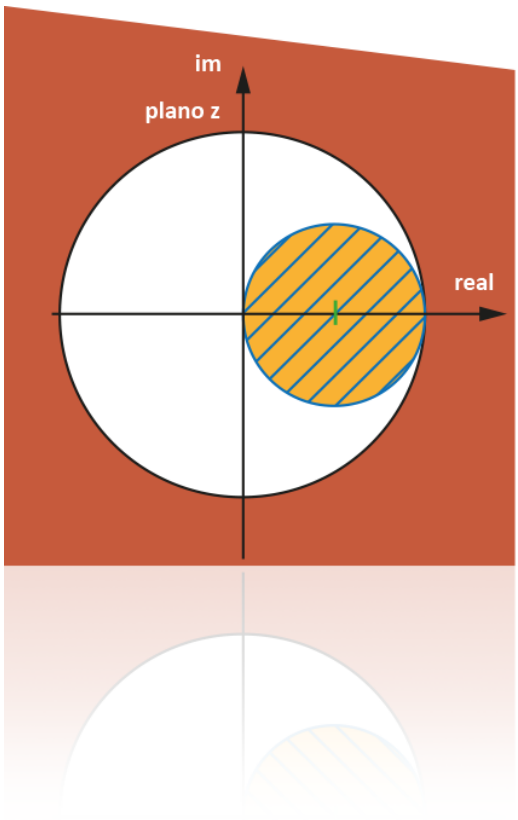
Control por computador

Ejercicios 3



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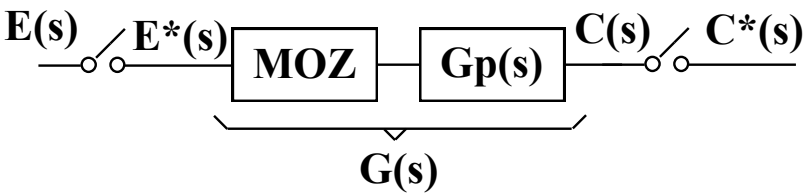
EJERCICIO 3.1

Encontrar la función de transferencia discreta para el siguiente sistema:



$$G_p(s) = \frac{1}{s(s+1)}$$

$$\frac{C(z)}{E(z)} = G(z)$$

$$G(z) = Z\left[\frac{1-e^{-Ts}}{s} G_p(s)\right] = (1-z^{-1})Z\left[\frac{G_p(s)}{s}\right]$$

$$G(z) = (1-z^{-1})Z\left[\frac{1}{s^2(s+1)}\right]$$

Por residuos:

- Residuo 1:

$$R_1 = \left[(s+1)\frac{1}{s^2(s+1)}\frac{1}{1-e^{Ts}z^{-1}}\right]_{s=-1} = \frac{z}{z-e^{-T}}$$

- Residuo 2:

$$R_2 = \left[\frac{d}{ds}\left(s^2\frac{1}{s^2(s+1)}\frac{1}{1-e^{Ts}z^{-1}}\right)\right]_{s=0} = \frac{-z(z-1-T)}{(z-1)^2}$$

$$G(z) = \left(\frac{z-1}{z}\right)\left[\frac{z}{z-e^{-T}} - \frac{z(z-1-T)}{(z-1)^2}\right]$$

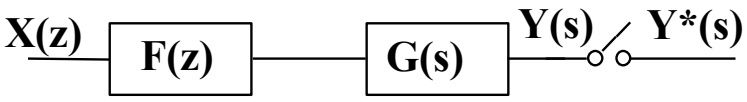
$$G(z) = \frac{z(T-1+e^{-T}) + (1-e^{-T}-Te^{-T})}{(z-1)(z-e^{-T})}$$

Si T=1:

$$G(z) = \frac{0.367z+0.264}{(z-1)(z-0.367)}$$

EJERCICIO 3.2

Encontrar la función de transferencia discreta para el siguiente sistema (T=1):



$$G_p(s) = \frac{5}{(s+2)(s+3)}$$

$$F(z) = \frac{z}{z-0.5}$$

$$\frac{Y(z)}{X(z)} = F(z)G(z)$$

$$G(z) = Z\left[\frac{1-e^{-Ts}}{s}G_p(s)\right] = (1-z^{-1})Z\left[\frac{G_p(s)}{s}\right]$$

$$G(z) = (1-z^{-1})Z\left[\frac{5}{s(s+2)(s+3)}\right]$$

$$G(z) = (1-z^{-1})Z\left[\frac{5}{s(s+2)(s+3)}\right]$$

Por Residuos:

- Residuo 1: $R_1 = \left(s\frac{5}{s(s+2)(s+3)}\frac{1}{1-e^{Ts}z^{-1}}\right)_{S=0} = \frac{5z}{6(z-1)}$

- Residuo 2: $R_2 = \left((s+2)\frac{5}{s(s+2)(s+3)}\frac{1}{1-e^{Ts}z^{-1}}\right)_{S=-2} = \frac{-5z}{2(z-e^{-2T})}$

- Residuo 3: $R_3 = \left((s+3)\frac{5}{s(s+2)(s+3)}\frac{1}{1-e^{Ts}z^{-1}}\right)_{S=-3} = \frac{5z}{3(z-e^{-3T})}$

$$G(z) = \left(\frac{z-1}{z}\right)\left[\frac{5z}{6(z-1)} - \frac{5z}{2(z-e^{-2T})} + \frac{5z}{3(z-e^{-3T})}\right]$$

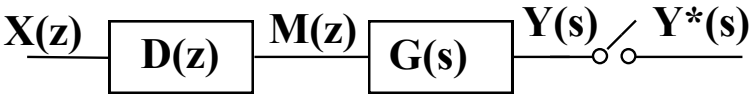
$$G(z) = \frac{0.58(z+0.18)}{(z-0.135)(z-0.05)}$$

$$\frac{Y(z)}{X(z)} = F(z)G(z)$$

$$\boxed{\frac{Y(z)}{X(z)} = \frac{z}{z-0.5}\frac{0.58(z+0.18)}{(z-0.135)(z-0.05)}}$$

EJERCICIO 3.3

Encontrar la función de transferencia discreta para el siguiente sistema



$$G_p(s) = \frac{1}{(s+1)}$$

$$2m_k - m_{k-1} = x_k$$

$$\frac{Y(z)}{X(z)} = D(z)G(z)$$

$$G(z) = Z\left[\frac{1-e^{-Ts}}{s}G_p(s)\right] = (1-z^{-1})Z\left[\frac{G_p(s)}{s}\right]$$

$$G(z) = (1-z^{-1})Z\left[\frac{1}{s(s+1)}\right]$$

Por Residuos:

- Residuo 1: $R_1 = \left(s\frac{1}{s(s+1)}\frac{1}{1-e^{Ts}z^{-1}}\right)_{S=0} = \frac{z}{(z-1)}$

- Residuo 2: $R_2 = \left((s+1)\frac{1}{s(s+1)}\frac{1}{1-e^{Ts}z^{-1}}\right)_{S=-1} = \frac{-z}{(z-e^{-T})}$

$$G(z) = \left(\frac{z-1}{z}\right)\left[\frac{z}{(z-1)} - \frac{z}{(z-e^{-T})}\right] = \frac{1-e^{-T}}{z-e^{-T}}$$

El Filtro digital:

$$2m_k - m_{k-1} = x_k$$

$$\left(2-z^{-1}\right)M(z) = X(z)$$

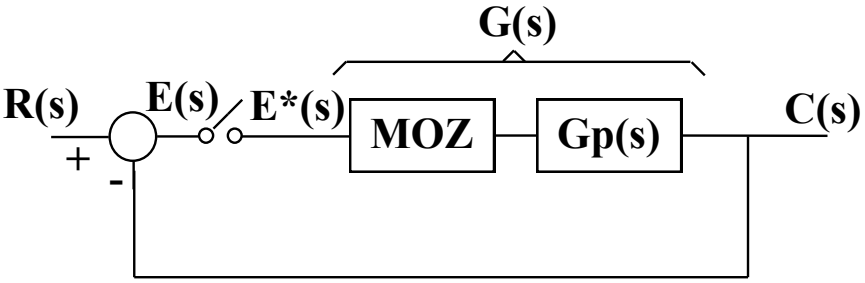
$$D(z)=\frac{M(z)}{X(z)}=\frac{1}{2-z^{-1}}=\frac{z}{2z-1}$$

$$\frac{Y(z)}{X(z)}=D(z)G(z)$$

$$\boxed{\frac{Y(z)}{X(z)}=\frac{z}{2z-1}\frac{1-e^{-T}}{z-e^{-T}}}$$

EJERCICIO 3.4

Encontrar la función de transferencia discreta para el siguiente sistema:



$$G_p(s)=\frac{1}{s+1}$$

$$\frac{C(z)}{R(z)}=\frac{G(z)}{1+G(z)}$$

$$G(z)=Z\left[\frac{1-e^{-Ts}}{s}G_p(s)\right]=(1-z^{-1})Z\left[\frac{G_p(s)}{s}\right]$$

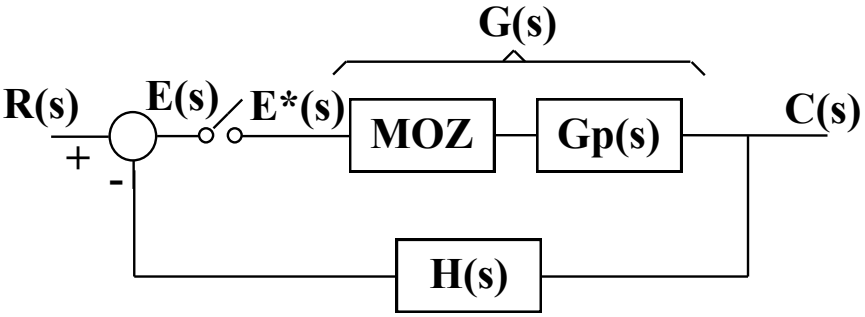
$$G(z)=(1-z^{-1})Z\left[\frac{1}{s(s+1)}\right]$$

$$G(z)=\left(\frac{z-1}{z}\right)\left[\frac{z}{(z-1)}-\frac{z}{(z-e^{-T})}\right]=\frac{1-e^{-T}}{z-e^{-T}}$$

$$\boxed{M(z)=\frac{C(s)}{R(s)}=\frac{\dfrac{1-e^{-T}}{z-e^{-T}}}{1+\dfrac{1-e^{-T}}{z-e^{-T}}}=\frac{1-e^{-T}}{z-2e^{-T}+1}}$$

EJERCICIO 3.5

Encontrar la función de transferencia discreta para el siguiente sistema (T=1) :



$$G_p(s) = \frac{1}{s+2}; \quad H(s) = \frac{5}{s+3}$$

$$M(z) = \frac{C(z)}{R(z)} = \frac{G(z)}{1+GH(z)}$$

$$G(z) = Z\left[\frac{1-e^{-Ts}}{s} G_p(s)\right] = (1-z^{-1})Z\left[\frac{G_p(s)}{s}\right]$$

$$G(z) = (1-z^{-1})Z\left[\frac{1}{s(s+2)}\right]$$

$$G(z) = \left(\frac{z-1}{z}\right)\left[\frac{0.5z}{(z-1)} - \frac{0.5z}{(z-e^{-2T})}\right] = 0.5\frac{1-e^{-2T}}{z-e^{-2T}}$$

$$\boxed{G(z) = \frac{0.43}{z-0.135}}$$

$$GH(z) = Z\left[\frac{1-e^{-Ts}}{s} G_p(s)H(s)\right] = (1-z^{-1})Z\left[\frac{G_p(s)H(s)}{s}\right]$$

$$GH(z) = (1-z^{-1})Z\left[\frac{5}{s(s+2)(s+3)}\right]$$

$$GH(z) = \left(\frac{z-1}{z}\right)\left[\frac{5z}{6(z-1)} - \frac{5z}{2(z-e^{-2T})} + \frac{5z}{3(z-e^{-3T})}\right]$$

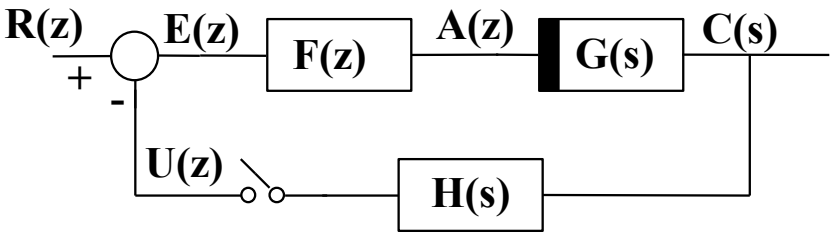
$$\boxed{GH(z) = \frac{0.58(z+0.18)}{(z-0.135)(z-0.05)}}$$

$$M(z) = \frac{G(z)}{1+GH(z)}$$

$$\boxed{M(z)=\frac{0.43(z-0.05)}{z^2-0.395z+0.111}}$$

EJERCICIO 3.6

Encontrar la función de transferencia discreta para el siguiente sistema (T=0.1):



$$F(z)=\frac{z}{z^2+z+1};\quad G_p(s)=\frac{3}{s+1};\quad ;H(s)=\frac{4}{s+2}$$

$$M(z)=\frac{F(z)G(z)}{1+F(z)GH(z)}$$

$$G(z)=Z\left[\frac{1-e^{-Ts}}{s}G_p(s)\right]=(1-z^{-1})Z\left[\frac{G_p(s)}{s}\right]$$

$$G(z)=(1-z^{-1})Z\left[\frac{3}{s(s+1)}\right]$$

$$G(z)=\left(\frac{z-1}{z}\right)\left[\frac{3z}{(z-1)}-\frac{3z}{(z-e^{-T})}\right]=\frac{3\left(1-e^{-T}\right)}{z-e^{-T}}$$

$$\boxed{G(z)=\frac{0.285}{z-0.9}}$$

$$GH(z)=Z\left[\frac{1-e^{-Ts}}{s}G_p(s)H(s)\right]=(1-z^{-1})Z\left[\frac{G_p(s)H(s)}{s}\right]$$

$$GH(z)=(1-z^{-1})Z\left[\frac{12}{s(s+1)(s+2)}\right]$$

$$GH(z)=\left(\frac{z-1}{z}\right)\left[\frac{6z}{(z-1)}-\frac{12z}{(z-e^{-T})}+\frac{6z}{(z-e^{-2T})}\right]$$

$$\boxed{GH(z) = \frac{0.05(z+1)}{z^2 - 1.72z + .74}}$$

$$M(z) = \frac{F(z)G(z)}{1 + F(z)GH(z)}$$

$$M(z) = \frac{\left(\frac{z}{z^2+z+1}\right)\left(\frac{0.285}{z-0.9}\right)}{1+\left(\frac{z}{z^2+z+1}\right)\left(\frac{0.05(z+1)}{z^2-1.72+0.74}\right)}$$

EJERCICIO 3.7

Siendo:

$$G(s) = \frac{1}{(s+1)(s+2)}$$

obtener la función de transferencia del equivalente discreto de la planta utilizando el mantenedor de primer orden.

Periodo de muestreo: $T = 0.1\text{seg}$

Mantenedor de primer orden: $B_1(s) = \left(\frac{1 - e^{-Ts}}{s}\right)^2 \cdot \frac{Ts+1}{T}$

$$G(z) = (1 - z^{-1})^2 \sum \text{Res}\left[\frac{G(s)}{s^2} \cdot \frac{Ts+1}{T} \cdot \frac{1}{1 - e^{sT}z^{-1}}\right] =$$

$$(1 - z^{-1})^2 \sum \text{Res}\left[\frac{1}{s^2(s+2)(s+1)}(s+10)\frac{1}{1 - e^{sT}z^{-1}}\right]$$

$$R_1 = \frac{d}{ds}\left[s^2 \frac{(s+10)}{s^2(s+2)(s+1)} \cdot \frac{1}{1 - e^{sT}z^{-1}}\right]_{s=0} = \frac{d}{ds}\left[\frac{(s+10)}{(s+2)(s+1)} \cdot \frac{1}{1 - e^{sT}z^{-1}}\right]_{s=0} =$$

$$= \frac{(s+2)(s+1) - (s+10)\left[(s+1) + (s+2)\right]}{(s+2)^2(s+1)^2} \cdot \frac{1}{1 - e^{sT}z^{-1}} + \frac{(s+10)}{(s+2)(s+1)} \cdot \frac{Te^{sT}z^{-1}}{(1 - e^{sT}z^{-1})^2}\Bigg|_{s=0} =$$

$$= -7\frac{1}{1 - z^{-1}} + 0.5\frac{z^{-1}}{(1 - z^{-1})^2} = \frac{-7z}{z-1} + \frac{0.5z}{(z-1)^2} = \frac{-7z(z-1) + 0.5z}{(z-1)^2} = \frac{z(-7z + 7.5)}{(z-1)^2}$$

$$R_2 = \left[(s+2)\frac{(s+10)}{s^2(s+2)(s+1)} \cdot \frac{1}{1-e^{sT}z^{-1}} \right]_{s=-2} = \left[\frac{(s+10)}{s^2(s+1)} \cdot \frac{1}{1-e^{sT}z^{-1}} \right]_{s=-2} = \frac{-2z}{z-0.8187}$$

$$R_3 = \left[(s+1)\frac{(s+10)}{s^2(s+2)(s+1)} \cdot \frac{1}{1-e^{sT}z^{-1}} \right]_{s=-1} = \left[\frac{(s+10)}{s^2(s+2)} \cdot \frac{1}{1-e^{sT}z^{-1}} \right]_{s=-1} = \frac{9z}{z-0.9048}$$

$$G(z) = \frac{(z-1)^2}{z^2}\left[\frac{z(-7z+7.5)}{(z-1)^2} - \frac{2z}{z-0.8187} + \frac{9z}{z-0.9048} \right] =$$

$$= \frac{1}{z}\left[-7z + 7.5 - \frac{2(z-1)^2}{z-0.8187} + \frac{9(z-1)^2}{z-0.9048} \right] =$$

$$= \frac{(-7z+7.5)(z-0.8187)(z-0.9048) - 2(z-1)^2(z-0.9048) + 9(z-1)^2(z-0.8187)}{z(z-0.8187)(z-0.9048)}$$

$$G(z) = \frac{0.0058(z^2 + 0.9655z - 0.4655)}{z(z-0.8187)(z-0.9048)}$$

$$\boxed{G(z) = \frac{0.0058(z-0.3530)(z+1.3185)}{z(z-0.8187)(z-0.9048)}}$$