Universidad de Cantabria
ETSII Telecomunicación
2º Curso de Ingeniería e Ingeniería Técnica de Telecomunicación
Teoría de la Comunicación

SOLUCIONES HOJA DE PROBLEMAS 5

1. a) K=4/(3ab), E[**XY**]=0, E[**X**]=0, E[**Y**]=0 ⇒ **X**,**Y** incorreladas

b) $f_X(x) = \begin{cases} 4/(3a) & -a/2 < x < -a/4 \\ 2/(3a) & -a/4 < x < a/4 \\ 4/(3a) & a/4 < x < a/2 \\ 0 & \text{resto} \end{cases}$ $\quad f_Y(y) = \begin{cases} 4/(3b) & -b/2 < y < -b/4 \\ 2/(3b) & -b/4 < y < b/4 \\ 4/(3b) & b/4 < y < b/2 \\ 0 & \text{resto} \end{cases}$

$f_{XY}(x,y) \neq f_X(x) f_Y(y)$ ⇒ **X** e **Y** no son Independientes

c) $C_{ZW} = ½(E[\mathbf{X}^2]-E[\mathbf{Y}^2])\mathrm{sen}(2\theta)$

si $E[\mathbf{X}^2]>E[\mathbf{Y}^2] \Rightarrow \theta=\pi/4$

si $E[\mathbf{X}^2]<E[\mathbf{Y}^2] \Rightarrow \theta=-\pi/4$

d) $C_{ZW}=0$ si $E[\mathbf{X}^2]=E[\mathbf{Y}^2]$

$E[\mathbf{X}^2]=5a^2/48$, $E[\mathbf{Y}^2]=5b^2/48 \Rightarrow a=b$

2. a) $E[\mathbf{UV}]=\cos\theta\,\mathrm{sen}\theta(\sigma_X^2-\sigma_Y^2+\eta_X^2-\eta_Y^2)+(\cos^2\theta-\mathrm{sen}^2\theta)(r\sigma_X\sigma_Y+\eta_X\eta_Y)$

$E[\mathbf{U}]E[\mathbf{V}]=\cos\theta\,\mathrm{sen}\theta(\eta_X^2-\eta_Y^2)+(\cos^2\theta-\mathrm{sen}^2\theta)(\eta_X\eta_Y)$

U,**V** Incorreladas ⇒ $E[\mathbf{UV}]=E[\mathbf{U}]E[\mathbf{V}] \Rightarrow \theta=½\mathrm{arcotg}(2r\sigma_X\sigma_Y/(\sigma_Y^2-\sigma_X^2))$

b) **U** es $N(\eta_U,\sigma_U)$ por ser combinación lineal de gaussianas con:

$\eta_U=\eta_X\cos\theta-\eta_Y\mathrm{sen}\theta$

$\sigma_U^2=\cos^2\theta\,\sigma_X^2+\mathrm{sen}^2\theta\,\sigma_Y^2$

3. a) $f_U(u) = u e^{-u^2/2}$ para $0 < u < \infty$ (Rayleigh)

$f_{UY}(u,y) = u e^{-u^2/2}$ para $\begin{cases} 0 < u < \infty \\ 0 < y < 1 \end{cases}$

b) $f_{ZW}(z,w) = \dfrac{1}{2\pi} e^{-\frac{(z^2+w^2)}{2}}$ para $\begin{cases} -\infty < z < \infty \\ -\infty < w < \infty \end{cases}$

$f_Z(z)$ y $f_W(w)$ son Gaussianas

4. a) $\Omega_{XY}=\{(0,0),(0,1),(1,1),(1,2)\}$

P(**X**=0,**Y**=0)=q^2 , P(**X**=0,**Y**=1)=pq , P(**X**=1,**Y**=1)=pq , P(**X**=1,**Y**=2)=p^2

P(**X**=0)=q , P(**X**=1)=p

P(**Y**=0)=q^2 , P(**Y**=1)=2pq , P(**Y**=2)=p^2

X e **Y** No son Independientes

b) $\eta_X=p$, $\eta_Y=2p$, $\sigma_X^2=pq$, $\sigma_Y^2=2pq$, $E[\mathbf{XY}]=p(1+p) \Rightarrow r_{XY}=1/\sqrt{2}$

X e **Y** No están Incorreladas

$\hat{\mathbf{Y}} = \mathbf{X} + p$

c) $\hat{\mathbf{Y}} = E[\mathbf{Y} \mid \mathbf{X} = x] = \mathbf{X} + p$

5. a) a=6/7 , b=2/7 $\Rightarrow$ $\hat{\mathbf{Y}}_1 = \frac{6}{7}\mathbf{X} + \frac{2}{7}$

b) $\hat{\mathbf{Y}}_2 = \frac{\mathbf{X}-1}{\ln \mathbf{X}}$

c) $r_{X\hat{Y}_1} = \frac{a}{|a|}$, $C_{X\hat{Y}_2} = \frac{1}{24}$

6. a) $f_Z(z)=pN(0,\sigma_1)+ qN(0,\sigma_2)$

b) $\eta_Z=0$, $\sigma_Z^2=p\sigma_1^2+q\sigma_2^2$

c) Estimador Lineal: $\hat{\mathbf{Z}} = (1-p)\mathbf{X}_2$, Estimador sin restricciones: $\hat{\mathbf{Z}} = (1-p)\mathbf{X}_2$

7. a) E[**Z**]=E[**X**]+cE[**Y**]=90ms

X, **Y** indep. $\Rightarrow \sigma_Z^2=\sigma_X^2+c^2\sigma_Y^2 =58.3ms^2$

$r_{XZ} = \sqrt{\frac{3}{7}}$

b) $f_Z(z) = \frac{1}{20}\left[G\left(\frac{z}{5}-16\right) - G\left(\frac{z}{5}-20\right)\right]$

c) Estimador Lineal: $\hat{\mathbf{Z}} = c\mathbf{Y} + \eta_X = 0.5\mathbf{Y} + 40$

Estimador sin restricciones: : $\hat{\mathbf{Z}} = c\mathbf{Y} + \eta_X = 0.5\mathbf{Y} + 40$

8. a) n=6000 veces, p=1/6 , q=5/6

aprox. DeMoivre-Laplace $P(k_1 \leq \mathbf{X} \leq k_2) \approx G\left(\frac{k_2 - np + 0.5}{\sqrt{npq}}\right) - G\left(\frac{k_1 - np - 0.5}{\sqrt{npq}}\right)$

$\eta=np=1000, \sigma=(npq)^{1/2}=28.86 \Rightarrow P(980 \leq \mathbf{X} \leq 1005) \approx G(0.19) - G(-0.71) = 0.3364$

b) $P(\mathbf{X} = 1005) = \binom{n}{1005} p^{1005} q^{n-1005} \approx \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(1005-\eta)^2}{2\sigma^2}} = 0.0136$ con $\eta=np$ y $\sigma^2=npq$