



## **D. EXAMPLES**



## *WORKED EXAMPLE I*

### *Cracked ship hull*

- **Introduction and objectives**
  - **Data**
  - **Analysis**



## INTRODUCTION AND OBJECTIVES

During a visual inspection of a ship hull, semi-elliptical surface cracks of 5 mm depth were observed. The working conditions cause a tensile state characterised by a stress of 110 MPa when the ship is unloaded and 350 MPa when the ship is loaded.

Knowing that in the marine environment, crack propagation can be provoked by stress corrosion cracking, and that in this steel this process takes place at a crack growth rate of  $1.2 \cdot 10^{-7}$  mm/s when the threshold of  $20 \text{ MPa} \cdot \text{m}^{-1/2}$  is passed:

- a) Represent in a FAD the state of the security conditions as a function of time. The ship is under unloaded condition for 3 months and under loaded condition for 7 months.
- b) Evaluate whether the critical conditions in the hull are produced by leak before break or brittle fracture in a plate of 20 mm in thickness.
- c) Determine the total life of the component.

### Hypotesis:

The cracks grow maintaining a constant relationship  $a/2c$  of 0.3 // General yielding is not considered



## DATA

*Material properties:*

$$\sigma_Y = 450 \text{ MPa}$$

$$K_{IC} = 120 \text{ MPa}\cdot\text{m}^{1/2}$$

$$K_{Isc} = 20 \text{ MPa}\cdot\text{m}^{1/2}$$

*SCC conditions:*

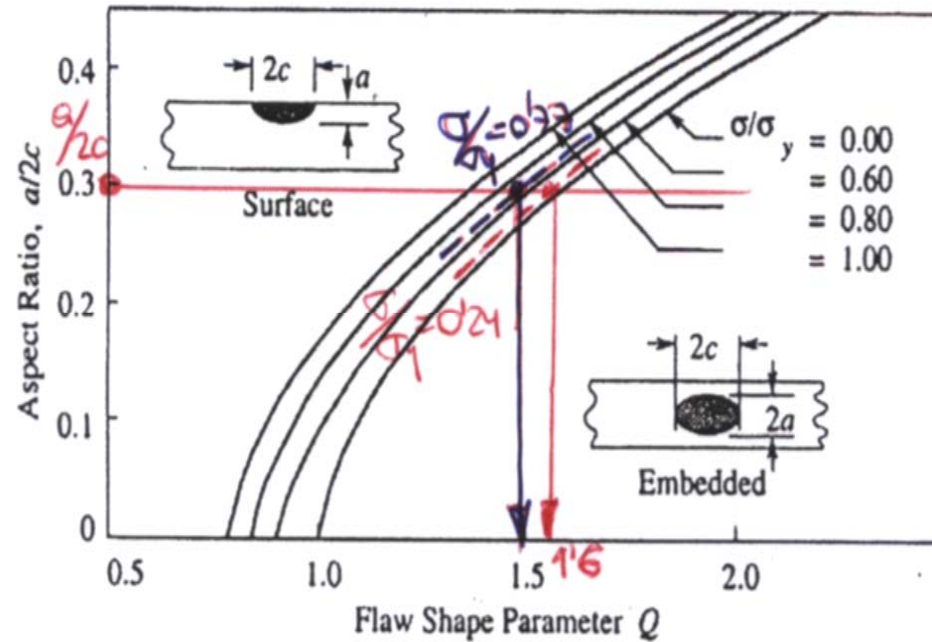
$$da/dt = 1.2 \cdot 10^{-7} \text{ mm/s}$$

$$K_{ISCC} = 20 \text{ MPa}\cdot\text{m}^{1/2}$$



## ANALYSIS

$$K_I = \sqrt{\frac{1.21}{Q}} \cdot \sigma \sqrt{\pi a}$$



Unloaded ship:  $\sigma = 110$  MPa       $\frac{\sigma}{\sigma_y} = 0.24$       in figure 1,  $Q = 1.6$

Loaded ship:  $\sigma = 350$  MPa       $\frac{\sigma}{\sigma_y} = 0.77$       in figure 1,  $Q = 1.5$

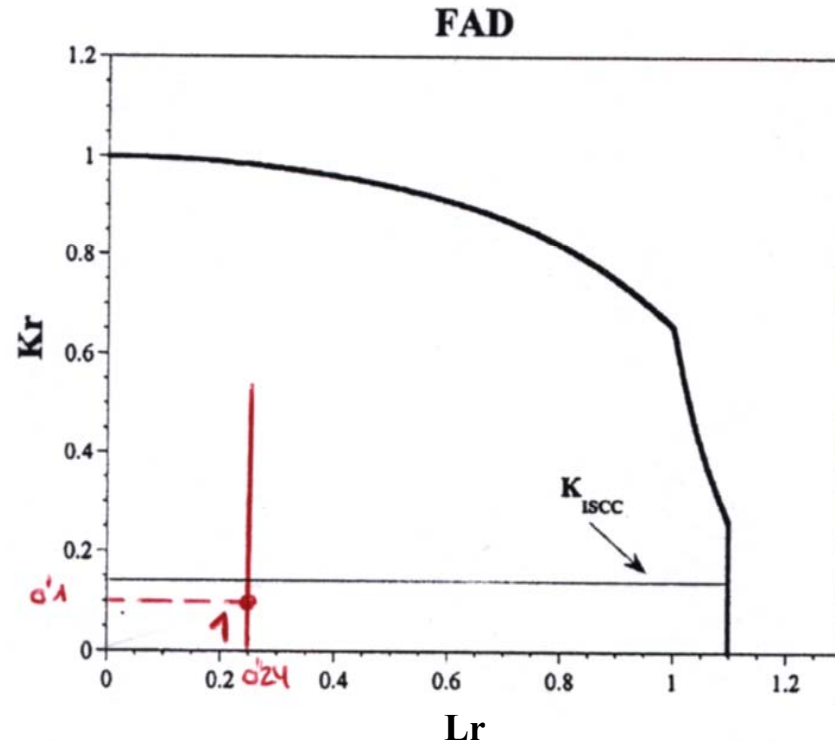


a) ANALYSIS

0 - 3 months: 
$$K_I = \sqrt{\frac{1,21}{1,6}} \cdot 110 \cdot \sqrt{\pi \cdot 0,005} = 11.53 \text{ MPa}\cdot\text{m}^{1/2} < K_{Isc}$$

$\left. \begin{matrix} K_r = 0.1 \\ L_r = 0.24 \end{matrix} \right\} \text{No Propagation}$

(1)





## ANALYSIS

**3 – 10 months:** 
$$K_I = \sqrt{\frac{1.21}{1.5}} \cdot 350 \cdot \sqrt{\pi \cdot 0.005} = 37.90 \text{ MPa} \cdot \text{m}^{1/2}$$

**3 months** 
$$\left. \begin{array}{l} K_r = 0.31 \\ S_r = 0.77 \end{array} \right\} \text{Propagation}$$

$$\Delta a = \overset{\circ}{a} \cdot t = 1.2 \cdot 10^{-7} \cdot 7 \cdot 30 \cdot 24 \cdot 3600 = 2.17 \text{ mm} \quad a_f = 7.17 \text{ mm} \quad K_{If} = 47.1 \text{ MPa} \cdot \text{m}^{1/2}$$

**10 months  
(2)** 
$$\left. \begin{array}{l} K_r = 0.39 \\ L_r = 0.77 \end{array} \right\}$$



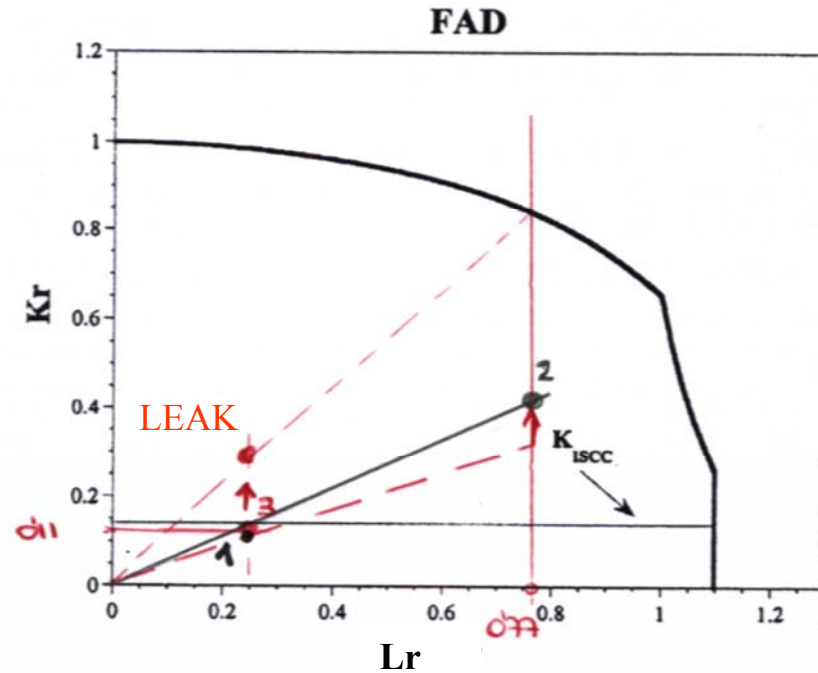
## ANALYSIS

**10 - 13 months:** 
$$K_I = \sqrt{\frac{1,21}{1,6}} \cdot 110 \cdot \sqrt{\pi \cdot 0,00717} = 14,30 \text{ MPa} \cdot \text{m}^{1/2} < K_{I_{SSC}}$$

$K_r = 0.119$   
 $L_r = 0.24$

**No Propagation**

**(3)**







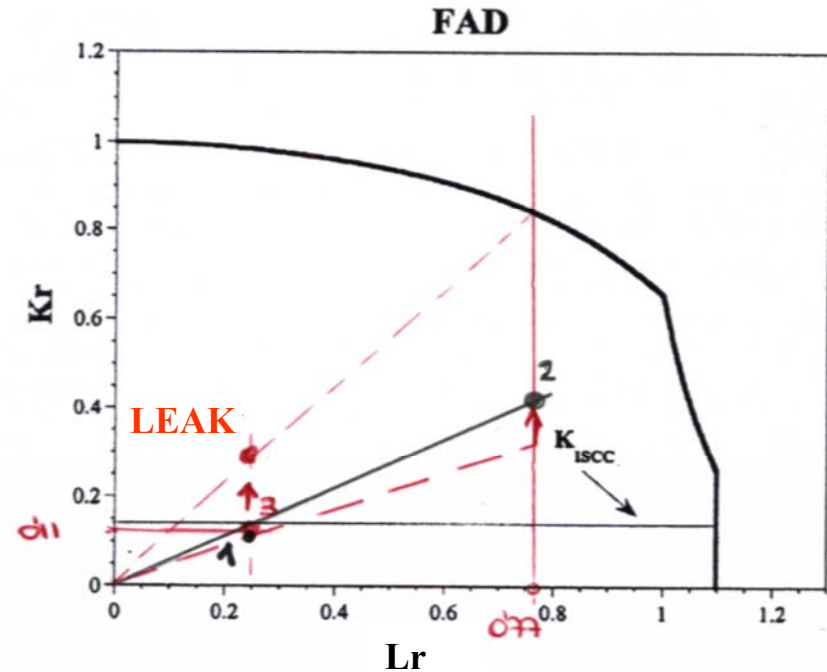
## ANALYSIS

In order to have propagation while the ship is unloaded, a minimum  $a_{ul}$  is needed:

$$K_I = \sqrt{\frac{1,21}{1,6}} \cdot 110 \cdot \sqrt{\pi \cdot a_{ul}} = 20 \text{ MPa} \cdot \text{m}^{1/2}$$

$$a_{ul} = 13 \text{ mm}$$

Then, for  $a < a_{ul}$  crack propagation is only produced during loading periods





**13 - 20 months:**

$$a_f = 7.17 + 2.17 = 9.34 \text{ mm} \quad (4)$$

**23 - 30 months:**

$$a_f = 9.34 + 2.17 = 11.51 \text{ mm} \quad (5)$$

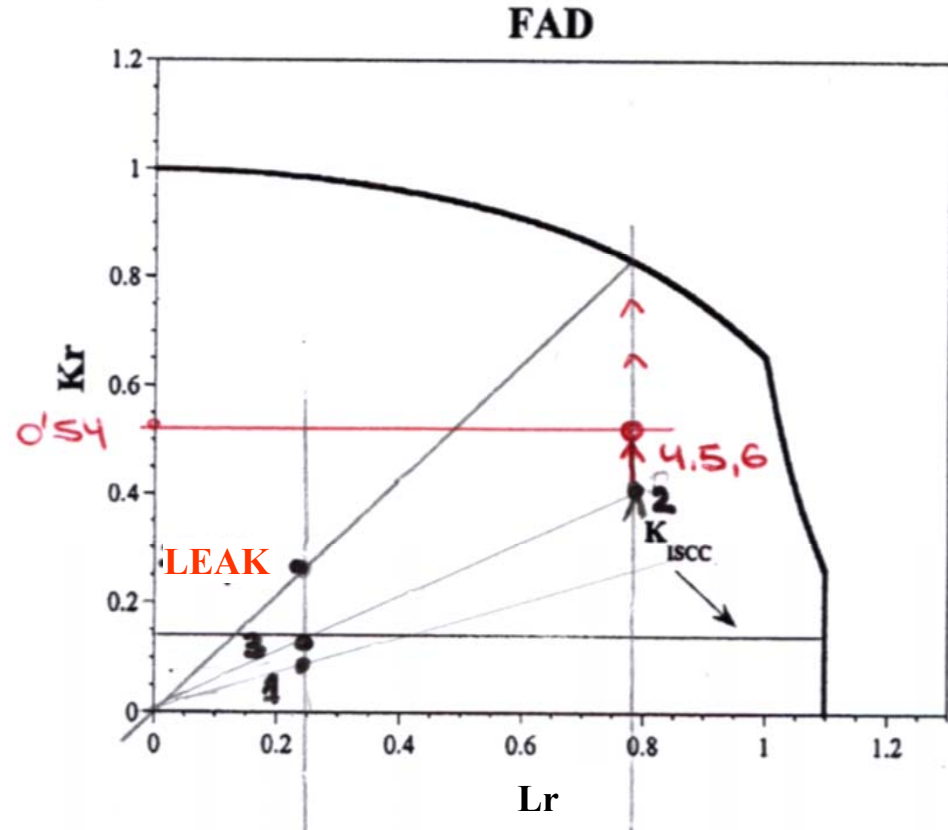
**33 - 40 months:**

$$a_f = 11.51 + 2.17 = 13.68 \text{ mm}$$

$$K_I = 65.15 \text{ MPa} \cdot \text{m}^{1/2}$$

$$\left. \begin{array}{l} K_r = 0.54 \\ S_r = 0.77 \end{array} \right\} (6)$$

**ANALYSIS**



From this moment, both loaded and unloaded conditions promote crack propagation cracking at the same time.



## ANALYSIS

b)

The FAL is reached when:  $K_r = 0.85$ ; then,  $K_I = 0.85 \cdot 120 = 102 \text{ MPa} \cdot \text{m}^{1/2}$

$$102 = \sqrt{\frac{1.21}{1.5}} \cdot 350 \cdot \sqrt{\pi \cdot a_c} \Rightarrow a_c = 0.034 \text{ m} = 3.4 \text{ mm} > 20 \text{ mm (thickness)}$$

then leak before break will happen

Leak  $a_{\text{leak}} = 20 \text{ mm}$

In theory, after leak, another propagation occurs until critical length is reached

$$K_I = \sigma \cdot \sqrt{\pi \cdot c} = 350 \cdot \sqrt{\pi \cdot c_c} = 0.85 \cdot K_{IC} = 102 \Rightarrow c_c = 0.027 \text{ m} = 27 \text{ mm}$$

But for  $a_{\text{leak}} = 20 \text{ mm}$ ,  $c = 33.3 \text{ mm}$ , which is bigger than 27 mm. Therefore, once leak happens, the component fails.



## ANALYSIS

c)

Knowing that  $\Delta a$  is 2.17 mm with the ship loaded and 0,93 mm with the ship unloaded:

Unloaded ship: 40-43  $\rightarrow a_f = 14,61$  mm

Loaded ship: 43-50  $\rightarrow a_f = 16,78$  mm

Unloaded ship: 50-53  $\rightarrow a_f = 17,71$  mm

Loaded ship: 53-60  $\rightarrow a_f = 19,88$  mm

Unloaded ship: 60-63  $\rightarrow a_f = 20,81$  mm  $\Rightarrow$  **LEAK AND FAILURE**



## *WORKED EXAMPLE II*

### *Plate under neutronic irradiation*

- **Introduction**
- **Objectives**
- **Analysis**



## INTRODUCTION

This case is an example of how the environment can change the mechanical properties of the material.

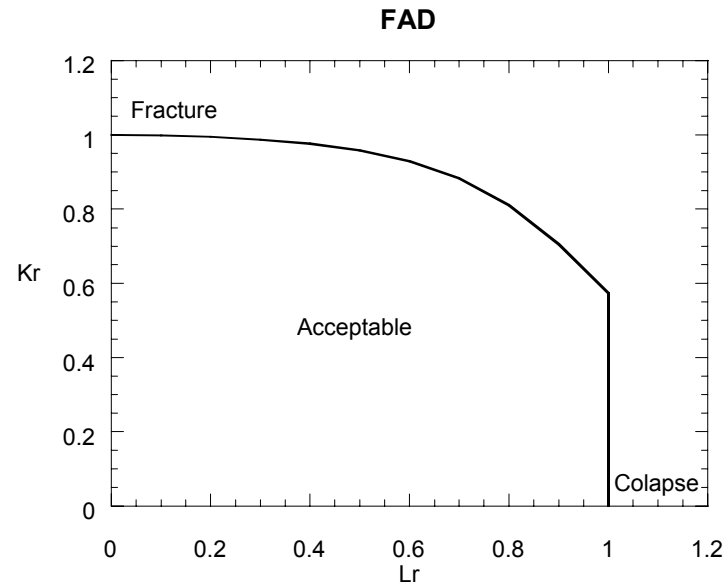
A metallic plate of big dimensions has cracks of  $2a = 20$  mm. The working conditions causes a tensile state characterised by a stress of 50, 150, 250 or 350 MPa. Because of an irradiation process, the mechanical properties of the material change with time in this manner:

T (years)	0	5	10	15	20
$\sigma_y$ (MPa)	500	510	540	565	585
$K_{IC}$ (MPa·m <sup>1/2</sup> )	150	135	120	100	85



## OBJECTIVES

- Represent in a FAD the state of the security conditions as a function of time (years 0, 10 and 20)
- Which one is more critical?
- Determine the period of time during which the safety factor is greater than 1.2.





## ANALYSIS

The stress intensity factor for a big plate is:

$$K_I = \sigma \sqrt{\pi \cdot a}$$

The crack geometry is  $2 \cdot a = 20$  mm, so:

$$K_I = \sigma \sqrt{\pi \cdot 0.01} = 0.177 \cdot \sigma$$

Using the expressions  $K_r (K_r = K_I / K_{IC})$  and  $L_r (L_r = \sigma / \sigma_Y)$  for the different working conditions, we can obtain for the years 0, 10 and 20:





## ANALYSIS

Year 0		
Working conditions	Kr	Lr
50 MPa	0.06	0.1
150 MPa	0.18	0.3
250 MPa	0.30	0.5
350 MPa	0.42	0.7

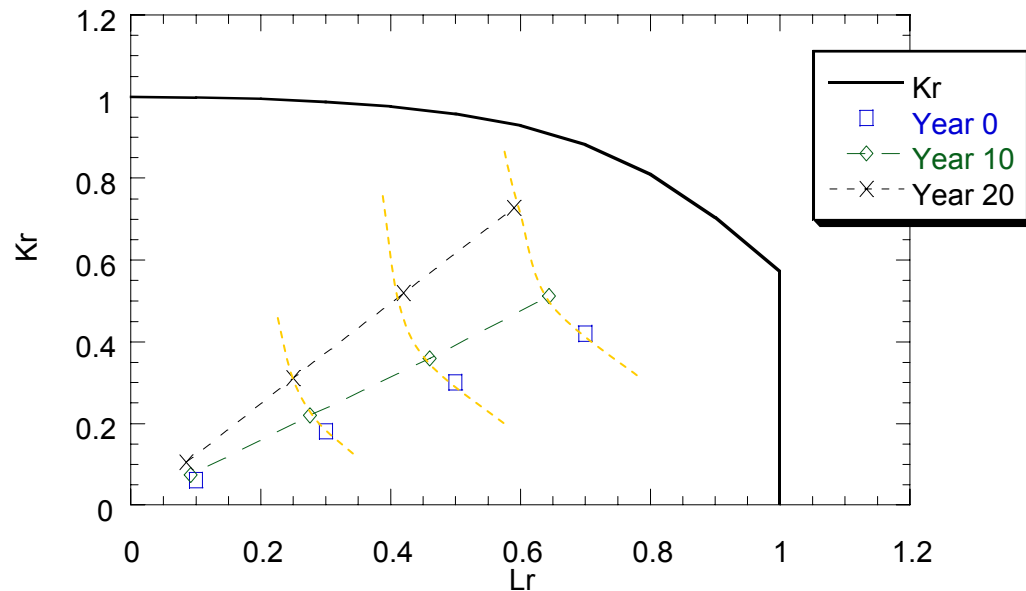
Year 10		
Working conditions	Kr	Lr
50 MPa	0.073	0.092
150 MPa	0.221	0.277
250 MPa	0.368	0.463
350 MPa	0.516	0.648

Year 20		
Working conditions	Kr	Lr
50 MPa	0.104	0.085
150 MPa	0.312	0.25
250 MPa	0.520	0.42
350 MPa	0.728	0.59



## ANALYSIS

It can be seen in the figure that the critical condition is reached in the year 20 when working conditions cause a tensile stress of 350 MPa. In this situation we have the lower safety factor.





## ANALYSIS

In the 20<sup>th</sup> year the safety factor can be obtained from the figure as:

$$S.F = \frac{OB}{OA} = 1.2$$

Therefore, the safety factor is greater than 1.2 during the first twenty years.

