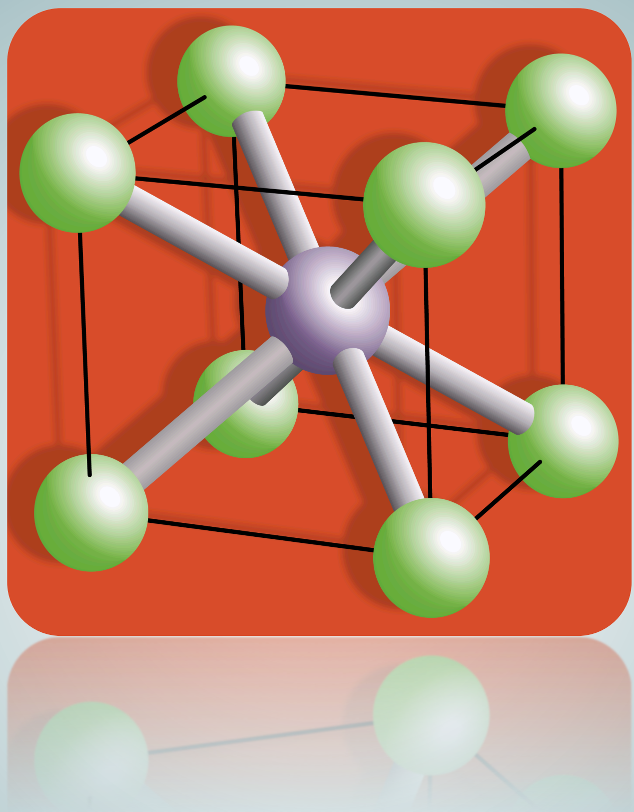
Materials

Exercises Topic 7. Fatigue



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FATIGUE

1. Fatigue data for a ferrous alloy are collected in the following table:

Amplitude (MPa)	*470*	*440*	*390*	*350*	*310*	*290*	*290*	*290*
Cycles to faylure	10^4	$3{\cdot}10^4$	10^5	$3{\cdot}10^5$	10^6	$3{\cdot}10^6$	10^7	10^8

a) Draw its WÖHLER diagram.
b) Estimate the fatigue limit (endurance) for this alloy.
c) A shaft made of this alloy is employed in a car's transmission that spins at 600 rpm. Estimate the life time of the shaft for the following stress levels working non-stop: 450 MPa, 310 MPa and 275 MPa

2. The clutch strand of a car, employed as a taxi, has a nominal diameter of 1.5mm and is made of the steel whose main characteristics are:

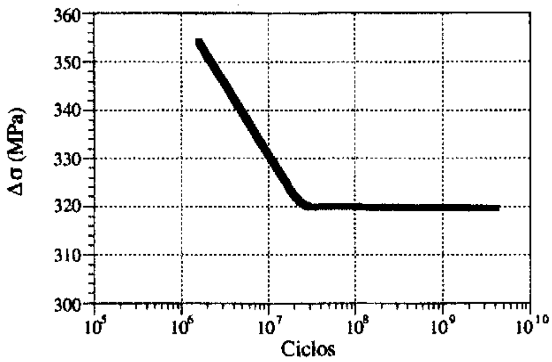
- *Yield stress:* $\sigma_Y = 250$ *MPa*
- *Fracture toughness:* $K_{IC} = 40$ *MPa·m*$^{1/2}$
- *Paris' law:* $da/dN = 8{\cdot}10^{-13}{\cdot}(\Delta K_I)^{2.8}$ *donde da/dN (m/ciclo) si* ΔK *se expresa en MPa·m*$^{1/2}$
- *Crack propagation threshold:* $\Delta Kth = 5$ *MPa·m*$^{1/2}$

For the gearing process, the taxi driver applies 30kg of force on the pedal, which is transmitted to the cable. This operation, in urban driving, is done an average of 4 times per kilometer. Assuming that the most probable crack type is an edge notch with an stress intensity factor of 3.9, andl that the cable manufacturer assures that there are not initial cracks deeper than 0.3mm:

a) Determine if there is any crack progation risk in the cable's working conditions.
b) Estimate after how many kilometers the failure of the cable will be expected, assuming that 80% of the use takes place in urban driving and during the remaining 20%, highway driving, crack propagation can be considered negligible.
c) Explain the effects that will have on the cable the fact that during its installation an accidental overload of 50kg would have taken place.

3. Pedro underwent a hip break while practicing sport, so he received a titanium alloy (Ti–6Al–4V) hip implant as a solution. The implant manufacturer assures that it was free of any defect; however, 100 days after the operation the hip implant broke without being exposed to any overload on it. The insurance company opened a research, obtaining the following information:

- The cross section of the hip implant where the failure occurred supported stresses oscillating for each one of Pedro's steps between in function of his weight (P):
  $\sigma_{máx} = P / 0,5$; $\sigma_{mín} = - \sigma_{máx}$ (stress in MPa if the weight is introduced in kg).
- The day of the surgery Pedro was 75 kg, but he put on 10 extra kg due to the rest afterwards, weight that he has maintained until the present date. For this reason, the first hypothesis of the insurance company was that the failure was caused by this overweight; in order to confirm this the following Wöler diagram was obtained from the manufacturer:



Assuming that from the operation Pedro has been doing a rehabilitation consisting of walking an average of 2km daily with short steps (0.4m/step).

a) Determine the life of the implant if Pedro would have maintained his initial weight.
b) Determine the life of the implant assuming the real extra weight that Pedro put on.

The second hypothesis of the insurance company was the possibility of the existence of some type of crack in the hip implant, for which the following characteristics were obtained:

- *Fracture toughness: K_{IC} = 55 MPa·$m^{1/2}$*
- *Paris' law: da/dN = $5\cdot 10^{-13}\cdot(\Delta K)^{2.5}$ (m/cycle) if ΔK is expressed in MPa·$m^{1/2}$*
- *Crack propagatin thesshold: ΔK_{th} = 20 MPa·$m^{1/2}$*
- *Stress intensity factor: $K_I = 3{,}3\cdot\sigma\cdot(\pi\cdot a)^{1/2}$*

c) Calculate what will be the initial crack length of the defect (when the implant was placed) considering that compression stresses (negative) do not contribute to the crack propagation ($\Delta\sigma = \sigma_{máx}$ for the present case).

d) Estimate the maximum weight that would have allowed Pedro to enjoy for unlimited service-life of the hip implant installed. ¿What will be the weight if there weren't any crack in the component?

4. A high strength steel (σ_Y = 1000 MPa) strap, whose cross section is 30 cm wide and 3 cm thick, is in service subjected to a cycling tensional state that varies between 2.7 MN and 1.8 MN. It is proposed that this structural element lasts 90.000 load cycles, and it is known that has an edge crack of a_0 = 8.5 mm deep $\left(K_I = 1.13\cdot\sigma\cdot\sqrt{\pi\cdot a}\right)$.

During the lab characterization of this type of straps (prior to their installation), it was concluded that when having initial edge cracks 2.5 mm deep, the failure occurred under a load of 7.2 MN. Also, by performing fatigue tests of the high strength steel employed in the manufacture the Paris' law $\frac{da}{dN} = 4.62\times 10^{-12}\left(\Delta K_I\right)^{3.3}$ (m/cycle if ΔK is in MPa·$m^{1/2}$) was fitted. In these conditions:

1) Determine the fracture toughness of the material.
2) Critical crack length that will produce the failure of the strap in service conditions.
3) Estimate if the strap will be able to last the 90.000 cycles pranned.
4) If the crack propagation threshold in fatigue is ΔK_{th} = 2.54 MPa·$m^{1/2}$, determine the maximum crack length that can be present in the strap when installing it in order to guarantee its infinite life facing fatigue for the aforementioned in-service conditions.
5) Determine the maximum life for the strap when installing it facing fatigue for the aforementioned in-service conditions, to last 90.000 cycles.

Tip: $\int x^n dx = \frac{x^{n+1}}{n+1}$ $(n \neq -1)$