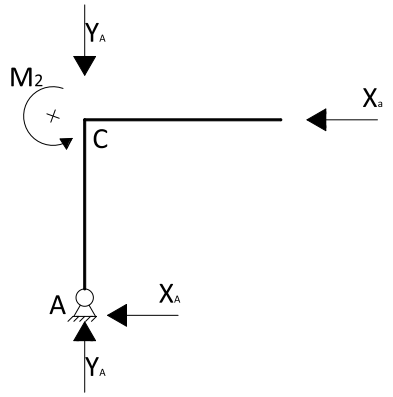


$$P.a = M_1 + M_2 = M_1 + M_1\frac{3l}{4h} = M_1(1+\frac{3l}{4h})$$

$$M_1 = \frac{P.a}{1+\frac{3l}{4h}} \qquad \text{Y} \qquad M_2 = \frac{P.a}{1+\frac{4h}{3l}}$$

Cálculo de las reacciones;

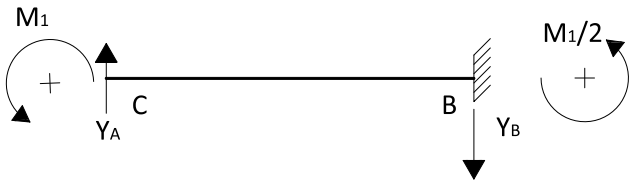
Para el soporte;



$$\sum M_e = 0 \rightarrow -X_a.h + M_2 = 0 \rightarrow X_A = \frac{M_2}{h}$$

$$X_A = \frac{P.a}{h(1+\frac{4h}{3l})}$$

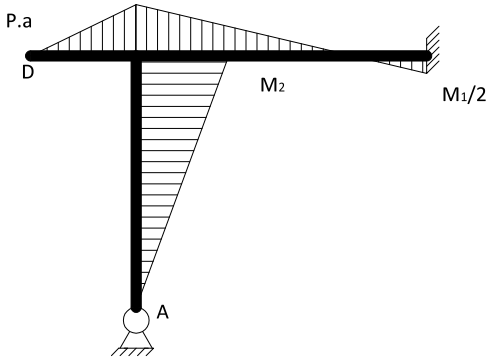
Para el dintel;



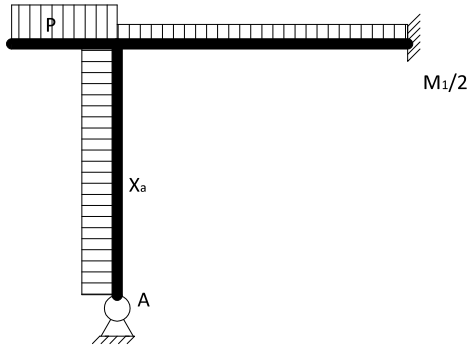
$$\sum M_B = 0 \rightarrow M_1 - Y_A.l + \frac{M_1}{2} = 0 \qquad Y_A = \frac{3Pa}{2l(1+\frac{3l}{h})}$$

$$Luego; Y_A = \frac{3M_1}{2l}$$

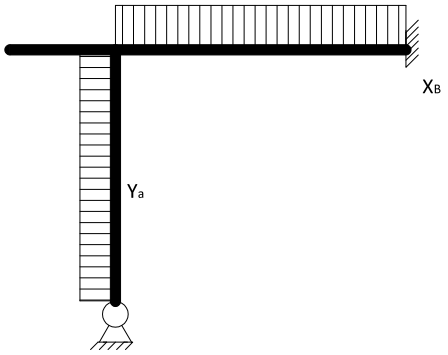
FLECTORES



CORTANTES

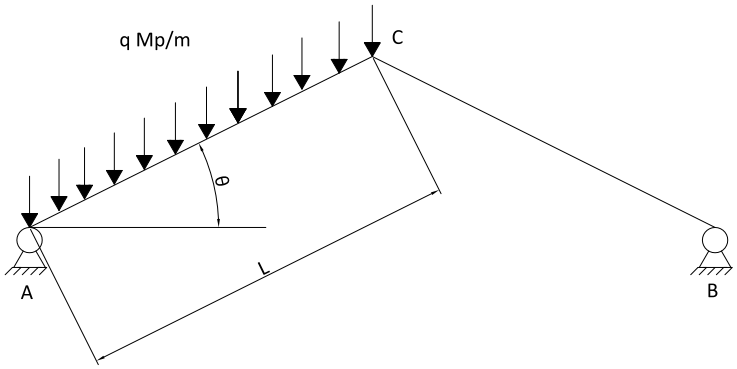


AXILES

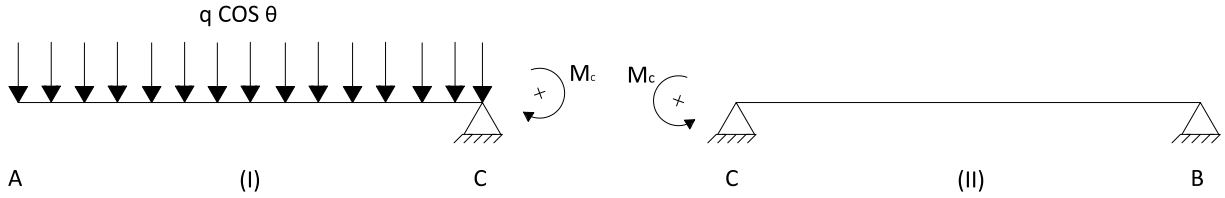


PROBLEMA 2º;

Determinar la ley de momentos flectores en el pórtico;



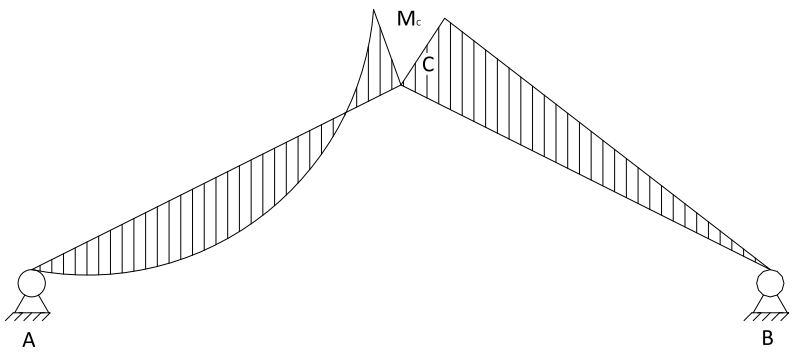
Igualamos los giros en C;



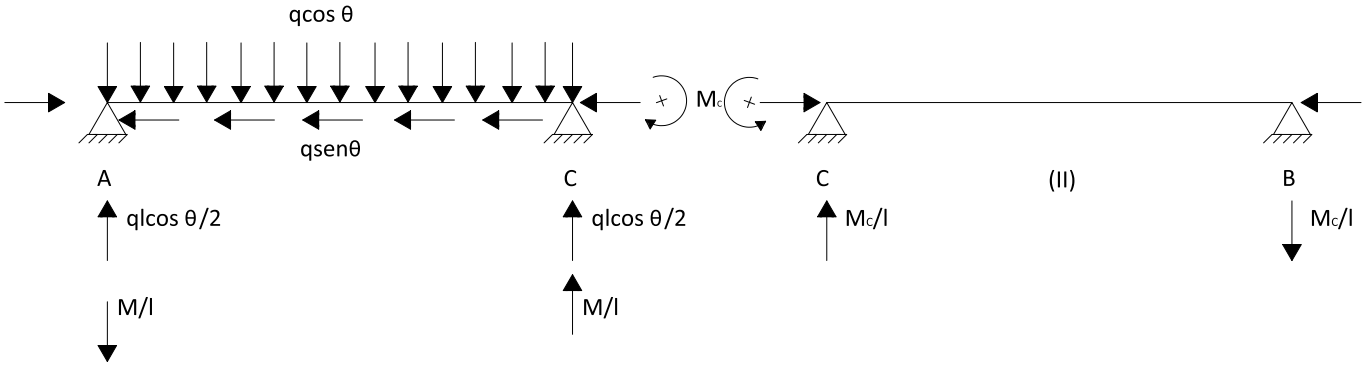
$$\vartheta_C^I = \frac{q\cos\vartheta . l^3}{24EI} - \frac{M_C . l}{3EI} \qquad \vartheta_C^{II} = \frac{M_C . l}{3EI}$$

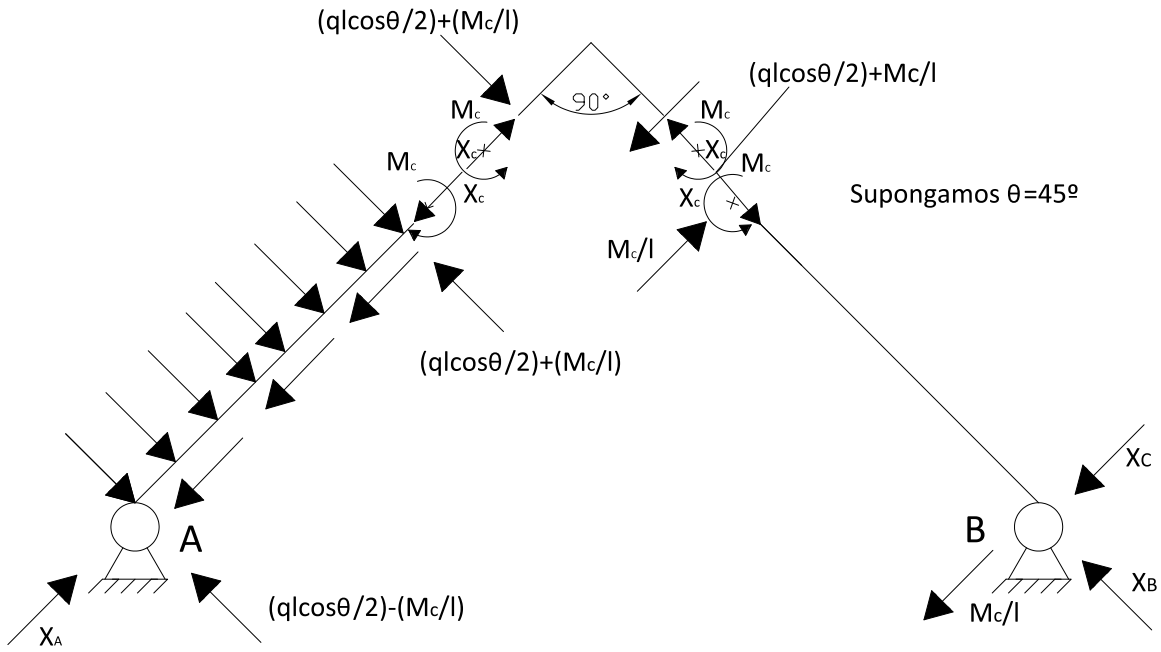
Luego; $\frac{q\cos\vartheta . l^3}{24EI} = \frac{2M_C . l}{3EI} \rightarrow M_C = \frac{q\cos\vartheta . l^2}{16}$

Luego la ley de flectores;



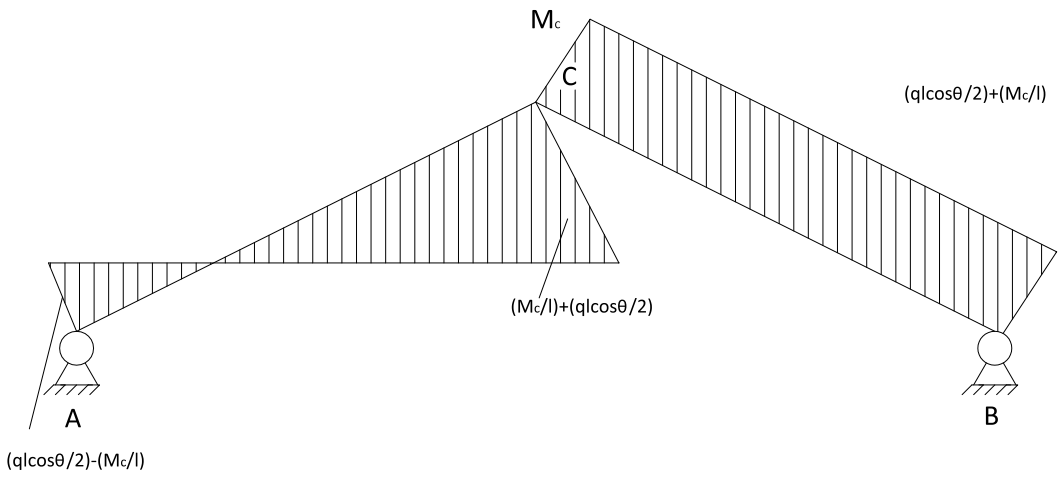
Reacciones;

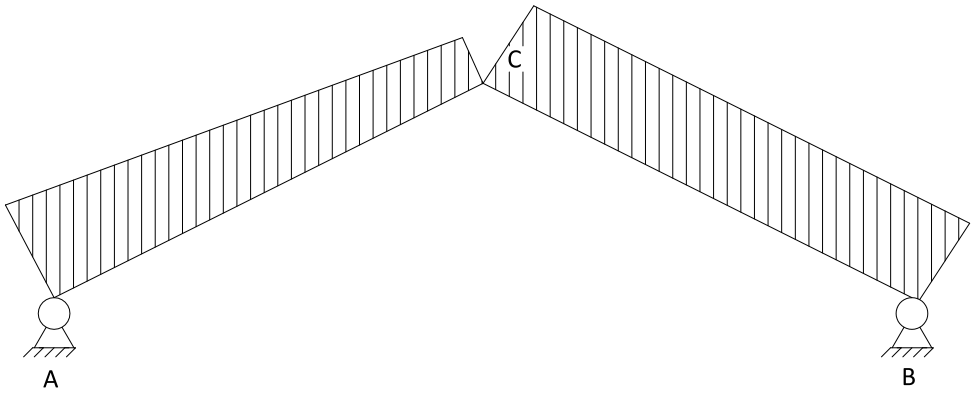




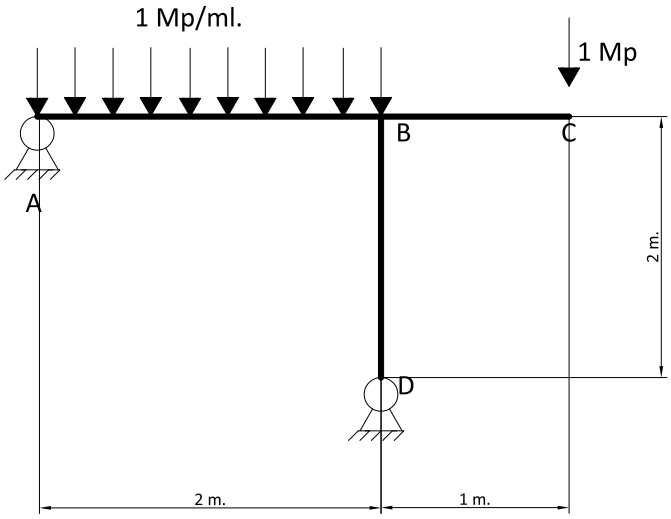
$$X_c = X_B = \frac{ql\cos\vartheta}{2} + \frac{M_c}{l} \rightarrow X_A = X_C + qlsen\vartheta$$

El resto de las leyes;

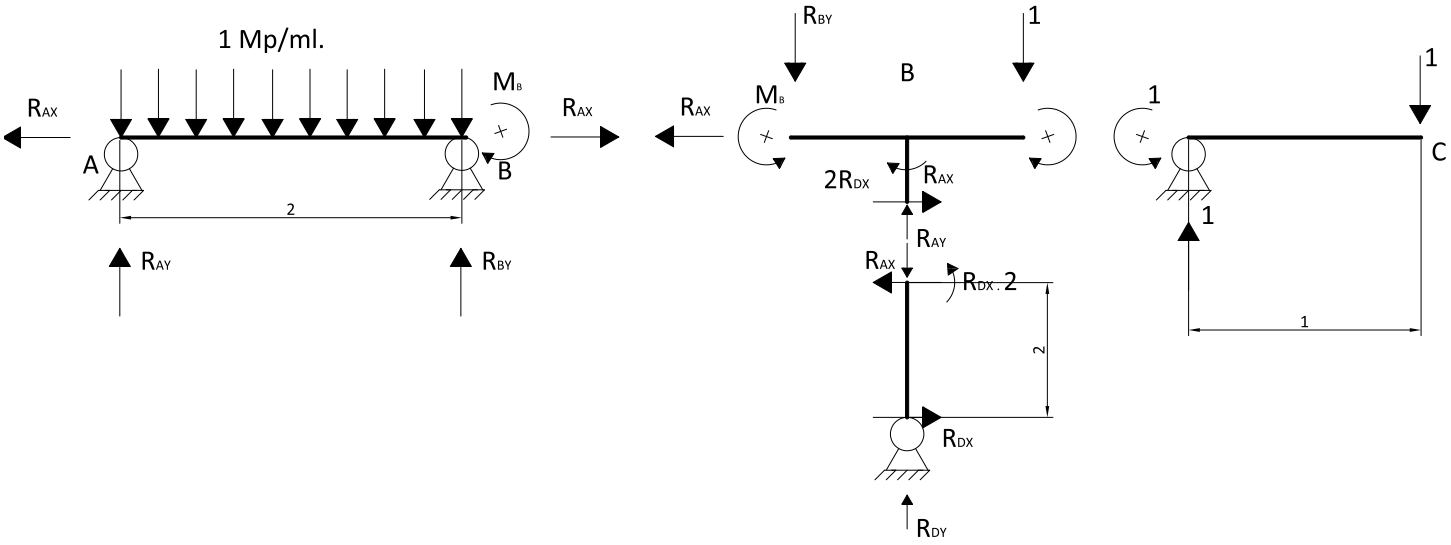



C
A
B

PROBLEMA 3º;



Descomponemos;



Tramo AB;

$$\sum F_y = 0 \rightarrow R_{AY} + R_{BY} = 2 \qquad \text{(I)}$$

$$\sum M_A = 0 \rightarrow -M_B + R_{By}.2 - 1.2.1 = 0 \text{ (II)}$$

$$R_{AX} = R_{BX}$$

Nudo B;

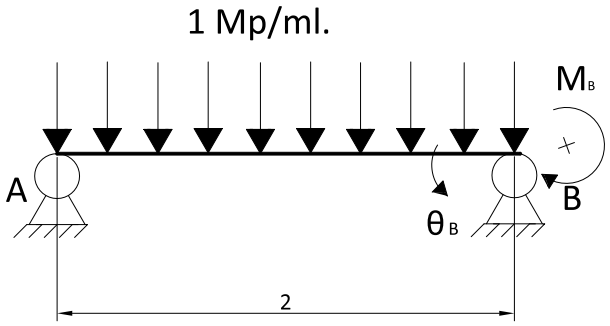
$R_{DY} = R_{BY} + 1$ (III)

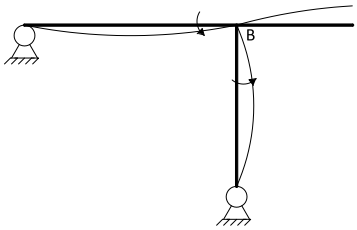
$-2R_{DX} + M_B = 1$ (IV)

Barra BD;

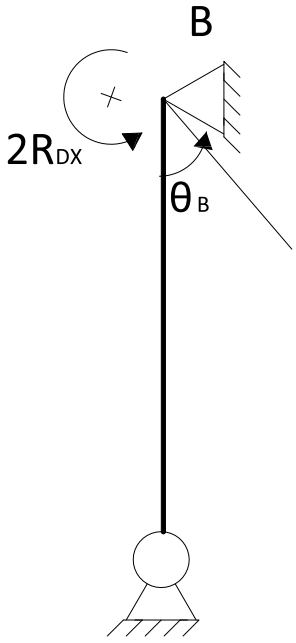
$R_{AX}.2 = R_{DX}.2$

Igualamos giros en B;





$$\vartheta_B = \frac{1.2^3}{24EI} - \frac{2M_B}{3EI}$$



$$\vartheta_B = \frac{2R_{DX}}{3EI}$$

Luego; $\frac{1.2^3}{24EI} - \frac{2M_B}{3EI} = \frac{2R_{DX}}{3EI}$

(IV) $\begin{matrix} 1 - 2M_B = 2R_{DX} \\ -1 + M_B = 2R_{DX} \end{matrix}$ ⇒ $M_B = 2/3$, $R_{DX} = -1/6$ (va en sentido contrario al Del dibujo)

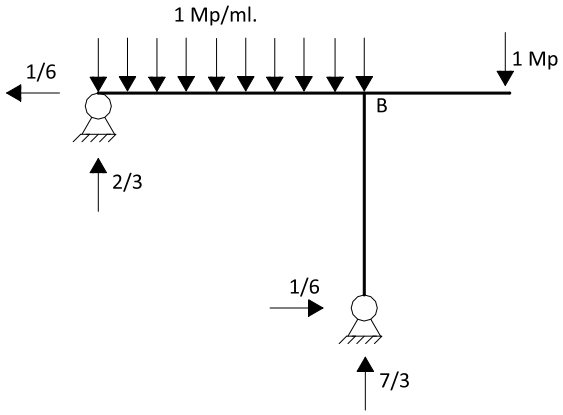
También:

$R_{AX} = 1/6$

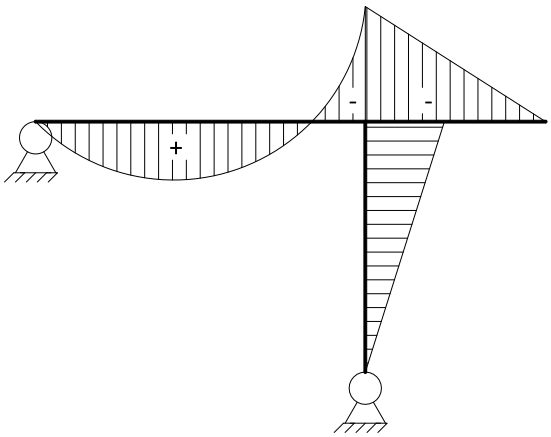
$De(II) \rightarrow R_{BY} = 4/3$

$De(I) \rightarrow R_{AY} = 2/3$

$De(III) \rightarrow R_{DY} = 7/3$

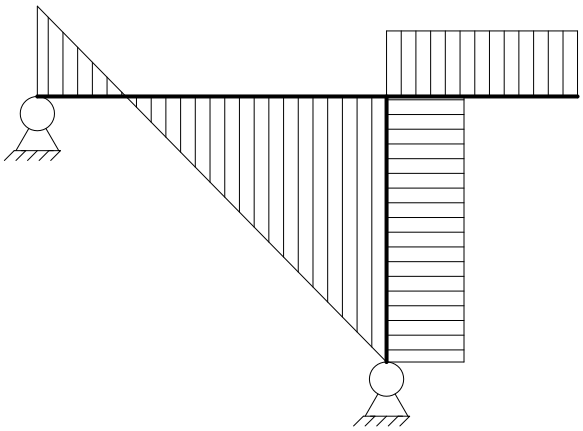


Flectores;

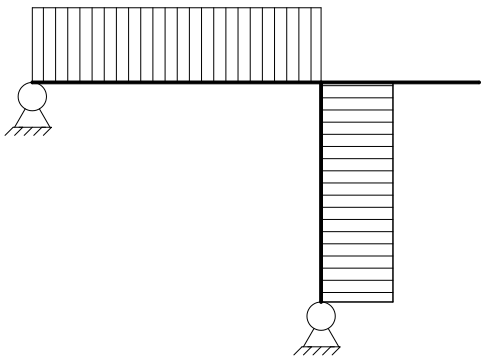


Acotar esta ley y calcular máximas.

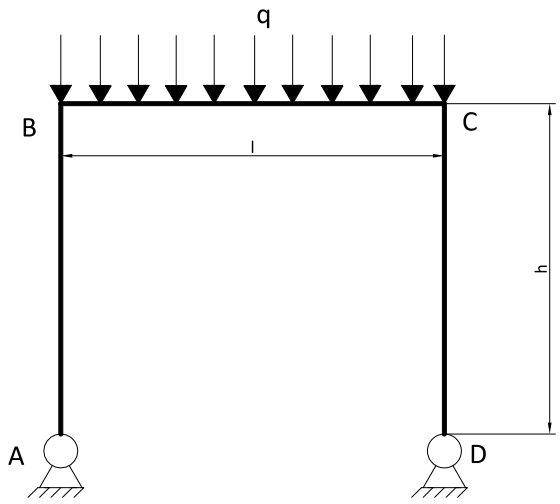
Cortantes;



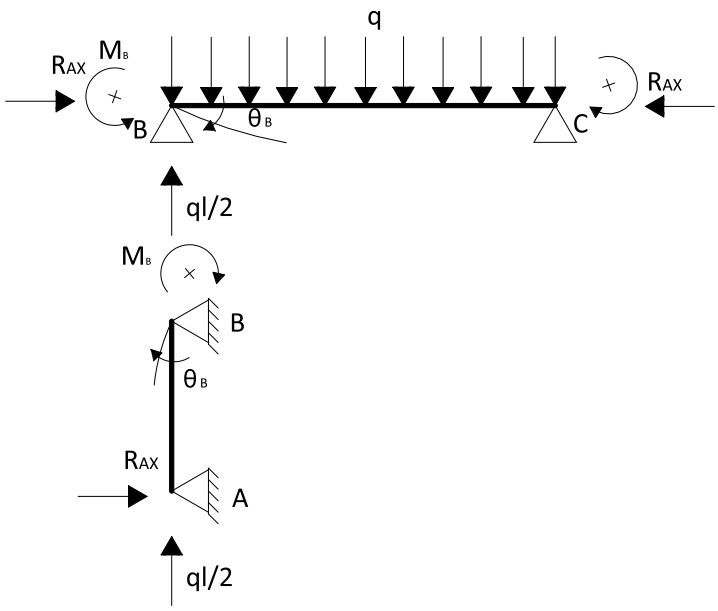
Axiles;



PROBLEMA 4º;



Descomponemos la estructura;



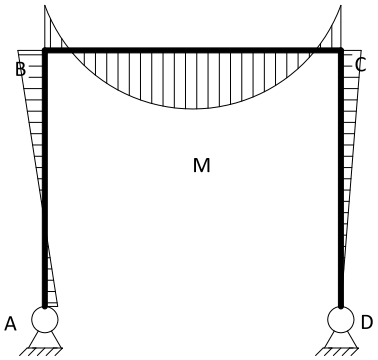
El pórtico es simétrico →M_B=M_C.

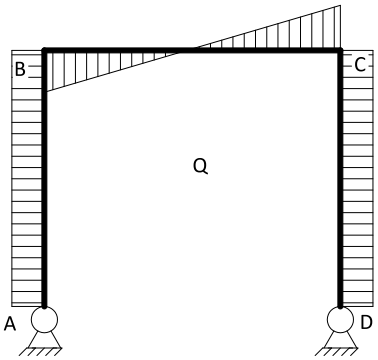
Igualamos el giro en B;

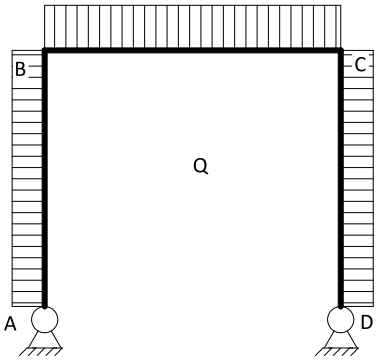
$$\frac{ql^3}{24EI} - \frac{M_B l}{3E} - \frac{M_B l}{6EI} = \frac{M_B h}{3EI}$$

$$M_B = \frac{ql^3}{4}\frac{l+2h}{4(l+h)^2 - l^2}$$

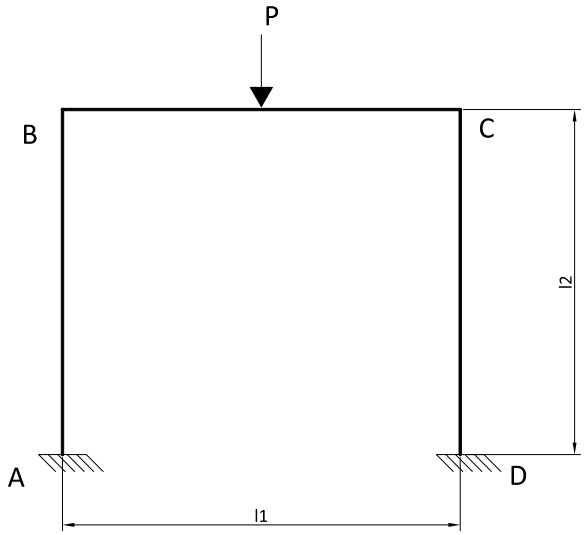
$$R_{AX}; h = M_B \rightarrow R_{AX} = \frac{M_B}{h}$$


B
C
M
A
D

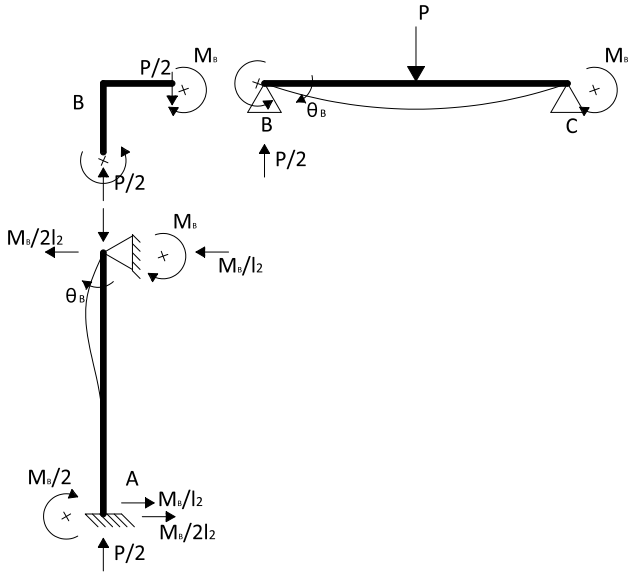

B
C
Q
A
D


B
C
Q
A
D

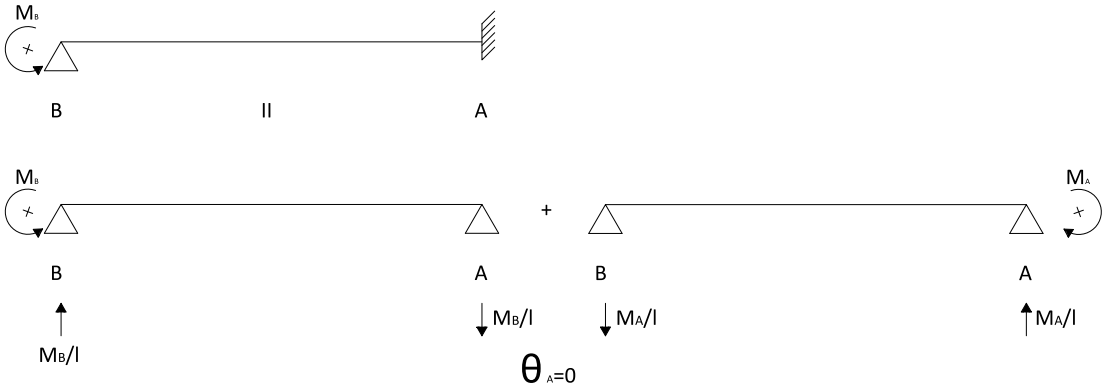
PROBLEMA 5º;



Descomponemos la estructura;



Recordatorio;

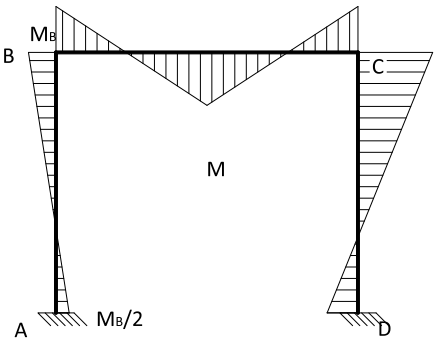


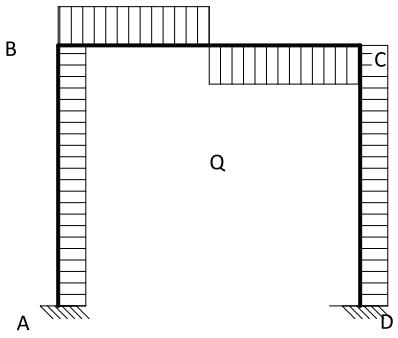
$$\frac{M_B l}{6EI} = \frac{M_A l}{3EI} \rightarrow M_A = \frac{M_B}{2}$$

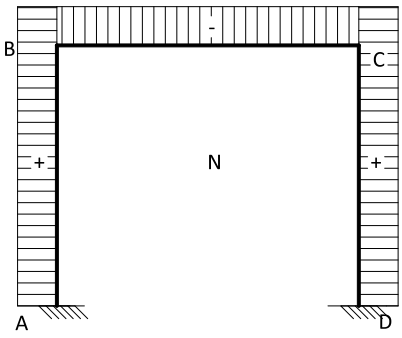
Igualamos el giro en B;

$$\frac{M_B l_2}{3EI} - \frac{M_B l_2}{2,6EI} = \frac{3Pl_1^2}{16EI} - \frac{M_B l_1}{3EI} - \frac{M_B l_1}{6EI}$$

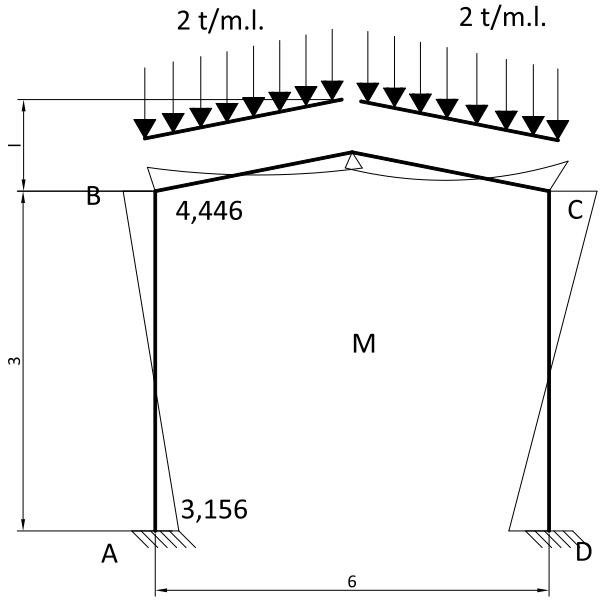
Operando; $M_B = \frac{Pl_1^2}{4(2l_1 + l_2)}$ $\quad R_{AX} = \frac{3}{2}\frac{M_B}{l_2}$

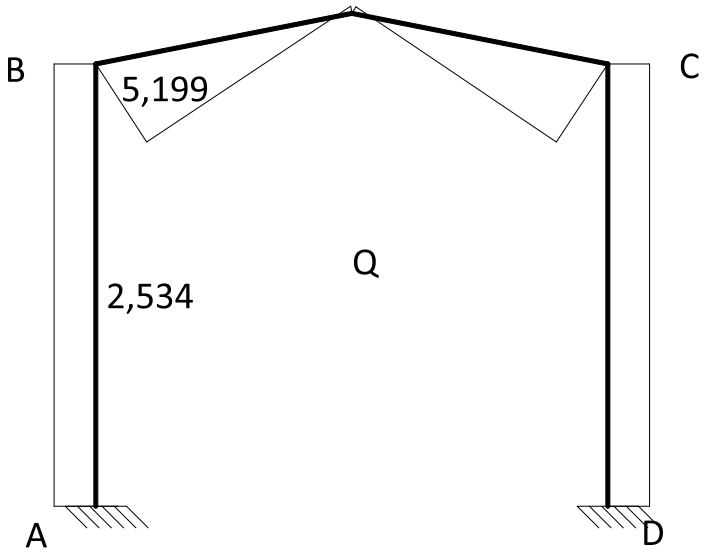


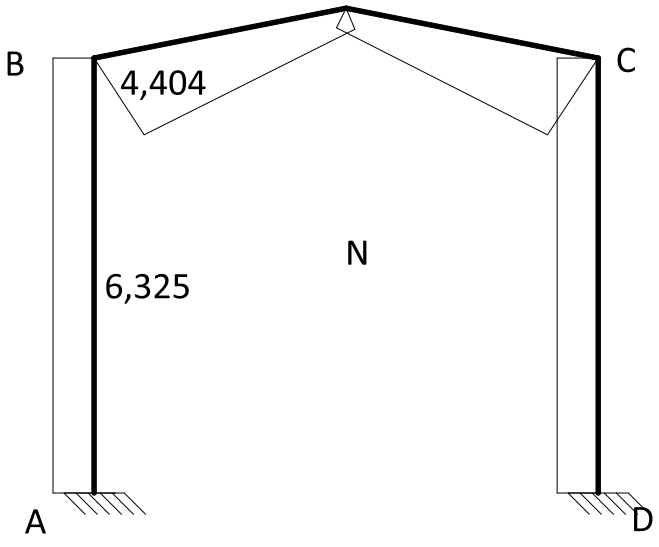




PROBLEMA 6º; Calcular las reacciones en D y acotar las leyes de flectores



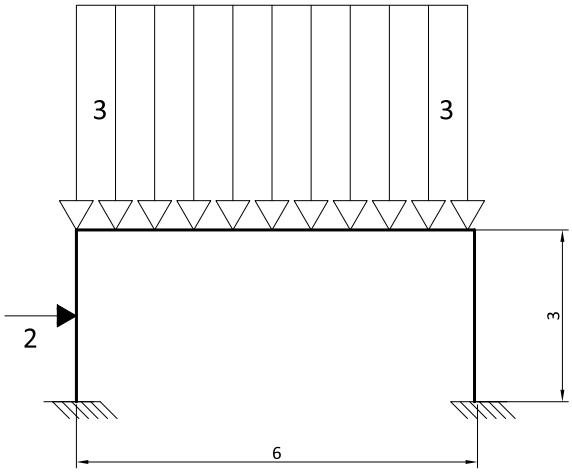


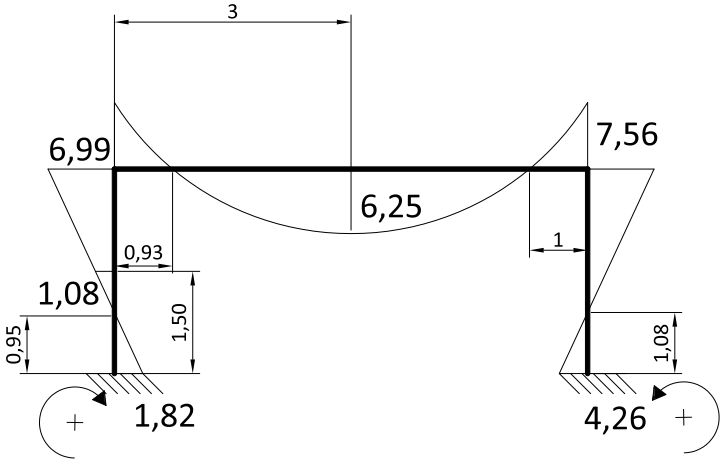


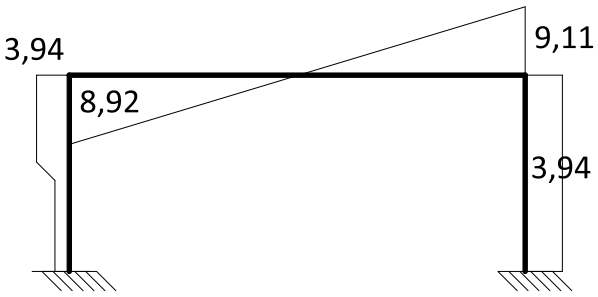
PROBLEMA 7º;

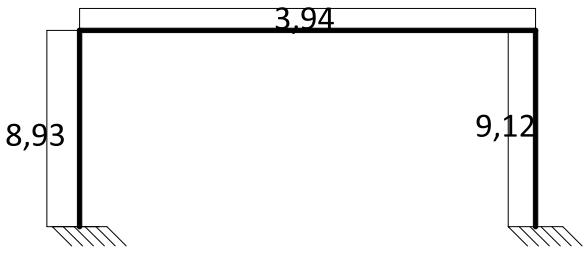
1º) REACCIONES EN APOYOS

2º) ACOTAR LEY DE FLECTORES

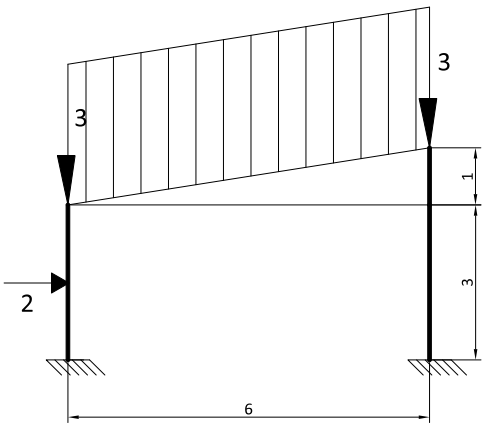


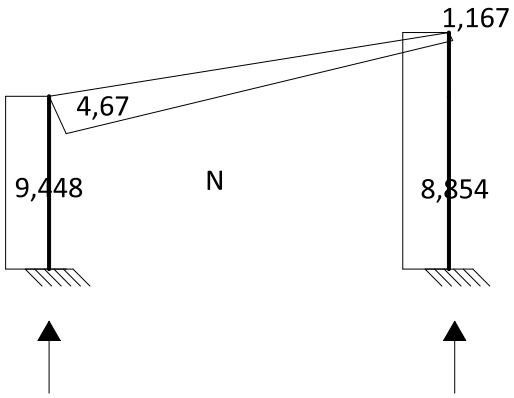


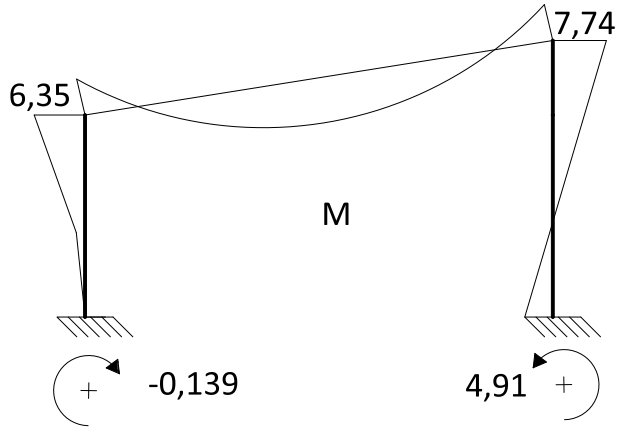


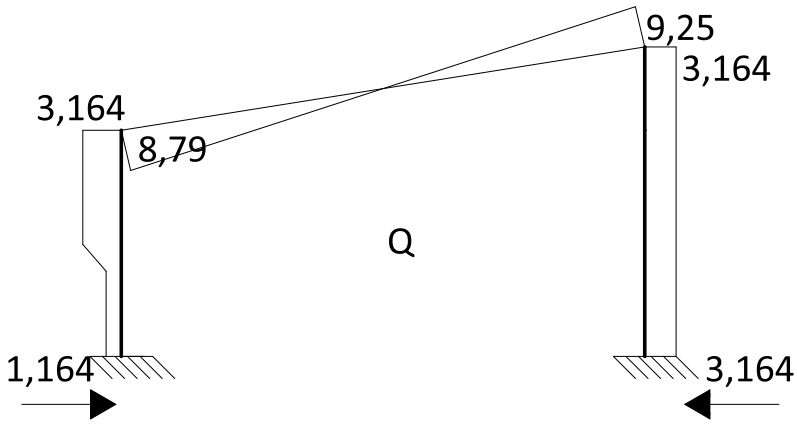


PROBLEMA 8º;


3
3
1
3
2
6


1,167
4,67
9,448
N
8,854


7,74
6,35
M
-0,139
4,91


9,25
3,164
3,164
8,79
Q
1,164
3,164

SEPTIEMBRE 2008

1º) REACCIONES EN D (3PUNTOS)

2º) ACOTAR LEYES DE FLECTORES Y CORTANTES;

2.1 – BARRA BC (2 PUNTOS)

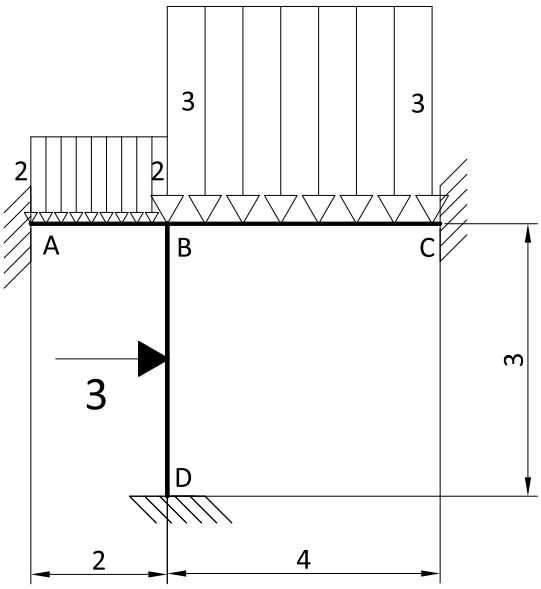
2.2 – BARRA AB (2 PUNTOS)

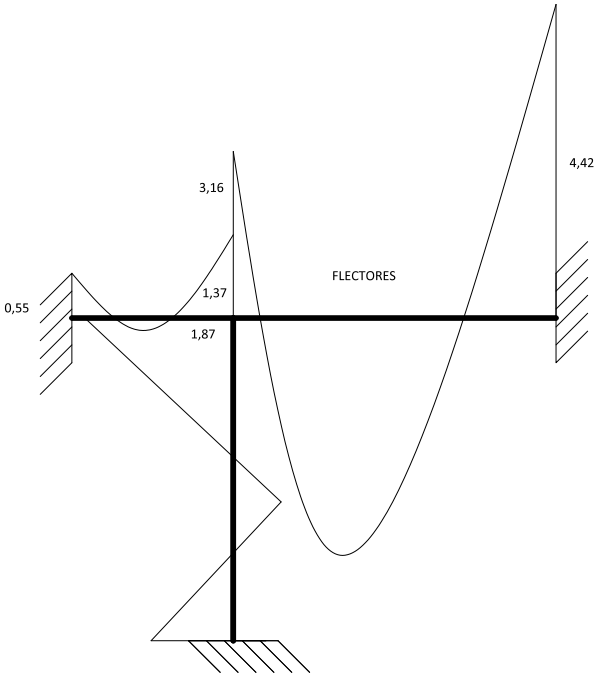
2.3 – BARRA BD (2 PUNTOS)

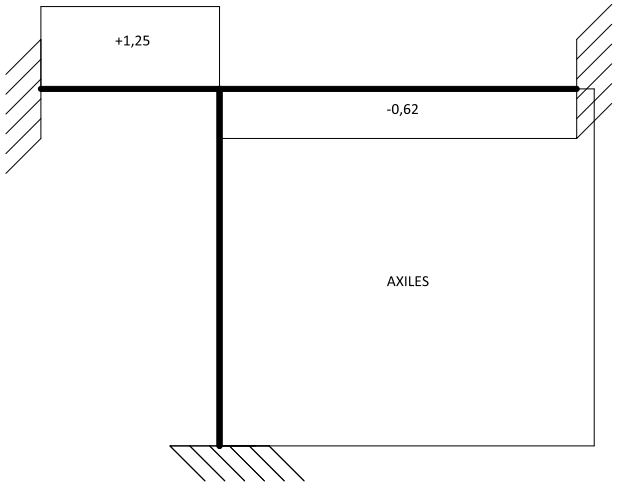
3º) DIMENSIONAR LOS PERFILES PARA UNA TENSION DE TRABAJO DE 2000 KP/CM2 (1 PUNTO)

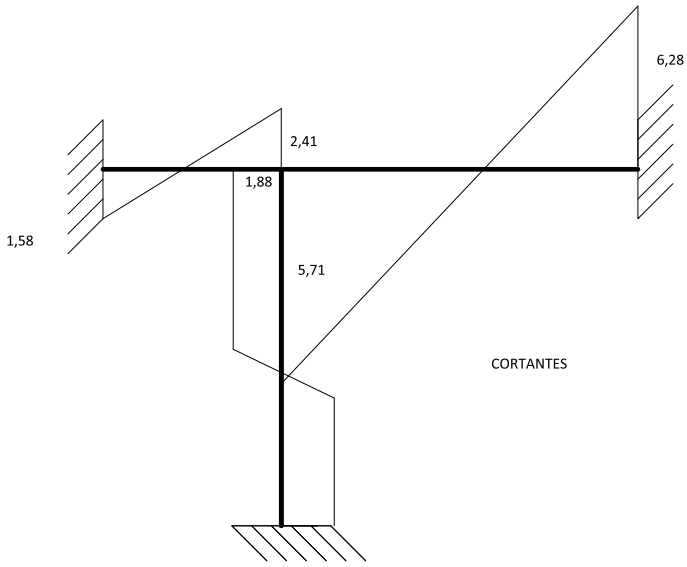
TOTAL: 10 PUNTOS

TIEMPO: 1H.45M.









CORTANTES